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# Fuzzy Max-Min Composition Technique in Medical Diagnosis

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**Abstract**

We propose a new technique named Fuzzy Max–Min Composition to Study the Sanchez's approach for Medical Diagnosis [13] and with the notion IFS [15].

**Keywords:** Fuzzy set; Intuitionistic fuzzy set (IFS); Intuitionistic fuzzy relation (IFR); Intuitionistic Medical diagnosis.

## 1 Introduction

The field of medicine is one of the most fruitful and interesting areas of applications for fuzzy set theory. In the discrimination analysis, the symptoms are ranked according to the grade of discrimination of each disease by a particular symptom and is represented in the form of a matrix called a frequency distribution matrix $F=(f_{ij})$ where $f_{ij}$ is the ratio of the patients with disease '$d_i$' and symptoms '$s_j$' to the total number of patients with disease '$d_i$'. This matrix model may not yield more accurate diagnosis in such cases where several diseases affect a single patient or when a single disease manifests quite differently in different patients and at different disease stages. Moreover, with the increased volume of information available to physicians from new medical technologies, the process of classifying different sets of symptoms under a single name of disease and determining the appropriate therapeutic actions becomes increasingly difficult. Recently, there are varieties of models of medical diagnosis under the general framework of fuzzy sets theory involving fuzzy matrices to deal with different complicating aspects of medical diagnosis.

In real world, we frequently deal with vague or imprecise information. Information available is sometimes vague, sometimes inexact or sometimes insufficient. Out of several higher order fuzzy sets, intuitionistic fuzzy sets (IFS) [1,2] have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Due to the some reason, evaluation of non-membership values is not also always possible and consequently there remains a part indeterministic on which hesitation survives. Certainly fuzzy sets theory is not appropriate to deal with such problem, rather IFS theory is more suitable. Out of several generalizations of fuzzy set theory for various objectives, the notion introduced by Atanassov [1] in defining intuitionistic fuzzy sets is interesting and useful. Fuzzy sets are intuitionistic fuzzy sets but the converse is not necessarily true [1]. In fact there are situations where IFS theory is more appropriate to deal with [8]. Besides, it has been cultured in [9] that vague sets [11] are nothing but IFS.

In the present paper we study Sanchez's method for medical diagnosis using the notion of IFS theory. The method of intuitionistic medical diagnosis involves intuitionistic fuzzy relations as defined in [7].

## 2 Preliminaries

We give here some basic definitions, which are used, in our next section.

**2.1 Definition**

Let a set E be fixed. An Intuitionistic fuzzy set or IFS A in E is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in E\}$ where the functions $\mu_A : E \to [0,1]$ and $\gamma_A : E \to [0,1]$ define the degree of membership and degree of non-membership respectively of the element $x \in E$ to the set A, which is a subset of E, and for every $x \in E$, $0 \le \mu_A(x) + \gamma_A(x) \le 1$. The amount $\Pi_A(x) = 1 - (\mu_A(x) + \gamma_A(x))$ is called the hesitation part, which may cater to either membership value or non-membership value or both.

**2.2 Definition**

If A and B are two IFS of the set E, then $A \subset B$ if and only if $\forall x \in E, [\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)]$ $A \supset B$ iff $B \subset A$,

$A = B$ iff $\forall$ $x \in E, [\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)]$,

$\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle / x \in E\}$,

$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$,

$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle / x \in E\}$.

Obviously every fuzzy set has the form

$$\left\{\left\langle x, \mu_A(x), \mu_{A^c}(x)\right\rangle / x \in E\right\}$$

**2.3 Definition**

Let $Q(X \to Y)$ and $R(Y \to Z)$ be two IFR. The max-min-max composition RoQ is the intuitionistic fuzzy relation from X to Z, defined by the membership function $\mu_{RoQ}(x,z) = \bigvee_y \left( \mu_Q(x,y) \wedge \mu_R(y,z) \right)$ and the

non – membership function.

$\gamma_{RoQ}(x,z) = \bigwedge_y \left(\gamma_Q(x,y) \vee \gamma_R(y,z)\right) \forall (x,z) \in X \times Z$ and $\forall\, y \in Y$.

## 3 Medical diagnosis

In this section we present an application of intuitionistic fuzzy set theory in Sanchez's approach [13, 14] for medical diagnosis. In a given pathology, suppose S is a set of symptoms, D a set of diagnoses, and P a set of patients.

Analogous to Sanchez's notion of "Medical Knowledge" we define "Intuitionistic Medical Knowledge" as an intuitionistic fuzzy relation R from the set of symptoms S to the set of diagnoses D (i.e., on S x D) which reveals the degree of association and the degree of non-association between symptoms and diagnosis.

Now let us discuss intuitionistic fuzzy medical diagnosis. The methodology involves mainly the following three jobs:

1. Determination of symptoms.
2. Formulation of medical knowledge based on intuitionistic fuzzy relations.
3. Determination of diagnosis on the basis of composition of intuitionistic fuzzy relations.

An intuitionistic fizzy relation Q is given from the set of patients P to the set of symptoms S and another intuitionistic fuzzy relation R is given from a set of symptoms S to the set of diagnoses D. The composition T of intuitionistic fuzzy relation R and Q.

**3.1 Algorithm**

[1] Compute $T = R \circ Q$

[2] Compute W, {Where W = $\{\mu_A(pi,s),\ \mu_{A^c}(pi,s)\}$

non-members in T $ie.\ \mu_{A^c}(pi,s) = 1 - \gamma_A(pi,s)\}$ converting as members in W.

[3] Find Min{ $\mu_A(pi,s),\ \mu_{A^c}(pi,s)$ }

[4] Find Max{Min{ $\mu_A(pi,s),\ \mu_{A^c}(pi,s)$ }} then we conclude that the patients pi is suffering from the disease dj (.i.e., j = 1,2,3,4,5).

## 4 Case study

Let there be four patients Ram, Mari, Sugu and Somu i.e., P = {Ram, Mari, Sugu, Somu} and the set of symptoms S = {temperature, headache, stomach pain, cough and chest - pain}. The intuitionistic fuzzy relation $Q(P \to S)$ is given as in Table 1. Let the set of Diagnosis be D = {Viral Fever, Malaria, Typhoid, Stomach Problem, Heart Problem}. The intuitionistic fuzzy relation $R(S \to D)$ is given as in Table 2.

**Table 1**

| Q | Temperature | Head Ache | Stomach Pain | Cough | Chest - Pain |
|---|---|---|---|---|---|
| Ram | (0.8, 0.1) | (0.6, 0.1) | (0.2, 0.8) | (0.6, 0.1) | (0.1, 0.6) |
| Mari | (0, 0.8) | (0.4, 0.4) | (0.6, 0.1) | (0.1, 0.7) | (0.1, 0.8) |
| Sugu | (0.8, 0.1) | (0.8, 0.1) | (0, 0.6) | (0.2, 0.7) | (0, 0.5) |
| Somu | (0.6, 0.1) | (0.5, 0.4) | (0.3, 0.4) | (0.7, 0.2) | (0.3, 0.4) |

**Table 2**

| R | Viral Fever | Malaria | Typhoid | Stomach Problem | Chest - Problem |
|---|---|---|---|---|---|
| Temperature | (0.4, 0) | (0, 7.0) | (0.3, 0.3) | (0.1, 0.7) | (0.1, 0.8) |
| Head ache | (0.3, 0.5) | (0.2, 0.6) | (0.6, 0.1) | (0.2, 0.4) | (0, 0.8) |
| Stomach Pain | (0.1, 0.7) | (0, 0.9) | (0.2, 0.7) | (0.8, 0) | (0.2, 0.8) |
| Cough | (0.4, 0.3) | (0.7, 0) | (0.2, 0.6) | (0.2, 0.7) | (0.2, 0.8) |
| Chest - Pain | (0.1, 0.7) | (0.1, 0.8) | (0.1, 0.9) | (0.2, 0.7) | (0.8, 0.1) |

**Table 3**

| T | Fever | Malaria | Typhoid | Stomach – Problem | Chest - Problem |
|---|---|---|---|---|---|
| Ram | (0.4, 0.1) | (0.7, 0.1) | (0.6, 0.1) | (0.2, 0.4) | (0.2, 0.6) |
| Mari | (0.3, 0.5) | (0.2, 0.6) | (0.4, 0.4) | (0.6, 0.1) | (0.1, 0.7) |
| Sugu | (0.4, 0.1) | (0.7, 0.1) | (0.6, 0.1) | (0.2, 0.4) | (0.2, 0.5) |
| Somu | (0.4, 0.1) | (0.7, 0.1) | (0.5, 0.3) | (0.3, 0.4) | (0.3, 0.4) |

**Table 4**

| W | Fever | Malaria | Typhoid | Stomach – problem | Chest - Problem |
|---|---|---|---|---|---|
| Ram | (0.4, 0.9) | (0.7, 0.9) | (0.6, 0.9) | (0.2, 0.6) | (0.2, 0.4) |
| Mari | (0.3, 0.5) | (0.2, 0.4) | (0.4, 0.6) | (0.6, 0.9) | (0.1, 0.3) |
| Sugu | (0.4, 0.9) | (0.7, 0.9) | (0.6, 0.9) | (0.2, 0.6) | (0.2, 0.5) |
| Somu | (0.4, 0.9) | (0.7, 0.9) | (0.5, 0.7) | (0.2, 0.6) | (0.3, 0.6) |

**Table 5**

| M | Fever | Malaria | Typhoid | Stomach – problem | Chest - Problem |
|---|---|---|---|---|---|
| Ram | 0.4 | **0.7** | 0.6 | 0.2 | 0.2 |
| Mari | 0.3 | 0.2 | 0.4 | **0.6** | 0.1 |
| Sugu | 0.4 | **0.7** | 0.6 | 0.2 | 0.2 |
| Somu | 0.4 | **0.7** | 0.5 | 0.3 | 0.3 |

From Table 5 it is obvious that, if the doctor agrees, then Ram, Sugu and Somu suffer from malaria whereas Mari faces Stomach Problem.

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