Applied Mathematical Sciences, Vol. 6, 2012, no. 36, 1791 - 1797

New Model for Determination Practical of Decision Making Units in DEA

Said Ebadi

Department of Mathematics, Ardabil Branch
Islamic Azad University, Ardabil, Iran
said_ebadisharafabad@yahoo.com

Abstract

Data envelopment analysis (DEA) assigns a score to each production unit (Decision Making Unit - DMU) considered in the analysis. Such score indicates whether the unit is efficient or not. For inefficient units, it also identifies a hypothetical value as the target and thus suggests improvements to their efficiency. These suggestions are not applicable to some DMUs because they have lower efficiency values and thus it becomes very difficult to reach the efficient frontier. On the other hand based on an *DEA* analysis no further improvement for efficient units can be indicated. Nevertheless, it is important for management to assign applicable target for units. Based on possible variations in the input and output levels of DMUs, for efficient DMUs, new units which are more efficient than DEA efficient units can be created to form a new improved frontier. In the new Production Possibility Set (PPS), inefficient DMUs, have new and high score efficiency. Available bank branch data was used to illustrate the applicability of this theoretical development. The sensitivity of the results to the parameters defined by management in the model was also examined, which proved the robustness of the proposed model.

Keywords: Data envelopment analysis, Linear programming.

1 Introduction

Data envelopment analysis (DEA) is a powerful technique in productivity management. It is a linear programming based on methodology for measuring the relative efficiency of decision making units (DMUs) (introduced by Charnes et al. [5]). A DEA analysis provides a variety of valuable information. It assigns a single score to each DMU that makes the comparison easy.

The method has the ability to simultaneously handle multiple inputs and outputs without requiring any judgments on their relative importance, so it does not need a parametrically driven input and output production function. More recently, stochastic input and output variations into DEA have been studied by; for example, Ebadi, Jahanshahloo, Monzeli and Aliev [6], Asgharian, Khodabakhshi, and Neralic [3], Khodabakhshi [8], and Khodabakhshi and Asgharian [9]. It establishes a best practice frontier among the units based on a comparison process. The units on this frontier are efficient units with an efficiency score of 1 and the rest are deemed inefficient. The level of inefficiency is measured by the unit's distance from this frontier.

One of the important advantages of DEA is its ability to identify performance targets for inefficient units and indicate what improvements can be made to achieve pareto-efficiency. In the real world, it might not be possible to adjust all inputs and outputs of inefficient units based on the DEA results; so we need to adjust all inputs and outputs for these DMUs so that it is feasible in practice to change their efficiencies.

For efficient units, no further improvement can be considered based on DEA, yet, increasing performance for even the best performers can be very important to management. Specifying targets for efficient units is of interest to operations analysts, management and industrial engineers. We have shown in our research that if the inputs and outputs of an efficient unit can be changed within a range, it is possible to find another combination of inputs and outputs within such constraints and define an artificial DMU that is more efficient compared to the DEA efficient unit from which it is derived. Therefore, although the "theoretical" frontier is not known, it is possible to define a "practical" one.

This new frontier envelops or touches the empirical frontier. The idea of introducing artificial "unobserved" DMUs was used by Allen [1] to capture value judgments in DEA. To overcome the issues related to complete weights flexibility in DEA, they used unobserved DMUs as an alternative approach to weight restrictions.

These units were constructed by varying the input–output levels of real DMUs in order to extend the production possibility set. In this paper, artificial DMUs were created using a linear programming model, so that the new frontier identifies the adjusted efficiency measures for DMUs and indicates targets for empirically efficient units. Section 3 presents the proposed model and methodology.

2 DEA Background

DEA provides a measure of the efficiency of a DMU relative to other such units, producing the same outputs with the same inputs. The units to be compared may be enterprisers, banks, schools, hospitals, etc. DEA is related to the concept of technical efficiency and can be considered as a generalization of efficiency measure.

Assume that there is a sample of n DMUs, each producing an s-dimensional row vector of outputs y, from an m-dimensional row vector of inputs x. Technology governs the transformation of inputs into outputs; the reference technology relative to which efficiency is assessed is given by the input requirement set $L(y) = \{x : x\ can\ produce\ y\}$. Farrell's [7] input-based measure of technical efficiency for each observation $t = 1,...,n$ is given by:

$$TE_t(x_t, y_t) = min\{\theta_t : \theta_t.x_t \in L(y_t)\}$$

that is, t^{th} DMU's observed input vector (x_t) is scaler $(0 \le \theta_t \le 1)$ until it is still just able to produce the observed level of output (y_t). The solution, $TE_t = \theta_t^*$, gives the proportion of the t^{th} DMU's actual input vector that is technologically necessary to produce its observed output vector given the best practice technology as revealed by the observed data. The vector $x_t^* = \theta_t^*.x_t$ would give the technically efficient (optimal) input vector for the t^{th} DMU.
One way to calculate this measure of technical efficiency is by the following linear programming problem (BCC-Model) [4] once for each DMU_t, $t = 1,...,n$:

$$\begin{aligned} &\min\ \theta_t \\ &\text{st:} \\ &\quad \lambda.Y \ge y_t \\ &\quad \lambda.X \le \theta.x_t \\ &\quad \textstyle\sum_{t=1}^{n}\lambda_t = 1, \\ &\quad \lambda_t \ge 0 \qquad t = 1,...,n \end{aligned}$$

Where Y is the n by s matrix of the observed outputs of all DMUs, X is the n by m matrix of the observed inputs for all DMUs, and λ is a n-dimensional row vector of weights that forms convex combination of observed DMUs relative to which the subject DMU's efficiency is evaluated. The constraint in this problem simply describe the input requirement set as given by the observed data.

3 Measures of Efficiency by using the new model

To explain the model, first consider the BCC ratio model:

$$
\begin{aligned}
Max \quad & h_0 = \frac{\sum_{r=1}^{s} u_r y_{ro} + u_0}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \\
s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj} + u_0}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad \forall j \\
& u_r \ge \varepsilon, \quad \forall r, \\
& v_i \ge \varepsilon \quad \forall i,
\end{aligned}
$$

In the above model x_{ij} and y_{rj} are the inputs and outputs of the j^{th} DMU; and u_r and v_i are the output and input weights, respectively. The objective is to obtain those weights that maximize the efficiency of the unit under evaluation, DMU_o, while the efficiency of all $DMUs$ must not exceed 1. The efficiency score and input output weights are the variables of the BCC model. The inputs and outputs of DMU_o are known. If DMU_o is efficient, then $h_o = 1$. In the real world, some of the factors (inputs and outputs) are fixed and it is not possible to vary their values, e.g. a store's floor space. However, changes in other factors are permitted within certain ranges, i.e., $L_{x_{io}} \le x_{io} \le U_{x_{io}}$ and $L_{y_{ro}} \le y_{ro} \le U_{y_{ro}}$.

Furthermore, some factors may have a specific relationship with some other factors. This information about inputs and outputs can be obtained from management. Suppose that there are upper and lower bounds for some or all inputs and outputs. Our goal is to look for the inputs and outputs of a new DMU within the specified range, but one that has an efficiency score greater than that of DMU_o, which is, at present, 1. We are attempting to create new $DMUs$ by adjusting the already efficient $DMUs'$ input and output variables according to the limits determined by management. This should produce units that could be used as models for the efficient $DMUs$ from which they were derived. In the following models x_{io} (inputs of the artificial DMU), y_{ro} (outputs of the artificial DMU), u_r and v_i are variables. The model then becomes:

$$
\begin{aligned}
Max \quad & h_o = \frac{\sum_{r=1}^{s} u_r \hat{y}_{ro} + u_o}{\sum_{i=1}^{m} v_i \hat{x}_{io}} \\
s.t: \quad & \\
& \frac{\sum_{r=1}^{s} u_r y_{rj} + u_o}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad \forall j \qquad (1)
\end{aligned}
$$

Efficiency of real units:

$$1 \le \frac{\sum_{r=1}^{s} u_r \hat{y}_{ro} + u_o}{\sum_{i=1}^{m} v_i \hat{x}_{io}} \le 1 + \delta$$

Efficiency of the new units:

$$\begin{array}{ll} L_{x_{io}} \le \hat{x}_{io} \le U_{x_{io}} & \forall i, \\ L_{y_{ro}} \le \hat{y}_{ro} \le U_{y_{ro}} & \forall r \\ u_r, v_i \ge \varepsilon & \forall r, i \\ u_o & free \end{array}$$

Note that in this model, unlike the standard DEA model, inputs and outputs are also variables. The objective function is to maximize the efficiency of the artificial DMU, while the weights must be feasible for all other units and factors can vary within the specified ranges. To have an improved unit, the efficiency score of the artificial unit is set to be greater than or equal to 1. DEA models which result in an efficiency score of more than 1 have been reported in the literature. Andersen and Petersen [2] developed modified versions of the DEA models for ranking efficient units in which the unit, a super efficient unit, could obtain an efficiency score of more than 1 by excluding such unit from the analysis. In this paper an upper limit, $(1 + \delta)$, is considered in the model for the efficiency of the new unit otherwise the model would be unbounded. The amount of possible increase in the efficiency of an empirically efficient unit, designated as δ, can be specified by management (for example: 5%).

The upper and lower bounds for factors and the possible improve in efficiency of an empirically efficient unit (δ) can be local or global based on the application; for example for comparing different branches of the same bank the information can be global, while it can be local if different banks are compared.
The ratio model, (1) can be transformed into a linear fractional programming model by substituting $\tilde{y}_{ro} u_r$ and $\tilde{x}_{io} v_i$ with new variables p_r and q_i, respectively, and replacing $L_{x_{io}} \le \tilde{x}_{io} \le U_{x_{io}}$ and $L_{y_{ro}} \le \tilde{y}_{ro} \le U_{y_{ro}}$ with $v_i L_{x_{io}} \le q_i \le v_i U_{x_{io}}$ and $u_r L_{y_{ro}} \le p_r \le u_r U_{y_{ro}}$, correspondingly. Then the linear fractional program can be transformed to a linear program, which is shown in (2), so that the linear programming method can be applied to solve the case. The process is relatively straightforward.

$$
\begin{aligned}
&Max \quad \sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} + u_o \\
&s.t: \\
&\qquad \frac{\sum_{r=1}^{s} u_r y_{rj} + u_o}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad \forall j \\
&\qquad \frac{\sum_{r=1}^{s} u_r \hat{y}_{ro} + u_o}{\sum_{i=1}^{m} v_i \hat{x}_{io}} = 1 + \delta \\
&\qquad L_{x_{io}} \le \hat{x}_{io} \le U_{x_{io}}, \qquad \forall i, \\
&\qquad L_{y_{ro}} \le \hat{y}_{ro} \le U_{y_{ro}}, \qquad \forall r, \\
&\qquad u_r, v_i \ge 1, \qquad \forall i, r, \\
&\qquad u_o \qquad free,
\end{aligned}
\tag{2}
$$

4 Conclusions

In this paper, a new model for determination of practical frontier of decision making units in DEA is discussed. In the new PPS, inefficient DMUs, have new and high score efficiency. Available bank branch data was used to illustrate the applicability of this theoretical development. Also, the sensitivity of the results to the parameters defined by management in the model was examined. This is proved the robustness of the proposed model.

References

[1] K. Allen, DEA in the ecological context -an overview. In Wassermann. G. (ED.) Data Envelopment Analysis in the service sector. Galber, Wiesbaden, (1999), 203 - 235.

[2] P. Andersen and N.C. Petersen, A procedure for ranking efficient units in data envelopment analysis, Management Science 39 (1993), 1261-1264.

[3] M. Asgharian, M. Khodabakhshi, and L. Neralic, Congestion in stochastic data envelopment analysis: An input relaxation approach, International Journal of Statistics and Management System, (2010), 84-106.

[4] R.D. Banker, A. Charnes and W.W. Cooper, Some method for estimating technical and scale inefficiencies in data envelopment analysis, Management Science, 30 (1984), 1078-1092.

[5] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, European Journal of Operation Research, 2 (1978), 429-444.

[6] S. Ebadi, G.R. Jahanshahloo, A.A. Mozeli and F. Aliev, Determination measure of efficiency using undesirable outputs of DEA. News of Baku University, Physico Mathematical sciences series 4 (2009), 86-91.

[7] M.J. Farrell, The measurement of productive efficiency, Journal of Royal Statistical Society A; 120; 3 (1957), 253-290.

[8] M. Khodabakhshi, Estimating most productive scale size with stochastic data in data envelopment analysis, Economic Modeling, 26 (2009), 968-973.

[9] M. Khodabakhshi and M. Asgharian, An input relaxation measure of efficiency in stochastic data envelopment analysis, Applied Mathematical Modeling, 33 (2009), 2010-2023.

Received: October, 2011