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An Evaluation of Sections Efficiency in Bo-ali and Arta Hospitals Using Data Envelopment Analysis

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Abstract

As costs of publicly funded hospitals are large and increasing, it is important to contain and possibly lower these costs. This can be done by identifying inefficient hospitals, so that their operations can be scrutinized and their costs decreased. The efficiency of hospitals is determined by a comparison of their inputs and outputs. For multiple inputs and outputs such a comparison is not straightforward but can be accomplished using Data Envelopment Analysis. This is a technique based on linear programming which compares each unit (sections of hospitals) with all the others and determines its efficiency in terms of other hospitals with comparable inputs and outputs. This technique is employed to evaluate the efficiency of 9 sections of Arta and Bo-ali hospitals in place Ardabil.

Mathematics Subject Classification: 90

Keywords: Data Envelopment Analysis, Decision Making Unit, Efficiency, Production Possibility Set

1 Introduction

A relatively recent technique, called Data Envelopment Analysis (DEA), developed by Charnes, Cooper and Rhodes [6] and Banker, Charnes and Cooper [4]. They extend the concept of technical efficiency from the case of one input and one output to that of multiple inputs and outputs. This approach is based on linear programming and allows the comparison of organizational units producing the same outputs using the same inputs, declaring one or more to be efficient, and others inefficient in the sense that the inefficient organizational unit use more inputs for the outputs they produce than the efficient ones.More recently, stochastic input and output variations into DEA have been studied by; for example, Asgharian, Khodabakhshi, and Neralic [3], Khodabakhshi [8] and Khodabakhshi and Asgharian [7]. In this study this technique is applied to inputs and outputs of 9 sections of Bo-ali and Arta Hospitals.

This study deals with the determination of technical efficiency, which is concerned with the production of given multiple outputs given minimal multiple inputs. For single output and single input, efficiency may be expressed as input per unit of output, and various organizational units may be compared, with the one having the lowest input per unit of output declared to be efficient, and the ones with lower ratios inefficient. For multiple inputs and outputs, such a measure is no longer straightforward, as it is not obvious how the multiple inputs and outputs should be aggregated.

In this study the following topics are discussed: Conventional approach for measuring hospital efficiency, Relation of the technical efficiency and DEA, Found efficiency by the solved a Linear Programming, Application of DEA to mentioned hospitals and Conclusions.

2 Conventional approaches for measuring hospital efficiency

Historically, there are approaches which have been widely used to evaluate hospital efficiency: ratio analysis and econometric regression techniques. A new measure of efficiency referred to as the Hospital Performance Index is the result of a recent project for hospital reimbursement in Ardabil.

2.1 Ratio analysis

Ratio analysis is a managerial accounting technique most often used in the private sector. However, this technique has also been used in the health care sector for comparison and measurement of efficiency.

In the hospital sector ratios are commonly published for use by managers and policy makers. This study provides, the following ratios:

- I) cost per patient day;
- II) average percentage occupancy (total patent days / maximum days of care);

- III) average daily number of patient day;
- IV) total cost per approved bed;
- V) average length of stay.

For these ratio to be useful for evaluation purposes, the following assumptions must be satisfied:

- i) A comparable set of hospital must be identified, based on closer analysis and / or experts judgement.
- ii) Various input-output or output-input relationships believed to be of importance must be identified. For example, cost per patient day is a common ratio identified for the use of evaluating efficiency.
- iii) If more than one ratio is used, the ratios should be considered simultaneously.

2.2 Econometric regression techniques

Many health related studies have used econometrics to estimate cost and production function. Two methods are employed to explore hospital costs and efficiency: the estimation of the cost function and the estimation of production. The cost function expresses the input in terms of the money as a function of the output, and the production function indicates the outputs as a function of the inputs.

The cost function can be estimated in the following form $TC = a_0 + a_1 x + a_2 x^2$, where TC represents total costs, a_0 is the constant term of the regression, x the total output, and a_1 and a_2 parameters indicating the change in total costs for changing output.

Instead of looking at a relationship between cost and output, the production examines the relation output and the various input measures in physical terms. A well-known form is that of the Cobb-Douglas production function $Q = AL^n K^m$, where Q stands for output, A is a constant to be estimated, L and K are the two inputs (traditionally labor and capital) and n and m the elasticities with respect to labor and capital. If $n + m < 1$, there are decreasing returns to scale, if $n + m = 1$, constant returns, and if $n + m > 1$, increasing returns. If the relationship is valid and the coefficients can be estimated, an insight can be obtained the efficiency of the hospital sectors.

2.3 The hospital performance index

The Hospital Performance Index (HPI) is used in Ardabil as a basis of funding of a select number of hospitals. This new methodology is based on the concept of relative efficiency measured as cost per case for given patient load. From this, each hospital's total cost of inpatient treatment is compared to the provincial average cost of the same inpatient treatment: both severity and volume are included. If actual costs exceed predicted costs, performance is rated poorly, if actual costs are less, performance is rated well.

3 Technical efficiency and DEA

DEA privies a measure of the efficiency of a Decision Making Unit (DMU) relative to other such units, producing the same outputs with the same inputs. The units to be compared may be enterprized, banks, schools, and this study they are sections of hospital. DEA is related to the concept of technical efficiency and can be considered as a generalization of efficiency measure.

Technical efficiency is concerned with the quantities of inputs and outputs of production processes employed by decision making units. If there is only one input and one output, a DMU is efficient if produces more per unit of input than other units. DMU's producing less than the maximum amount of output per unit of input are inefficient. To obtain a formulation that can be generalized for multiple inputs and outputs, this is reformulated as follows.

Let the DMU to be evaluated be unit o and let k be the index of all DMU's to be considered including unit o. Let y_k be the output of unit k and x_k its input. The efficiency of DMU_o is then $\frac{y_o/x_o}{Max_k(y_k/x_k)}$, which cannot exceed 1.

This efficiency can also be found as the solution of the linear programming problem:

$$\begin{aligned} \max_{w} f \quad & = (y_o/x_o)w \\ s.t: \quad & \\ (y_k/x_k)w \quad & \leq 1 \quad \text{for all} \quad k \\ w \quad & \geq 0. \end{aligned}$$

Obviously w is determined by binding constraint(s) corresponding $w = \frac{1}{Max_k(y_k/x_k)}$. The following nonlinear programming problem has the same solution

$$\begin{aligned} \max_{u,v} \quad f \quad & = \frac{y_o u}{x_o v} \\ s.t: \quad & \\ \frac{y_k u}{x_k v} \quad & \leq 1 \quad \text{for all} \quad k \\ u, v \quad & \geq 0. \end{aligned}$$

u can be interpreted as the value of the output and v as the value of the input. DMU_o can select u and v to maximize its efficiency, but the constraints stipulate that u and v must be chosen such that the efficiency of each DMU, including that of DMU_o, cannot exceed 1. This problem has multiple optimal solutions, as only $w = u/v$ matters.

The problem can be generalized for multiple inputs and outputs. u is the weight or value assigned to the single output. If there are multiple outputs, represented by the index i, there are corresponding weights u_i. For multiple inputs, represented by the index j, the weights are v_j. The nonlinear programming problem is then:

$$\begin{aligned} \max_{u_i,v_j} \quad f \quad & = \tfrac{\sum_i y_{0i}u_i}{\sum_j x_{0j}v_j} \\ s.t: & \\ \frac{\sum_i y_{ki}u_i}{\sum_j x_{kj}v_j} & \leq 1 \quad \text{for all} \quad k \\ u_i, v_j & \geq 0 \quad \text{for all} \quad i, j. \end{aligned}$$

The interpretation of this problem is as follows. In evaluation of DMU_o, weights u_i and v_j are used for its the outputs and the inputs respectively. These weights are chosen to give the best possible efficiency rating for DMU_o, subject to the constraints that these weights result in efficiencies of all DMU's don't be exceeded 1. Note that in perfectly competitive markets the weights would be given by the equilibrium prices of inputs and outputs. For nonprofit organization such prices are not available and often no prices are available at all. Instead weights are used, which are selected to give the highest efficiency rating possible for unit o, given the performance of all DMU's considered. Such an evaluation is totaly and absolutely fair to DMU_o, because the weights replacing prices are chosen to maximize its efficiency rating.

It can be proved that the solution of this nonlinear programming problem is equivalent to that of the following linear programming problem, which will be referred to as the *primal* problem:

$$\begin{aligned} \max_{u_i,v_j} \quad f \quad & = \textstyle\sum_i y_{oi}u_i \\ s.t: & \\ \sum_j x_{oj}v_j & = 1 \\ \sum_i y_{ki}u_i & \leq \textstyle\sum_j x_{kj}v_j \quad \text{for all} \quad k \\ u_i, v_j & \geq 0 \quad \text{for all} \quad i, j. \end{aligned}$$

The problem that is the dual of this linear programming problem is as follows (the *dual* problem):

$$
\begin{aligned}
\min_{f,l_k} \quad g \quad & = f \\
s.t: \quad & \\
\sum_k x_{ki} l_k \quad & \leq x_{oi} f \quad \text{for all} \quad i \\
\sum_k y_{kj} l_k \quad & \geq y_{oj} \quad \text{for all} \quad j \\
l_k \quad & \geq 0 \quad \text{for all} \quad k.
\end{aligned}
$$

This problem can be interpreted as that of finding a linear combination of all DMU's producing at least the same output as DMU_o but using at most a fraction f of its inputs, with f to be minimized. Both the primal and dual problem can be used for computational purposes.

The formulation given above is the one given in Charnes, Cooper, and Rhodes [6], (the CCR formulation) which assumes that the productions have constant returns to scale. This is frequently not realistic. In the case of hospitals, a small hospital may be not made comparable to a large one by simply reducing input by some factor. This is avoided in a formulation give by Banker, Charnes, Cooper [4], (the BCC formulation) allowing for increasing and decreasing returns to scale. This is achieved by adding to the dual linear programming problem the convexity constraint $\sum_k l_k=1$, and in the primal linear programming by including a constant term $-u_c$ to the objective function and all except the first constraint.

4 Application of DEA to sections of Arta and Bo-ali Hospitals

This section discuses the use of DEA to evaluate the efficiency of 9 general sections in Arta and Bo-ali hospitals, both for the CCR and the BCC formulation.

4.1 Selection of Inputs and Outputs

Of great impotence is the choice of inputs and outputs. The following items were considered:

Inputs :

1)Number of beds staffed and in operation

2)Number of nuesrs

3)Total operation expenditure

4)Number of physicians
5)Floor area
6)Energy use
⋮

Outputs :
1)Number of sicks
2)Quality of care
3)Physicians trined
4)Nurses trained
⋮

In this study first two inputs and outputs are considered. Therefore inputs and outputs data [1, 5], for 9 units DMU's given as follows:

$DMUs$	$input_1$	$input_2$	$output_1$	$output_2$
DMU_1	10	2	2	1
DMU_2	3	1	3	4
DMU_3	15	2	12	2
DMU_4	20	4	20	3
DMU_5	17	2	15	3
DMU_6	37	4	30	1
DMU_7	45	4	31	1
DMU_8	9	3	8	4
DMU_9	9	4	4	3

Table 1. Inputs and Outputs data

4.2 Calculation

To evaluate for each the 9 sections, a linear programming problem must be solved, and since two formulations ware used, a total of 18 problems ware solved. Since the primal problem, which is formulated in terms of weights, has a row for each DMU and a column for each input and output, whereas for the dual problem the reverse is true, the dual problem, having fewer rows, is easier to solve for the simplex method.

4.2.1 Results for Constant Returns to Scale (CCR Formulation)

First the results for the CCR formulation which assumes constant returns to scale, are discussed, which is followed by the results for the BCC formulation, in which increasing or decreasing returns to scale are allowed.

The results of the CCR model for *lambda* form summarized as follows:

$DMUs$	f	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9
DMU_1	0.211	0	0.186	0	0.032	0.053	0	0	0	0
DMU_2	1	0	1	0	0	0	0	0	0	0
DMU_3	0.884	0	0	0	0.126	0.632	0	0	0	0
DMU_4	1	0	0	0	1	0	0	0	0	0
DMU_5	1	0	0	0	0	1	0	0	0	0
DMU_6	0.991	0	0	0	0	1.441	0	0.27	0	0
DMU_7	1	0	0	0	0	0	0	1	0	0
DMU_8	0.889	0	2.667	0	0	0	0	0	0	0
DMU_9	0.444	0	1.333	0	0	0	0	0	0	0

Table 2: The results of the CCR model for *lambda* form

and, the results of the CCR model for u , v form summarized as follows:

$DMUs$	f	u_1	u_2	v_1	v_2
DMU_1	0.211	0.098	0.015	0.073	0.135
DMU_2	1	0	0.250	0.333	0
DMU_3	0.884	0.074	0	0.053	0.105
DMU_4	1	0.050	0	0.050	0
DMU_5	1	0.067	0	0.048	0.095
DMU_6	0.991	.033	0	0.003	0.222
DMU_7	1	0.032	0	0.003	0.217
DMU_8	0.889	0.111	0	0.111	0
DMU_9	0.444	0.111	0	0.111	0

Table 3: The results of the CCR model for u, v form

In this models it is possible that one DMU is found with efficiency equal to 1, but it is practically inefficient. It is better to use *epsilon* form of the models for solving this problem that is explained as follows:

$$\begin{aligned} \min_{f,l,s^+,s^-} \quad g \quad &= f - e(S^+ + S^-) \\ s.t: & \\ Yl - S^+ \quad &= Y_o \\ fX_o - Xf - S^- \quad &= 0 \\ l, S^+, S^- \quad &\geq 0. \end{aligned}$$

where $10^{-6} \leq e \leq 1/max_i \sum_j x_j$.

DMU$_o$ is efficient if and only if the following two conditions are satisfied:

1) f=1

2) All slacks are zero.

Results of calculations with *epsilon* form method is summarized as following table:

$DMUs$	f	l_2	l_4	l_5	l_7	S_1^+	S_2^+	S_1^-	S_2^-
DMU_1	0.211	0.186	0.032	0.053	0	0	0	0	0
DMU_2	1	1	0	0	0	0	0	0	0
DMU_3	0.884	0	0.126	0.632	0	0	0.274	0	0
DMU_4	1	0	1	0	0	0	0	0	0
DMU_5	1	0	0	1	0	0	0	0	0
DMU_6	0.991	0	0	1.441	0.270	0	3.595	0	0
DMU_7	1	0	0	0	1	0	0	0	0
DMU_8	0.889	2.667	0	0	0	0	2.667	0	0
DMU_9	0.444	1.333	0	0	0	0	2.333	0	0.444

Table 4. Result of CCR *epsilon* form

where for all DMUs have: $l_1 = l_3 = l_6 = l_8 = l_9$=0. It is observed that two mentioned conditions are held for: DMU_2, DMU_4, DMU_5 and DMU_7. A rating below 1 means that the DMU is inefficient because there are other DMUs that can produce at least the same quantities of outputs with less inputs.

Of the 9 DMUs, 44 *percent* have an efficiency rating of 1, so that the remaining 56 *percent* are deemed to be inefficient. The number of efficient DMUs depends on the number of inputs and outputs and how different the DMU's are in terms of these.

4.2.2 Results for Increasing and Decreasing Returns to Scale (BCC Formulation)

If there are constant returns to scale, DMU's of different sizes may be compared, in the sense that units may be scaled up and scaled down to obtain comparable inputs and outputs. This means that the sum of the l 's does not have to be 1 in the dual problem. If the constraint $\sum_k l_k$=1 is added, this is no longer true, and the production function may have increasing or/and decreasing returns to scale, see Banker, Charnes and Cooper [4] and Adolphson, Cronia and Walters [2]. Two orientations are possible for formulation, an input orientation and an output orientation, giving different results in this case, of which the former is used here. The dual variable of convexity constraint indicates whether increasing or decreasing returns are present.

The linear program for the BCC model are given below:

$$\begin{aligned}
\min_{f,l,s^+,s^-} \quad g \quad &= f - e(S^+ + S^-)\\
s.t:&\\
Yl - S^+ \quad &= Y_o\\
fX_o - Xl - S^- \quad &= 0\\
\sum_j l_j \quad &= 1\\
l, S^+, S^- \quad &\geq 0.
\end{aligned}$$

where $10^{-6} \leq e \leq 1/max_i \sum_j x_j$
Results of calculation for this model (BCC) summarized as following table:

$DMUs$	f	l_2	l_4	l_5	l_6	l_7	l_8	S_1^+	S_2^+	S_1^-	S_2^-
DMU_1	0.5	1	0	0	0	0	0	1	3	2	0
DMU_2	1	1	0	0	0	0	0	0	0	0	0
DMU_3	0.895	0.261	0.025	0.714	0	0	0	0	1.261	0	0
DMU_4	1	0	1	0	0	0	0	0	0	0	0
DMU_5	1	0	0	1	0	0	0	0	0	0	0
DMU_6	1	0	0	0	1	0	0	0	0	0	0
DMU_7	1	0	0	0	0	1	0	0	0	0	0
DMU_8	1	0	0	0	0	0	1	0	0	0	0
DMU_9	0.444	0.941	0.059	0	0	0	0	0	0.941	0	0.601

Table 5: Results of BCC model

where for all DMUs: $l_1 = l_3 = l_9 = 0$.
Of the 9 DMUs, 67 *percent* have an efficiency rating of 1, so that the remanding 33 *percent* are deemed to be inefficient. The number of DMUs with a rating of 1 has now increased from 44 *percent* to 67 *percent* which is explained by the units of different size being less easily comparable.

5 conclusions.

Data envelopment analysis was used evaluate 9 sections of Arta and Bo-ali Hospitals in Ardabil with as inputs number of beds and nurses, and as outputs number of sicks and quality of care. For CCR formulation it was found that the efficiency rating varied from 0.211 to 1, and for the BCC formulation from 0.444 to 1, that the average rating was much lower for smaller hospitals.

If the existence of increasing and decreasing returns is taken into account, as the BCC formulation does, almost all efficiency ratings go up considerably, but the high number of hospital with a rating of 1 diminish the usefulness of the results, as apparently no comparable hospitals could be found in many situations. But again it was found that the larger hospitals were on average more efficient than the smaller ones.

The result may be used both for corrective action and for policy purposes. For example, the DMU_1 and DMU_3, which have CCR ratings of 0.211 and 0.884 , and BCC ratings of 0.5 and 0.895, may be compared with their reference group, to find out how their management may be improved. The results for small hospitals may give rise to reconsidering the minimum size of the hospital. The BCC formulation is useful here because it gives an estimate of the constant costs of a hospital.

References

[1] Arta Hospital: Statistical Supplement in the year, 2009.

[2] D.L. Adolphson, G.C. Cronia and L.C. Walters, A unified framework for classifying DEA models, *Pergamon Press, New York, 1991.*

[3] M. Asgharian, M. Khodabakhshi, and L. Neralic, Congestion in stochastic data envelopment analysis: An input relaxation approach, *International Journal of Statistics and Management System,* **5** (2010), 84-106.

[4] R.D. Banker, A. Charnes and W.W. Cooper, Some method for estimating technical and scale inefficiencies in data envelopment analysis, *Management Science,* **30** (1984), 1078-1092.

[5] Bo-ali Hospital: Statistical Supplement in the year, 2009.

[6] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, *European Journal of Operation Research,* **2** (1978), 429-444.

[7] M. Khodabakhshi and M. Asgharian, An input relaxation measure of efficiency in stochastic data envelopment analysis, *Applied Mathematical Modeling,* **33** (2009), 2010-2023.

[8] M. Khodabakhshi, Estimating most productive scale size with stochastic data in data envelopment analysis, *Economic Modeling,* **26** (2009), 968-973.

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