Applied Mathematical Sciences, Vol. 6, 2012, no. 52, 2559 – 2566

Coupled Synchronization of the Newton-Leipnik Chaotic System

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Abstract. This paper investigates the synchronization of two Newton-Leipnik Chaotic Systems. A new sufficient condition of global asymptotic synchronization is attained from the theory of stability of time-varying systems. In addition , compared with the previously proposed method , the sufficient condition for the synchronization of two linearly coupled unified chaotic systems is simpler and less conservative , and the range of coupled coefficients is wider. Numerical simulation shows the effectiveness and feasibility of this method.

Keywords: Newton-Leipnik Chaotic System; chaos control; coupled synchronization; numerical simulation; linear time-varying system

1. Introduction

Chaos has already had an enduring effect on science, and issues of chaos control and synchronization have been extensively investigated since the early 1990s [1-3]. Meanwhile, various effective schemes, including the OGY method[4], adaptive control, feedback control, backstepping design [2] and active control [3], nonlinear control [5],etc., have been widely applied in this field. Since Pecoro Carroll and put forward the chaotic synchronization method in 1990 [6].At home and abroad, a lot of different chaotic synchronization methods were put forward [3,7], such as completely synchronization, phase synchronization, generalized synchronization, synchronization, projection synchronization and so on.

Recently, because of the application of chaos in the Secure communication, the coupling synchronization in two same chaotic systems have caused considerable concern. Agiza [8] applied the activities controller implement into synchronization of two Rossler chaotic systems and two Chen chaotic systems. Li [9] used an active controller to achieve nonlinear coupled synchronization of Chua's circuit. They achieve synchronizat- ion of unified chaotic system and of Lorenz chaotic system with the linear coupling approach in papers[10-12]. But the basis for the coupling coefficient selection is different. According to Routh-Hurwitz criterion and the derivation of rigorous mathematical theory, Lü obtain a sufficient condition for synchronization of two linear coupling chaotic systems. Li proved a negative definite symmetric matrix by the mathematical theory,and there were negative eigenvalues in the matrix. By the asymptotically stable Error system, To derive a sufficient condition of coupled chaotic systems' synchronization was gained. With the spectral lyapunov stability theory and linear matrix inequalities and Matlab Control System Toolbox, Park Calculated the solution of the optimal linear matrix inequalities and obtained the stability criterion of unified chaotic system's gradual synchronization.

This paper considers two linear coupling Newton-Leipnik Chaotic Systems and the theory of linear time-varying continuous system stability. Without too cumbersome derivation of the mathematical theory, we can get to make two different initial value of chaotic systems achieve synchronization by a sufficient condition. There aren't many constains in criterion expressions by comparing, and a broader range of coupling coefficient. This method is applied to the Newton-Leipnik Chaotic System, and satisfactory results were achieved. Thus the effectiveness and feasibility of the method wereconfirmed.

2. System description and its basic properties

The Newton-Leipnik chaotic system, derived in [1], is described by the following differential equation system

$$\begin{cases} \dot{x} = -ax + y + 10yz \\ \dot{y} = -x - 0.4y + 5xz \\ \dot{z} = bz - 5xy \end{cases} \tag{1}$$

where x , y and z are the state variables, parameters a and b are the two positive real constants.

The divergence of this flow is given by

$$\nabla \cdot V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a - 0.4 + b \tag{2}$$

Obviously, the system is dissipative if $-a - 0.4 + b < 0$. When $a = 0.4$ and $b = 0.175$, the system converges to a set ofmeasure zero exponentially.In fact, for the initial conditions $(5,1,20)$, one can obtain a double strange attractor,the upper and the lower attractor, respectively (see Fig. 1). Therefore, this system has a attractor with parameters $a = 0.4$ and $b = 0.175$.

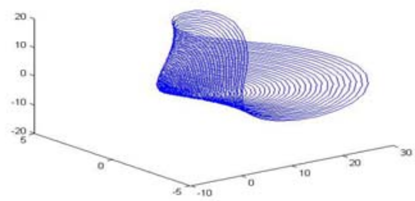

Fig. 1. a attractor of the chaotic system with $a = 0.4$ and $b = 0.175$.

3. Linear coupled synchronization of Newton-Leipnik chaotic system

3.1 Linear coupled Newton-Leipnik chaotic system

Consider the linear coupling of two identical Newton-Leipnik chaotic system:

$$\begin{cases} \dot{x}_1 = -ax_1 + x_2 + 10x_2x_3 + d_1(y_1 - x_1) \\ \dot{x}_2 = -x_1 - 0.4x_2 + 5x_1x_3 + d_2(y_2 - x_2) \\ \dot{x}_3 = bx_3 - 5x_1x_2 + d_3(y_3 - x_3) \\ \dot{y}_1 = -ay_1 + y_2 + 10y_2y_3 + d_1(x_1 - y_1) \\ \dot{y}_2 = -y_1 - 0.4y_2 + 5y_1y_3 + d_2(x_2 - y_2) \\ \dot{y}_3 = by_3 - 5y_1y_2 + d_3(x_3 - y_3) \end{cases}, \tag{3}$$

where $x_i, y_i (i = 1,2,3)$ are the state variables,and $d_i (i = 1,2,3)$ are coupling coefficients to ensure that need to synchronize two chaotic systems. Here do not like the literature [10-12] in for $d_i > 0$ limit.

Defined as shown in the error signal:

$$\begin{cases} e_1(t) = x_1(t) - y_1(t) \\ e_2(t) = x_2(t) - y_2(t) \\ e_3(t) = x_3(t) - y_3(t) \end{cases} \tag{4}$$

Error system of equation (3) is

$$\begin{cases} \dot{e}_1 = -ae_1 + e_2 + 10x_2e_3 + 10y_3e_2 - 2d_1e_1 \\ \dot{e}_2 = -e_1 - 0.4e_2 + 5x_1e_3 + 5y_3e_1 - 2d_2e_2 \\ \dot{e}_3 = be_1 - 5x_1e_2 - 5y_2e_1 - 2d_3e_3 \end{cases}. \tag{5}$$

The coefficient matrix of equations (6) is

$$J(t) = \begin{pmatrix} -a-2d_1 & 1+10y_3 & 10x_2 \\ -1+5y_3 & -0.4-2d_2 & 5x_1 \\ b-5y_2 & -5x_1 & -2d_3 \end{pmatrix}. \tag{6}$$

Obviously, just let the coupling coefficients satisfy certain conditions, and the error system (5) tends to infinity in time asymptotically stable. We can achieve synchronization of two coupled Newton-Leipnik chaotic systems of different initial conditions and same structures.

As the error system is linear, and its coefficient matrix is time-varying, so by using stability theory of a continuous time-varying linear system, determine to make the error system (5) asymptotically stable, and the coupling coefficients must satisfy .

Here the stability of linear time-varying continuous system theory is demonstrated.

3.2 Stability theory of continuous time-varying linear system

Consider the following form of third-order linear time-varying systems:

$$\begin{cases} \dot{x}_1 = a_{11}(t)x_1 + a_{12}(t)x_2 + a_{13}(t)x_3 \\ \dot{x}_2 = a_{21}(t)x_1 + a_{22}(t)x_2 + a_{23}(t)x_3 \\ \dot{x}_3 = a_{31}(t)x_1 + a_{32}(t)x_2 + a_{33}(t)x_3 \end{cases}. \tag{7}$$

Assuming the coefficients $a_{ij}(t)$ are continuous and bounded, and set $a_{ij}(t) \le -a < 0$ $(i=1,2,3)$, $\forall t \ge t_0$ .The coefficient matrix $A(t)$ is divided into 2 × 2 blocks, as follows.

$$A(t) = \left(\begin{array}{cc:c} a_{11}(t) & a_{12}(t) & a_{13}(t) \\ a_{21}(t) & a_{22}(t) & a_{23}(t) \\ \hline a_{31}(t) & a_{32}(t) & a_{33}(t) \end{array}\right) \tag{8}$$

Let $b_{11} = \sup\limits_{t_0 \le t < +\infty} \{a_{11}(t) + |a_{21}(t)|, a_{22}(t) + |a_{12}(t)|\}$,

$$b_{12} = \sup_{t_0 \le t < +\infty} \{|a_{13}(t)|, |a_{23}(t)|\},$$

$$b_{21} = \sup_{t_0 \le t < +\infty} \{|a_{31}(t)|, |a_{32}(t)|\},$$

$$b_{22} = \sup_{t_0 \le t < +\infty} \{|a_{33}(t)|\},$$

$$B=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}. \tag{9}$$

Consider auxiliary equations:

$$V_1^* = b_{11}V_1^* + b_{12}V_2^*,$$

$$V_2^* = b_{21}V_1^* + b_{22}V_2^*. \tag{10}$$

Theorem If the variable coefficient linear system (7) continuous and bounded, $a_{ij}(t) < 0 \quad (i = 1, 2, 3)$ and these coefficients of its auxiliary equations (9) satisfy:

1) $b_{11} \le -\beta < 0$, $b_{22} \le -\beta < 0$, β is a positive constant;

2) All eigenvalues have negative real parts.

The zero solution of the system (7) is asymptotically stable.

3.3 Case analysis and numerical simulation

By the stability theory of linear time-varying system, study stability of error system (5), and obtain range of the coupling coefficient. Then we give out a sufficient condition of the global incremental synchronization of two identical Newton-Leipnik chaotic systems. Finally, numerical simulation is given out. Above illustrate the correctly theoretical analysis.

First, the error system corresponding to the symmetric matrix $J(t)$ corresponds with time-varying matrix $A(t)$, then get the following expression:

$$b_{11} = \sup_{t_0 \le t < +\infty} \{-a - 2d_1 + |-1 + 5y_3|, -0.4 - 2d_2 + |1 + 10y_3|\},$$

$$b_{12} = \sup_{t_0 \le t < +\infty} \{|10x_2|, |5x_1|\},$$

$$b_{21} = \sup_{t_0 \le t < +\infty} \{|b - 5y_2|, |-5x_1|\},$$

$$b_{22} = \sup_{t_0 \le t < +\infty} \{-2d_3\}.$$

Based on the above stability of linear time-varying system theory, the error system (5) is global asymptotic stability, as long as inequality $b_{11} < 0, b_{22} < 0, b_{11}b_{22} > b_{21}b_{12}$, b_{12}, b_{21} are non-negative numbers, Apparently b_{12}, b_{21} is satisfied and a sufficient condition will be met as follows:

$-a-2d_1+\left|-1+5y_3\right|<0,$

$-0.4-2d_2+\left|1+10y_3\right|<0$,

$-2d_3<0$,

$\left(-a-2d_1+\left|-1+5y_3\right|\right)\cdot\left(-2d_3\right)>m^2$,

$\left(-0.4-2d_2+\left|1+10y_3\right|\right)\cdot\left(-2d_3\right)>m^2$.

Where $m=\max(\left|5x_1\right|,\left|10x_2\right|,\left|b-5y_2\right|)$.

As the chaotic system is bounded, we can easily select the appropriate coupling coefficients to satisfy the above inequality. For initial values are different, the same structure of two Newton-Leipnik chaotic systems is global complete synchronization.

Fourth-order Runge-Kutta method for the simulation, set the initial value $\left(x_1(0),x_2(0),x_3(0),y_1(0),y_2(0),y_3(0)\right)=(1,5,5,1,2,0)$ and simulation results are shown in Figure 2, where $d_1=-1.3,d_2=-1.5,d_3=0.4$.

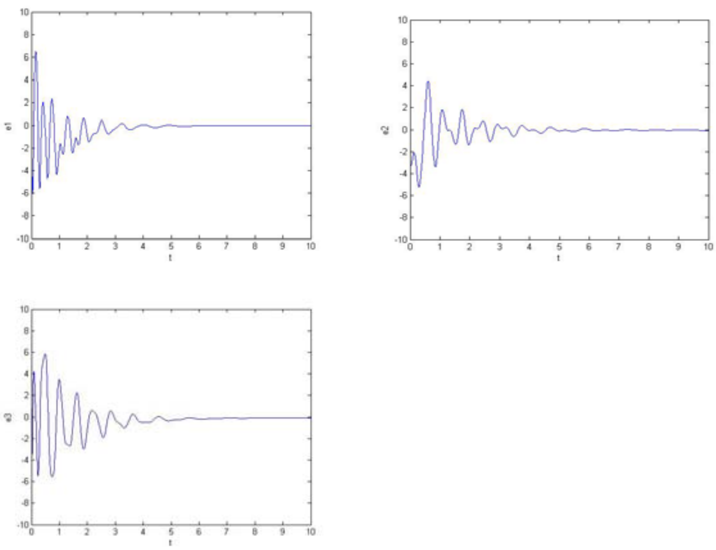

Fig. 2. The time response of error states (e_1 , e_2 , e_3) .

The above is just sufficient condition of two linear coupling Newton-Leipnik chaotic systems' synchronization, and it means that some do not meet the conditions of the coupling coefficient, but also make a global gradual synchronization of Newton-Leipnik chaotic system.

Finally, sufficient condition of two Newton-Leipnik chaotic systems' ynchronization is compared with these literatures [10-12]. First, theories and methods are not the same. Because the error system of two coupled chaotic systems synchronization is linear time-varying continuous system, which is used here to analyze its stability. The sufficient condition of two identical Newton-Leipnik chaotic systems for linear coupling, the method may be more reasonable than other methods, so that is to get a new sufficient condition for chaos synchronization. Second, these literatures [10-12] in the derivation of sufficient conditions of chaotic systems synchronization , these coupling coefficients are required these preconditions of $d_i > 0 (i = 1, 2, 3)$, but here we does not require. For example, [11] requires $d_2 > 14$, there is no need. Here the coupling coefficient d_i is broader range of options.

4. Conclusion

In this paper, the linear coupling of the two unified chaotic systems were studied. By the stability theory of Linear time-varying continuous system, new progressive sufficient conditions of the global chaos synchronization have been obtained. By comparing with proposed several methods, we found that this mathod is not only simple, less constraints, lack of conservation, and a wider range of coupling coefficient. The method is applied to Newton-Leipnik chaotic system, two identical chaotic systems can quickly achieve the global incremental synchronization, and numerical simulations show the effectiveness and feasibility of the method. In addition, this analysis method was applied to other chaotic systems and control. This method has good universality, and it is worth further studying.

Acknowledgement. Project supported by the Special Funds of the National Natural Science Foundation of China (Grant No. 11141003)

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Received: December, 2011