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Blind Separation of Ground Reaction Force Signals

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Abstract

Blind source separation presents an interest for several applications fields. In this study, focus is on the blind separation of biomechanical signals namely ground reaction force signals. We showed recently in [1, 2], that ground reaction force signals are cyclostationary. Thus, the idea of this paper is to make use of the cyclostationary character of ground reaction force signals to separate it and then compare the separation results with those based on stationary statistics. To this end, we introduce frequency domain approaches for either blind source separation or MIMO system identification excited by cyclostationary inputs. These approaches exploit the cyclic spectra matrices of the whitened measurements to identify the mixing system at each frequency bin, up to constant diagonal, frequency dependent permutation and phase ambiguity matrices. Two efficient algorithms to fix the permutation problem and to remove the phase ambiguity based on cyclostationarity are also presented. The approaches exploit the fact that the input signals are cyclostationary with the same cyclic frequency. Simulation examples are presented to illustrate the performances of these approaches. Furthermore, the effectiveness of the proposed methods are tested over real signals and compared with existing methods, as well.

Keywords: Cyclostationarity, Cyclic spectra, Ground reaction force signals, Blind source separation, Permutation problem, Phase ambiguity

1 Introduction

The objective of Blind Source Separation (BSS) is to separate multiple sources, mixed through an unknown mixing system, using the spatial diversity which is given by the system outputs (measurements). Certain methods combine the spatial diversity with other kind of diversities (namely spectral, time-frequency, ...). However, in such cases some *a priori* information about the source signals is necessary. In many real-world situations, man-made signals

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such as those encountered in rotating machines, biomechanics, communications, telemetry, radar, and sonar systems are non-stationary and very often cyclostationary [3, 4, 5, 1].

In a cyclostationary context, BSS has motivated much research which we summarize as follows: Liang et al. in [6, 7], were the first, to our knowledge, to make use of the information offered by the cyclic frequencies in order to separate signals. Furthermore, the approach in [6] does not impose any restriction on the cyclic frequencies of the inputs. Abed-Meraim et al. [3] introduce two Second-Order Statistics (SOS) based criteria to blindly separate sources with or without distinct cyclic frequencies. However, for convolutive mixtures, the authors in [8], separate signals by selecting successively their respective cyclic frequencies and thanks to the spatial diversity they can estimate the channel transfer function using only the output Second-Order Cyclic Statistics (SOCS) (namely cyclic correlation). In [9], Dapena et al. proposed an adaptive algorithm that simultaneously utilizes Higher-Order Statistics and cyclic moments. Ferréol and Chevalier (see [10] and references therein) show that, in a cyclostationary context, the Fourth-Order (FO) empirical estimators generate asymptotically biased estimates of the temporal mean of the quadri-covariance and therefore, they propose an alternative FO statistics estimator to overcome these problems. Ypma et al. [11] showed that, using the bilinear form framework, it is possible to include several types of signal characteristics simultaneously for separation of instantaneously mixed signals. Specifically, they combine SOBI (Second-Order Blind Identification) [12] with SCORE (Spectral Self-Coherence Restoral) [13] to give rise to CycloSOBI which actually exploits both time coherence and cyclostationarity of the source signals. Xiang et al. [14, 15] present a method which jointly utilizes the phase and frequency redundancy of source signals. Jafari et al. in [16, 17] show that involving the cyclostationarity in the Natural Gradient Algorithm improves the performances of the separation in term of convergence speed. Jallon et al. [18] introduce an FO based contrast function whose estimation does not require the knowledge of the cyclic frequencies. Boustany and Antoni [19, 20], propose two methods for the separation of cyclostationary signals mixed in a dynamic environment using SOCS: CYCLOBLISS (CYCLOstationary BLInd Source Separation) and CYMOD (CYclic Multiple Output Deconvolution). Rhioui et al. propose in [21], a method to identify the mixture matrix, even in the under-determined case, of cyclostationary signals with different cyclic frequencies. Pham proposes in [22] a method based on joint block approximate diagonalization of of cyclic interspectral density.

Blind separation and identification of Multiple Input Multiple Output (MIMO) systems which are excited by cyclostationary inputs has been addressed in the frequency domain as well. Starting from a convolutive time domain mixture, one can manage to set the frame size N of the Discrete Fourier Transforms

(DFT) much larger than the channel length L i.e. $N >> L$ in order to approximate linear convolution by circular convolution. In this case, the DFT allows us to have an instantaneous complex mixture at each frequency bin. The major problem associated with this kind of approach however, is to recover the permutation [23] and the phase [24] at each frequency bin. Thus, at each frequency, the unmixing system is identified, then the sources are recovered at the outputs and a time domain channel is formed by the Inverse DFT (IDFT) with respect to some constraints like in [25, 26]. Bradric et al. in [27, 28] addressed the problem of blind identification of Finite Impulse Response Multiple-Input Multiple-Output (FIR MIMO) systems driven by cyclostationary inputs by using the correlation of Short-Time DFT. Antoni et al. make use of the cyclic spectral density matrices for the blind identification [29] and signal separation [30] of sources with the same cyclic frequency. Hanson et al. present in [31] a technique for operational modal analysis of MIMO systems excited by at least one cyclostationary input, with a unique cyclic frequency, in the the cepstrum domain. In addition, a special issue [32], of 14 papers, was dedicated to BSS applied to mechanical signals. several papers of this special issue exploit cyclic statistics, in time or frequency domains, for the separation of vibratory signals.

The originality of our study lies in dealing with biomechanical signals that present second order cyclostationarity properties [1, 2]. The objective of this experimentation is to analyze and to characterize a runner's step from the Ground Reaction Forces (GRF) measured during a running on a treadmill. Mechanical parameters were measured for each step using treadmill dynamometer. The reason behind this application is to separate the contribution of the impact and the propulsive force, that share the same cyclic frequency, for both legs from the GRF signals. These are the motivations behind the proposition of BSS approaches exploiting the fact that GRF signals are cyclostationary with the same cyclic frequency. The methods offered in this paper are frequency domain techniques that exploit the cyclic spectrum matrices of the measurements to identify the mixing channel and separate convolved cyclostationary sources with the same cyclic frequency. Such MIMO problems find applicability also in blind channel estimation in rotating machines -where the sources may share the same cyclic frequency- for vibratory diagnostic. The main contribution regards four respects. First, The freedom to choose either Singular Value Decomposition (SVD) or Eigenvalue Decomposition (EVD) based methods if the sources of interest have no energy on the harmonics of the cyclic frequency or else the diagonalization of a positive definite linear combination (PDLC) or Approximate Joint Diagonalization (AJD) [33] based methods if the sources of interest have energy in the harmonics of the cyclic frequency. Second, an efficient method for addressing the permutation problem inherent with frequency domain based approaches is proposed. This method exploits the cyclic spectrum matrices of the restored signals situated apart from each other at a distance equal to the cyclic frequency. Third, the phase ambiguity which is inherent in frequency domain approaches is addressed in this paper. The method exploits the correlation between the phases of the channels separated from each other with the cyclic frequency. Fourth, the application of the proposed methods as well as existing BSS methods to real signals, namely GRF signals for separation ends.

The organization of this paper which is mainly composed of 2 sections is as follows: Section 2 addresses the BSS problem in frequency domain based on cyclic spectra of GRF signals. The problem statement including the set of required assumptions is presented in Subsection 2.1. Subsection 2.2 presents the algorithms to estimate the magnitude of MIMO system as well as the methods to correct the permutation problem and to cancel the phase ambiguity. Subsection 2.3 discusses the BSS issue. Simulation experiments illustrating the performance of the proposed approaches are described in Subsection 2.4. We report some encouraging results on biomechanical signals in Subsection 2.5. Section 3 of the paper concerns the application of certain existing instantaneous BSS methods, based on stationarity statistics, to GRF signals. Subsection 3.1 explains the motivations behind the use of instantaneous BSS methods. The separation results are discussed in Subsection 3.2. Ultimately conclusions and perspectives are presented in section 4.

2 BSS of GRF signals based on cyclic statistics

2.1 Problem formulation

We consider n-source m-sensor MIMO linear model for the received signals for the convolutive mixing problem:

$$\mathbf{x}(t) = \sum_{k=0}^{L-1} \mathbf{H}(k)\ \mathbf{a}(t-k) + \mathbf{b}(t) \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), ..., x_m(t)]^T$ is the vector of measurements, $\mathbf{a}(t) = [a_1(t), ..., a_n(t)]^T$ is the source vector which the components are real-valued, CycloStationary Independent Temporally (CSIT) i.e. a cyclostationary signal where the samples are independent, mutually statistically uncorrelated, centered (in the ensemble-average sense) and with the same cyclic frequency β. $\mathbf{H}(t)$ is the $m \times n$ impulse response matrix whose elements are $\{h_{ji}(t)\}$ and $\mathbf{b}(t) = [b_1(t), ..., b_m(t)]^T$ is the additive noise vector which is assumed to be stationary, temporally and spatially white, zero mean and independent of the source signals. The superscript T denotes the transpose and L is the length of the real-valued channels. By taking the DFT of (1) over N lines, we obtain:

$$\mathbf{X}(f_k) = \mathbf{H}(f_k)\ \mathbf{A}(f_k) + \mathbf{B}(f_k) \tag{2}$$

where $\mathbf{X}(f_k)$, $\mathbf{H}(f_k)$, $\mathbf{A}(f_k)$ and $\mathbf{B}(f_k)$ are the DFT of $\mathbf{x}(t)$, $\mathbf{H}(t)$, $\mathbf{a}(t)$ and $\mathbf{b}(t)$, respectively. From (2) it is obvious that at each frequency bin f_k, the estimation of $\mathbf{A}(f_k)$ can be seen as an instantaneous complex BSS problem.

Let introduce the cyclic spectra of the source vector [34]:

$$\begin{aligned}\mathbf{S}_a^{q\beta}(f_k) &= \mathbb{E}\{\mathbf{A}(f_k)\mathbf{A}^H(f_k - q\beta)\} \\ &= Diag\left([\sigma_1^2(q\beta), \ldots, \sigma_i^2(q\beta), \ldots, \sigma_n^2(q\beta)]\right)\end{aligned} \tag{3}$$

where $q \in \mathbb{Z}^*$, $i = 1, \ldots, n$, the superscript H denotes the complex conjugate transpose of a matrix and $\sigma_i^2(q\beta)$ stands for the cyclic spectrum of the signal $a_i(t)$, at the qth multiple of β, which is frequency independent since $a_i(t)$ is CSIT. Furthermore, we suppose that the source signals have different cyclic spectra at a given cyclic frequency $q\beta$. Equ. (3) means that the cyclic spectrum matrices of the inputs at different harmonics are diagonals, with different diagonal elements, under the uncorrelation hypothesis.

2.2 MIMO Identification

2.2.1 Proposed approaches for system estimation

The algorithms presented hereafter aim to separate the sources, at each frequency, from the blind identification of the steering vectors. This identification requires the prewhitening of the data, aiming to orthogonalize the source steering vectors so as to search for the latter through a unitary matrix. To do so, we first start by the SVD of $\mathbf{H}(f_k)$, this leads to:

$$\mathbf{H}(f_k) = \mathbf{U}(f_k)\Sigma(f_k)\mathbf{V}(f_k) \tag{4}$$

where $\mathbf{U}(f_k)$ and $\mathbf{V}(f_k)$ are $m \times n$ and $n \times n$ unitary matrices respectively, and $\Sigma(f_k)$ is $n \times n$ diagonal matrix. Let us introduce the cyclic spectrum matrix of the measurements [34]:

$$\mathbf{S}_x^{q\beta}(f_k) = \mathbf{H}(f_k)\mathbf{S}_a^{q\beta}(f_k)\mathbf{H}^H(f_k - q\beta) + \mathbf{S}_b^{q\beta=0}(f_k) \tag{5}$$

where $\mathbf{S}_x^{q\beta}(f_k)$, $\mathbf{S}_a^{q\beta}(f_k)$ and $\mathbf{S}_b^{q\beta=0}(f_k)$ are the cyclic spectrum matrices of the measurements, sources and noise respectively. The cyclic spectrum of the noise is null for all nonzero cyclic frequencies since the noise is stationary, as mentioned above.

The Eigenvalue Decomposition (EVD) of the cyclic spectrum matrix of the measurements evaluated when $\beta = 0$ allows us to compute the whitening matrix $\mathbf{W}(f_k)$ as explained below:

$$\mathbf{W}(f_k) = \Sigma^{-1}(f_k)\mathbf{U}^H(f_k) \tag{6}$$

To identify $\mathbf{V}(f_k)$, we shall use the cyclic spectrum matrices of the whitened processes, $\mathbf{Z}(f_k)$, at each $q\beta$.

$$\mathbf{S}_z^{q\beta}(f_k) = \mathbf{V}(f_k)\mathbf{S}_a^{q\beta}(f_k)\mathbf{V}^H(f_k - q\beta) \tag{7}$$

Actually, one can proceed in one of the following manners to identify $\mathbf{V}(f_k)$.

SVD based approach It is apparent from Equ. (7) that the SVD allows the identification of $\mathbf{V}(f_k)$ for a given $q\beta$.

EVD based approach Let us define the matrix $\tilde{\mathbf{S}}_z^{q\beta}(f_k)$ of (7) in order to have a symmetric expression in the right-hand side of Equ. (7), thus giving:

$$\begin{aligned} \tilde{\mathbf{S}}_z^{q\beta}(f_k) &= \mathbf{S}_z^{q\beta}(f_k)\mathbf{S}_z^{q\beta}(f_k)^H \\ &= \mathbf{V}(f_k)\tilde{\mathbf{S}}_a^{q\beta}(f_k)\mathbf{V}^H(f_k) \end{aligned} \tag{8}$$

The EVD of (8) makes possible the estimation of $\mathbf{V}(f_k)$ for a given $q\beta$.

Diagonalization of a PDLC As long as $\tilde{\mathbf{S}}_z^{q\beta}(f_k)$ is different for a set of cyclic frequencies $\{q\beta,\ q \neq 0\}$, $\mathbf{V}(f_k)$ can be estimated as the matrix that diagonalizes the PDLC:

$$\overline{\mathbf{S}}_z^{\beta}(f_k) = \sum_{q=-Q}^{Q} \lambda_{q\beta_i}\, \tilde{\mathbf{S}}_z^{q\beta}(f_k),\ q \neq 0 \tag{9}$$

where $2Q$ is the number of matrices and the coefficients $\lambda_{q\beta_i}$'s are chosen so as the matrix $\overline{\mathbf{S}}_z^{\beta}(f_k)$ be positive definite.

AJD based approach Instead of diagonalizing one or a PDLC matrix of $\tilde{\mathbf{S}}_z^{q\beta}(f_k)$. One can simultaneously diagonalize $\tilde{\mathbf{S}}_z^{q\beta}(f_k)$ for a set of $\{q\beta,\ q \neq 0\}$. Thus, the application of the well-known AJD to (8) at different values of q will be suitable, and hence enables the estimation of $\mathbf{V}(f_k)$.

Once $\mathbf{V}(f_k)$ is identified, by one of the four proposed approaches, one can estimate the mixing MIMO system $\mathbf{H}(f_k)$ based on the Equ. (4) as shown below:

$$\hat{\mathbf{H}}(f_k) = \mathbf{H}(f_k)\Delta\mathbf{P}(f_k)\mathbf{D}(f_k) \tag{10}$$

where $\hat{\mathbf{H}}(f_k)$ is an estimation of $\mathbf{H}(f_k)$ with constant diagonal matrix, Δ, which introduces a scaling ambiguity, frequency dependent permutation $\mathbf{P}(f_k)$, and phase ambiguity $\mathbf{D}(f_k)$ (the *ith* diagonal element of $\mathbf{D}(f_k)$ is of the form $e^{\mathrm{j}\phi_D^i(f_k)}$).

Certainly, the estimation offered by (10) can no longer provide a good estimation of the actual system $\mathbf{H}(f_k)$ neither in the frequency domain nor in the time one by involving the Inverse DFT (IDFT). Hereafter, we propose two techniques to cancel the permutation problem and phase ambiguity, so that the mixing MIMO system will be estimated up to uniform permutation, general constant diagonal matrices and a linear phase, thus giving:

$$\hat{\mathbf{H}}_{opt}(f_k) = \mathbf{H}(f_k)\Delta\mathbf{P}e^{\mathrm{j}f_k\mathbf{M}} \tag{11}$$

where $\mathbf{M}$ is a constant diagonal matrix that introduces circular shifts to the sources.

2.2.2 Permutation correction

It is worth emphasizing that in the literature of BSS in the frequency domain, the problem of adjusting permutations received a lot of interest. In [35], it has been proposed a reorganization criterion to remove the permutation indeterminacy. A constraint on the size of the separating filter is made in [25] to solve the permutation problem. The paper [26] presents an algorithm that exploits the cross power spectral density matrices of nonstationary sources between adjacent frequencies. We have proposed adjusting permutation techniques based on either cyclic cepstra, for the case where the sources have different cyclic frequencies, [36] or cyclic spectra matrices [37].

An interesting property of cyclostationary signals is their spectral redundancy. In our case, the cyclic spectra of sources are frequency independent (since the sources are *CSIT*), different for each source and will overlap because they share the same cyclic frequency, as well. This is the property that we seek to exploit in our effort to derive an algorithm for the correction of permutations across all the frequency bins. Let us first consider the estimating sources:

$$\hat{\mathbf{A}}(f_k) = \hat{\mathbf{H}}^{\#}(f_k)\mathbf{X}(f_k) = \mathbf{D}^{*}(f_k)\mathbf{P}^{T}(f_k)\Delta^{-1}\,\mathbf{A}(f_k) \tag{12}$$

where the superscripts $\#$ and $*$ denote the pseudo-inverse and the complex conjugate of a matrix, respectively. As the signals at the output of $\hat{\mathbf{H}}^{\#}(f_k)$ have the same cyclic frequency as well as the actual sources, then (3) holds that:

$$|\mathbf{S}_{\hat{a}}^{q\beta}(f_k)| = \mathbf{P}^{T}(f_k)|\Delta^{-1}\mathbf{S}_{a}^{q\beta}(f_k)\Delta^{-1}|\mathbf{P}(f_k - q\beta) \tag{13}$$

where the absolute value $|.|$ is applied to each element of the matrix $\mathbf{S}_{\hat{a}}^{q\beta_i}(f_k)$. We notice that the phase effect has disappeared in (13) because of the absolute value. It is apparent therefore from (13) that performing the SVD of $|\mathbf{S}_{\hat{a}}^{q\beta}(f_k)|$ leads to the identification of both matrices $\mathbf{P}(f_k)$ of and $\mathbf{P}(f_k - q\beta)$. This is because $\mathbf{P}(f_k)$ and $\mathbf{P}(f_k - q\beta_i)$ are orthogonal matrices. The SVD of $|\mathbf{S}_{\hat{a}}^{q\beta}(f_k)|$ is given by:

$$|\mathbf{S}_{\hat{a}}^{q\beta}(f_k)| = \Pi_1(f_k)\,\Gamma^{q\beta}(f_k)\,\Pi_2^T(f_k) \tag{14}$$

where $\Pi_1(f_k) = \mathbf{P}^T(f_k)$ and $\Pi_2(f_k) = \mathbf{P}^T(f_k - q\beta)$. Actually, we have $\Gamma^{q\beta}(f_k) = \mathbf{P}^T|\Delta^{-1}\mathbf{S}_a^{q\beta}(f_k)\Delta^{-1}|\mathbf{P}$, there subsists only a single unknown permutation concerning the initial labelling of the sources because the SVD will sort the singular values of $\Gamma^{q\beta}(f_k)$ in ascending or descending order over all frequency bins. Hence, no connection with the true order of the singular values $\mathbf{S}_a^{q\beta}(f_k)$ is made. Therefore, the Magnitude of the MIMO system is estimated up to a uniform permutation matrix $\mathbf{P}$ and constant diagonal matrix Δ. As well-known in blind source separation, this indeterminacy is inevitable.

At the output of the algorithm, the estimated sources will have a uniform permutation, $\mathbf{P}$, over all frequency bins. Moreover, the relationship (10) will be simplified to:

$$\hat{\mathbf{H}}(f_k) = \mathbf{H}(f_k)\Delta\mathbf{P}\mathbf{D}(f_k) \tag{15}$$

2.2.3 Phase retrieval

The phase retrieval of Single-Input Single-Output (SISO) systems, excited by cyclostationary inputs, has been studied in several papers [38, 39, 40, 41].

Let us introduce the overall mixing-unmixing system after correcting the permutation problem:

$$\hat{\mathbf{H}}^{\#}(f_k)\,\mathbf{H}(f_k) = \mathbf{D}^*(f_k)\mathbf{P}^T\Delta^{-1} \tag{16}$$

The cyclic spectrum matrix of the signals, at the output of the unmixing system $\hat{\mathbf{H}}^{\#}(f_k)$ of (15), $\hat{\mathbf{a}}(t)$ for a given $q\beta$ is as follows:

$$\mathbf{S}_{\hat{a}}^{q\beta}(f_k) = \mathbf{D}^*(f_k)\mathbf{P}^T\Delta^{-1}\mathbf{S}_a^{q\beta}(f_k)\Delta^{-1}\mathbf{P}\mathbf{D}(f_k - q\beta) \tag{17}$$

where $\mathbf{S}_{\hat{a}}^{q\beta}(f_k)$ is diagonal. The following relation holds for the phase of the quantities involved in (17):

$$\Phi_D(f_k - q\beta) - \Phi_D(f_k) = \Psi(f_k) - \Theta(q\beta) \tag{18}$$

where $\Theta(q\beta) = \arg\{\mathbf{P}^T\Delta^{-1}\mathbf{S}_a^{q\beta}(f_k)\Delta^{-1}\mathbf{P}\}$ and $\Psi(f_k) = \arg\{\mathbf{S}_{\hat{a}}^{q\beta}(f_k)\}$. Furthermore, $\Theta(q\beta)$ being frequency independent since the sources are CSIT and can be estimated by summing up (18) over all discrete frequencies which yield a zero right-hand side, thus we get:

$$\Theta(q\beta) = \frac{1}{N}\sum_{k=1}^{N} \Psi(f_k) \tag{19}$$

In this way, $\Theta(q\beta)$ can actually be computed from $\Psi(f_k)$. Let take the IDFT of Equ. (18), thus giving:

$$\Phi_D(t) = \frac{\Theta(q\beta)\delta(t) - \Psi(t)}{(1 - \exp(\mathrm{j}2\pi q\beta t))}, \qquad t \neq \frac{l}{q\beta} \tag{20}$$

Actually, $\Phi_D(t)$ cannot be completely identified whenever $t = \frac{l}{q\beta}$, a way to fix this is to interpolate the missing information assuming the continuity of $\Phi_D(t)$. The identification of $\Phi_D(f_k)$ is possible by taking the DFT of (20). This is the key relationship that can lead to the estimation of $\Phi_D(f_k)$. Finally, the following relationship yields to the elimination of the phase ambiguity from (10):

$$\hat{\mathbf{H}}_{opt}(f_k) = \hat{\mathbf{H}}(f_k)e^{-\mathrm{j}\Phi_D(f_k)} \tag{21}$$

Hence, the MIMO system is identified as in (11).

2.3 BSS issue

At each frequency bin an estimate of the vector of source signals is computed by applying to the received signal $\mathbf{X}(f_k)$ the pseudo-inverse of $\hat{\mathbf{H}}_{opt}(f_k)$, thus we get:

$$\begin{aligned} \hat{\mathbf{A}}(f_k) &= \hat{\mathbf{H}}^{\#}_{opt}(f_k)\mathbf{X}(f_k) \\ &= \left(e^{\mathrm{j}f_k\mathbf{M}}\right)^{-1}\mathbf{P}^T\Delta^{-1}\mathbf{A}(f_k) + \\ &\quad \hat{\mathbf{H}}^{\#}_{opt}(f_k)\mathbf{B}(f_k) \end{aligned} \tag{22}$$

In general, this procedure is not optimal for recovering the source signals because of the vector of noises. The Least Mean Square (LMS) algorithm which allows to get a maximal SNR at the output of $\mathbf{G}(f_k)$ can be used as a multidimensional separator. The optimum solution for such separator is given by:

$$\mathbf{G}(f_k) = \mathbf{S}^0_x(f_k)^{-1}\mathbf{S}^0_{xa}(f_k) \tag{23}$$

where

$$\mathbf{S}^0_{xa}(f_k) = \mathbb{E}\{\mathbf{X}(f_k)\mathbf{A}^H(f_k)\} = \mathbf{H}(f_k) \tag{24}$$

Using (11) and (24) in (23), we get an estimate of $\mathbf{G}(f_k)$:

$$\hat{\mathbf{G}}(f_k) = \mathbf{S}_x^0(f_k)^{-1}\hat{\mathbf{H}}_{opt}(f_k) = \mathbf{G}(f_k)\Delta\mathbf{P}e^{\mathrm{j}f_k\mathbf{M}} \tag{25}$$

Hence, the estimate of the source vector $\mathbf{A}(f_k)$ is given by:

$$\hat{\mathbf{A}}(f_k) = \hat{\mathbf{G}}^H(f_k)\mathbf{X}(f_k) \approx \left(e^{\mathrm{j}f_k\mathbf{M}}\right)^{-1}\mathbf{P}^T\Delta^{-1}\,\mathbf{A}(f_k) \tag{26}$$

As a result, in the frequency domain, the sources are recovered up to a scale factor and a permutation.

2.4 Simulation results

The proposed methods are tested using synthetic signals in order to evaluate its effectiveness. The simulation is made with $n = 2$ cyclostationary signals, of 10^5 samples each, which are actually amplitude modulated white gaussian noises: $a_1(t) = m_1(t)(0,8cos(\frac{2\pi}{T}t)+0,6cos(\frac{4\pi}{T}t))$ and $a_2(t) = m_2(t)(square(2\pi\frac{t}{T},9)+1)$ where the function *square*, which is actually a function of Matlab, generates a square wave with period 2π. The cyclic period T is equal to 50 samples. The normalized Second-Order cyclic frequency is: $\beta = \frac{1}{T}$. The signals are then filtered by a, 3×2, ARMA-MIMO system. The transfer function of the $(ij)th$ element of the MIMO system is given as: $H_{ij}(z) = \frac{1+b'_{ij}\,z^{-1}+b''_{ij}\,z^{-2}}{1+a'_{ij}\,z^{-1}+a''_{ij}\,z^{-2}}$ where $b'=\begin{pmatrix} -0.2 & -0.443 \\ 0.7 & 0.57 \\ 0.5 & 0.9 \end{pmatrix}$, $b''=\begin{pmatrix} 0.8 & 0.6 \\ 0.6 & 0.1 \\ 0.3 & 0.2 \end{pmatrix}$, $a'=\begin{pmatrix} 0.64 & 0.8 \\ -0.92 & -1.64 \\ -1.2 & -1.7 \end{pmatrix}$ and $a''=\begin{pmatrix} 0.1 & 0.5 \\ 0.21 & 0.74 \\ 0.740818 & 0.8 \end{pmatrix}$. The length of the hanning window and the DFT are set to 128 and 256, respectively. The number of matrices for AJD and PDLC approaches is $2Q = 8$.

Comparison between the proposed methods: We provide here a comparison between the proposed approaches. For each value of the SNR, $M_c = 20$ Monte Carlo runs where implemented.

As a performance index to measure the identification performance and to make comparison as well, we here used the Normalized Mean-Square Error (NMSE). For M_c Monte Carlo runs, the NMSE_{ji} for the subchannel $h_{ji}(r)$ is given by the formula:

$$\mathrm{NMSE}_{ji} = \frac{\sum_{l=1}^{M_c}\left[\sum_{r=1}^{L}(\hat{h}_{ji}^{(l)}(r) - h_{ji}(r))^2\right]}{M_c\sum_{r=1}^{L}(h_{ji}(r))^2} \tag{27}$$

The overall NMSE is obtained by averaging over all subchannels:

$$\mathrm{ONMSE}_{ji} = \frac{1}{mn}\sum_{j=1}^{m}\sum_{i=1}^{n}\mathrm{NMSE}_{ji} \tag{28}$$

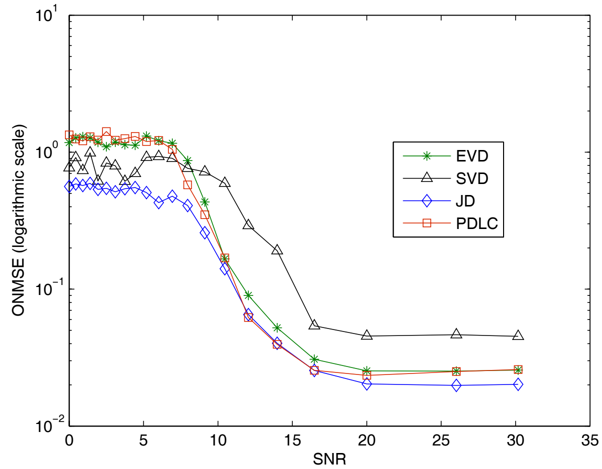

Figure 1: Performance comparison of the proposed methods: ONMSE versus SNR.

Figure (1) shows the variation of each method' ONMSE with the SNR. As can be seen from this figure, by decreasing the SNR, the ONMSE, for the proposed methods, improves (decreases). However, it shows the proposed approaches, specially the AJD based approach, perform well and provide better ONMSE even for low SNR. Moreover, EVD and PDLC based approaches have almost the same performances. Actually, the SVD based approach provides high values of ONMSE specially for high SNR in comparison with the other approaches.

2.5 Application to biomechanical signals

This subsection concerns a real application of the proposed approaches for characterizing mechanical step variability during treadmill running. The human locomotion, in particular the walk and the running, are defined by sequences of cyclic gestures and repeated. The variability of these sequences is able to reveal the faculties or the motor failings.

The reason behind this experimentation is to analyze and to characterize a runner's step from the GRF measured during a running on a treadmill. Mechanical parameters were measured for each step using treadmill dynamometer (for details, see [42]). Vertical GRF signal (see Figure 2) and belt velocity were sampled at a rate of $1000Hz$.

The objective of this application is to separate the contribution of the impact force (passive pick) and the propulsive force (active pick) for both legs from the GRF signals (Figure 2). Actually, the active and passive picks happen

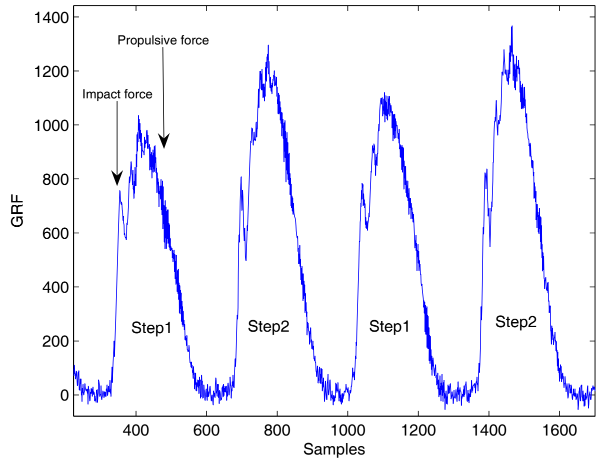

Figure 2: Vertical GRF in time domain

independently i.e. there is no correlation between these actions for both legs. This leads to a total of 4 independent sources since the active and passive picks are different and proportional to each leg. We have 12 sensors at our disposal which are positioned on the 4 corner of the treadmill dynamometer i.e. 3 sensors by corner.The projection of the cyclic spectrum of a vertical GRF signal on the cyclic frequency axe (see Figure 3) shows that the vertical GRF signals are second-order cyclostationary with 689.6897 samples as cyclic period. The corresponding normalized cyclic frequency shared by the sources is 0.0014. Thus, the 4 sources happen at the same cyclic frequency which is equals to the step frequency. However, this makes difficult their separation from only one sensor. This is the reason why we use the BSS tools for the separation.

Indeed, The separation results enable doctors to extract parameters for a biomechanical study of running (kinematics analysis, machine works, mechanical characteristics of lower limbs, impacts analysis, ...).

We apply the AJD based approach for the separation. The parameter $2Q$ is set to 4 i.e. the number of cyclic spectrum matrices of the whitened data is 8. We choose the AJD based approach rather than the other proposed approaches because it is the most efficient. Figure 4 reports the separated signals. As we can see, the impact force of the step1 is alternate with the one of the step2. However, the propulsive forces of both steps are not completely separated since they are not alternate.

Discussion The separation results (Fig. 4) might be justified by the hypothesis made on the source signals (whites or colored) and also by the separation

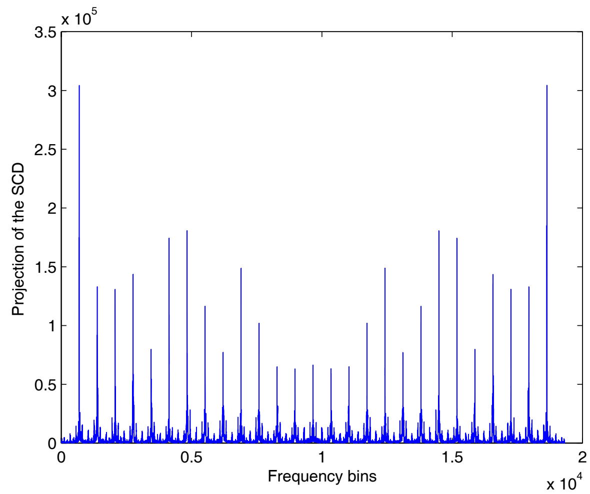

Figure 3: The cyclic spectrum of the GRF signal

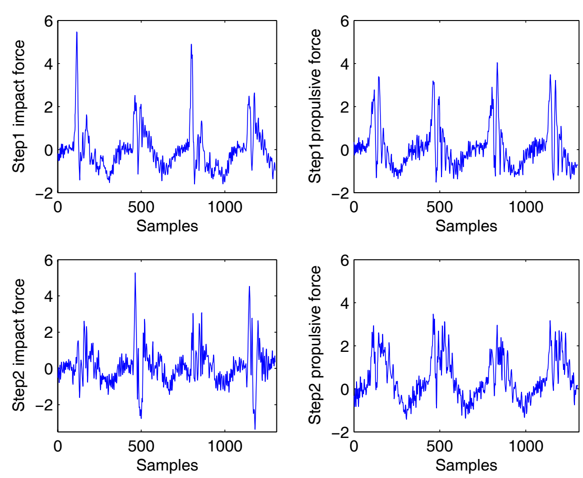

Figure 4: Separated signals

criterion choice (SOS or HOS). This leads us to test some existing BSS methods that make use of different hypothesis on the mixture system as well as on the the source signals (stationarity or cyclostationarity, time coherence, ...).

3 BSS of GRF signals based on stationary statistics

This section aims to apply some existing BSS methods based either on SOS or HOS.

3.1 Separation with existing BSS methods

In low frequencies, we notice an almost constant gain of the frequency response, which introduces little convolution between the input force and the impulse response. The instantaneous model, which is a particular case of the convolutive BSS model, is apparently the most appropriate to model the force signal mixtures. This leads us to consider instantaneous BSS methods, SOBI [12], cycloSOBI [11] and JADE [33], for GRF signals' separation.

3.2 Separation results

After applying the SOBI, cycloSOBI and JADE to the set of GRF signals. We observed that only JADE method, which based on HOS, offers good results in comparison with the methods based on SOS, SOBI and cycloSOBI, which cannot correctly separate the contribution of each leg. Figure 5 displays the separation results using JADE algorithm. It is noteworthy that the contributions of each step have been correctly separated from the overall mixture of the GRF signals. In comparison with the separation result using the proposed AJD based approach, the advantage with JADE separation is certainly the good restoration of the propulsive forces for each leg.

Discussion To justify the reason why JADE algorithm performs well for GRF signals, we calculate the kurtosis and the PSD of the JADE' estimates. The kurtosis of the 4 sources estimates are greater than 0 and equal to : 6.9171, 7.4008, 1.2567, and 0.5122. This shows that the sources are not gaussians. Figures 6 shows the PSD of the 4 estimates, we notice that the spectra are almost superposed over the whole frequency axe. Hence, this justify why SOBI and cycloSOBI algorithms perform poorly for GRF signals.

As a matter of fact, the separation result, of active and passive picks of each leg, using the AJD based approach might be justified by the non impulse like nature the propulsive sources i.e. they present a time coherence, and

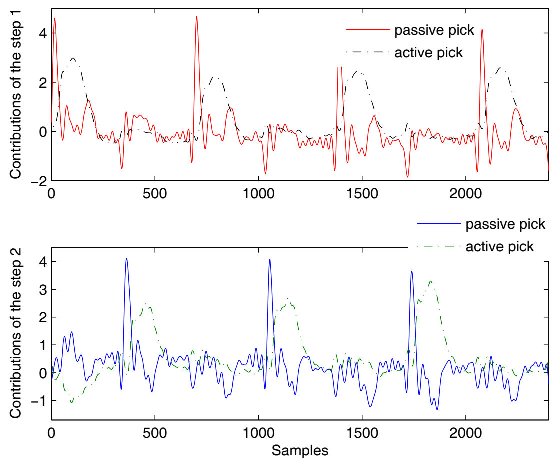

Figure 5: Separation results using JADE.

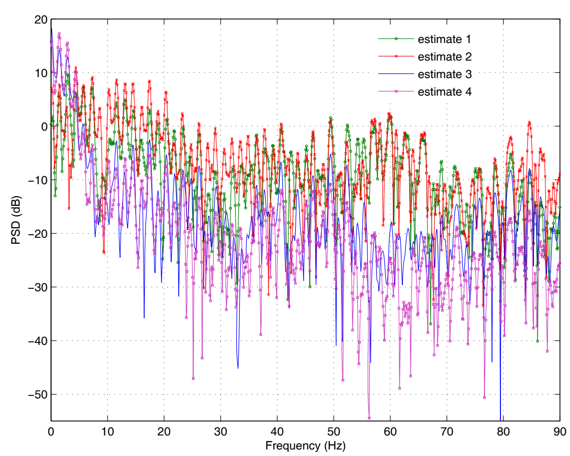

Figure 6: Zoom on the PSD of JADE estimates.

hence, their cyclic spectra will be frequency dependent. This leads to a serious problem for the correction of permutation problem and phase ambiguity since they suppose frequency independent cyclic spectra of the source signals.

4 Conclusion

The spectral redundancy allowed us to apply either SVD, EVD, PDLC or AJD algorithms to the cyclic spectrum matrices of the whitened processes, for each cyclic frequency and for every frequency bin as well, in order to identify the MIMO system up to certain ambiguities (namely constant diagonal, frequency dependent permutation and phase matrices). Two robust algorithms to overcome the frequency dependent permutation and phase ambiguities, based on cyclostationarity, were presented. The performance of the algorithms was demonstrated, it is apparent therefore that the proposed methods perform well specially the AJD based approach. Furthermore, the AJD based method was tested through real signals. The method is able to completely separate the impact forces but not the propulsive forces of GRF signals that share the same cyclic frequency.

In the aim of better understanding the GRF structure, we tested some existing instantaneous BSS methods that use different assumption on the source signals. Jade algorithm, which makes use of HOS, performs well and separate the contribution of each leg. These results open some perspectives, to explore in the future, that encourage the use of HOS for the separation and characterization of GRF signals. Indeed, it will be better to jointly utilize JADE algorithm or other contrast functions with the cyclostationarity property of GRF signals to more increase the separation quality. It will be also interesting to explore the contribution of cyclic HOS (namely the cyclic bispectrum) to the BSS of GRF signals.
Finally, we hope that our study contributes to better characterize and understand the mechanical phenomena behind the GRF signals' behavior.

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