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Radiation Effects on Unsteady Flow through a Porous Medium Channel with Velocity and Temperature Slip Boundary Conditions

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Abstract

In a horizontal porous layer bounded by impermeable plates, the unsteady flow produced by an oscillatory motion of the lower plate, is considered. Channel plates are kept at different but constant temperatures. Both velocity and temperature slips are assumed partially at the channel walls. To account for radiation, Cogley equilibrium model is employed for radiative heat flux. Viscous dissipation is also considered in the analysis. The initial boundary value problem is solved numerically by the use of Crank-Nicolson implicit difference scheme. Effects of the pertinent parameters such as velocity and temperature slip parameters, radiation parameter, permeability of the porous medium and Brinkman number, are investigated on the flow and temperature fields and discussed graphically.

Keywords: Radiation, porous medium, unsteady flow, permeability, velocity and temperature slip conditions.

1 Introduction

The study of flow and heat transfer through a porous medium has become of main interest in science and technology because of several engineering applications, particularly when the fluid flow is caused by shearing motion of a plate. Stokes investigated such flow of a viscous fluid caused by an impulsively started or oscillating shearing motion of a flat plate. These problems are termed as Stokes first and second problem respectively [Schlichting (1968)]. Panton (1968) presented exact solutions corresponding to transient effects for Stokes' oscillating plate problem in terms of tabulated function-forms. The unsteady flow of a viscous fluid caused by the cosine and sine oscillations of a plate was discussed in depth by Erdogan (2000) and presented exact transient solutions. He also indicated the importance of such study in several applied problems in vibrating media and acoustics. Fetecau et al. (2008) discussed Stokes second problem for Newtonian fluids.

Channel flows are important and one of the fundamental flow situations is the well known Couette flow which occurs in many industrial and engineering applications. Such flows in parallel plate channels filled or partially filled with a porous medium have been investigated by several researchers, e.g. Bhargava and Sacheti (1989), Daskalakis (1990), Nakayama (1992), Kuznetsov (1998, 2000), Hayat et al. (2005), Bég et al. (2008), Chauhan and Kumar (2010), Chauhan and Agrawal (2011).

In all fluid flow problems, a common assumption is that of the no-slip condition at the impermeable boundary wall. But in certain cases, this condition does not hold and it should be replaced in the analysis by a partial slip boundary condition. In fact, slip at the wall occurs when the fluid is a rarefied gas, suspension or emulsion, and fluid slips also on hydrophobic surfaces, see Yoshimura and Prud'homme (1988), Eijkel (2007). It is interesting to investigate Couette flow in a parallel plate channel under fluid slippage at the plates. The effects of fluid slippage at the channel plates for Couette flow of gases are examined by Marques et al. (2000). It is also important to examine the Couette flow in a channel bounded by two plates in which one is stationary and other is oscillating. Khaled and Vafai (2004) investigated the effects of slip condition on Stokes flow and Couette flow due to one wall oscillating and other one is fixed of a channel. In these studies, effects of radiation are neglected. However these effects are significant in some engineering and industrial applications where the system operates at higher temperatures.

In this paper, unsteady flow through a horizontal porous layer bounded by impermeable plates, is considered which is caused by an oscillatory motion of the lower channel plate. Heat transfer is also determined with radiation effects. The initial boundary value problem is solved numerically under the velocity and temperature slip boundary conditions and the effects of the pertinent parameters on the flow and temperature fields are examined and discussed.

2 Formulation of the problem

We consider an unsteady flow of a viscous incompressible fluid through a horizontal porous layer of high permeability bounded by impermeable plates. The fluid and the channel plates are initially at rest, and the whole system is kept at a constant temperature T_0. The channel boundary plates are separated by a distance *h*. A Cartesian coordinate system is assumed where *x*-axis is taken along the lower horizontal plate, while *y*-axis is taken normal to the plates. At time $t > 0$, the lower plate located at $y = 0$ is given an oscillatory motion in its own plane with a velocity $u_0 \sin \omega t$, and maintained at a constant temperature T_0, while the upper plate is raised with a constant temperature T_1. Here $T_1 > T_0$. The flow inside the porous channel is induced by the lower oscillatory plate. Velocity and temperature slips are assumed partially at both the channel plates.

It is further assumed that the medium is optically thin and with relatively low density. Following Cogley et al. (1968) equilibrium model, we therefore take expression of the radiative heat flux as follows:

$$\frac{\partial q_r}{\partial y} = 4\left(T - T_0\right)\int_0^\infty K_{\lambda\omega}\left(\frac{\partial e_{b\lambda}}{\partial T}\right)_\omega d\lambda = 4I^*\left(T - T_0\right), \tag{1}$$

where $K_{\lambda\omega}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is the plank constant. By introducing the following non-dimensional quantities:

$$\eta = \frac{y}{h}, \tau = \omega t, u = \frac{u'}{h\omega}, K = \frac{K_0}{h^2}, \theta = \frac{T - T_0}{T_1 - T_0}. \tag{2}$$

The governing momentum and energy equations in the non-dimensional form are given by

$$\frac{\partial^2 u}{\partial \eta^2} - \frac{1}{K\phi_1}u = \frac{R}{\varepsilon\phi_1}\frac{\partial u}{\partial \tau}, \tag{3}$$

$$\phi_2\frac{\partial^2 \theta}{\partial \eta^2} + \phi_1 Br\left(\frac{\partial u}{\partial \eta}\right)^2 + \frac{Br}{K}u^2 - F\theta = Pe\frac{\partial \theta}{\partial \tau}. \tag{4}$$

The initial and boundary conditions for the present problem are given by

For $\tau \le 0$; $\quad u = \theta = 0$, $\quad$ for all η.

For $\tau > 0$; $\quad u - \beta\frac{\partial u}{\partial \eta} = R_0 \sin\tau$, $\quad \theta = \alpha\frac{\partial \theta}{\partial \eta}$, $\quad$ at $\eta = 0$

$$u + \beta\frac{\partial u}{\partial \eta} = 0, \quad \theta = 1 - \alpha\frac{\partial \theta}{\partial \eta}, \quad \text{at } \eta = 1. \tag{5}$$

Where, $\phi_1 = \bar{\mu}/\mu$, the viscosity-ratio parameter; $R = h^2\omega/\nu$, the vibrational Reynold's number; $Br = \mu h^2\omega^2 / k\left(T_1 - T_0\right)$, the Brinkman number; $F = 4I^* h^2/k$, the radiation parameter; $Pe = \rho C_p h^2\omega/k$, the Péclet number; $\phi_2 = \bar{k}/k$, the thermal conductivity-ratio; $\beta = \beta'/h$, the velocity slip parameter; $\alpha = \alpha'/h$, the temperature slip parameter; $R_0 = u_0/h\omega$, the dimensionless amplitude of the lower plate velocity; ρ, the density; ε, the porosity; μ, the viscosity of the clear fluid; $\bar{\mu}$, the effective viscosity of the fluid in porous medium; K_0, permeability of the porous medium; C_p, specific heat at constant pressure and $\bar{k}$ is the thermal conductivity of the porous medium.

3 Numerical Solution

The governing equations (3) and (4) subject to the initial and boundary conditions (5) are solved by the Crank-Nicolson implicit finite difference scheme which is appropriate for the parabolic type of equations. The computational domain $\left(0<\tau<\infty\right)$ and $\left(0<\eta<1\right)$ is divided into a mesh of lines parallel to τ and η axes.

By substituting the finite difference approximations to derivatives in equations (3) and (4), the governing equations are transformed to the following algebraic equations:

$$\lambda u_{i+1,j}-\left(2\lambda+\frac{\Delta\tau}{K\phi_1}-\frac{2R}{\varepsilon\phi_1}\right)u_{i,j}+\lambda u_{i-1,j}$$
$$=-\lambda u_{i+1,j+1}+\left(2\lambda+\frac{\Delta\tau}{K\phi_1}+\frac{2R}{\varepsilon\phi_1}\right)u_{i,j+1}-\lambda u_{i-1,j+1}, \tag{6}$$

$$-\lambda\phi_2\theta_{i+1,j+1}+\left(2\lambda\phi_2+2Pe+F\Delta\tau\right)\theta_{i,j+1}-\lambda\phi_2\theta_{i-1,j+1}-\lambda\phi_2\theta_{i+1,j}$$
$$+\left(2\lambda\phi_2-2Pe+F\Delta\tau\right)\theta_{i,j}-\lambda\phi_2\theta_{i-1,j}=R_{i,j}, \tag{7}$$

where, $R_{i,j}=\lambda\phi_1 N_{Br}\left[\left(u_{i+1,j}-u_{i,j}\right)^2+\left(u_{i+1,j+1}-u_{i,j+1}\right)^2\right]+\frac{N_{Br}}{K}\Delta\tau\left[u_{i,j}{}^2+u_{i,j+1}{}^2\right]$,

$\lambda=\Delta\tau/\left(\Delta\eta\right)^2$, $\Delta\tau$ and $\Delta\eta$ are mesh sizes.

The unknowns $u_{i,j+1}$ and $\theta_{i,j+1}$ are not expressed in the known quantities namely, $u_{i-1,j}, u_{i,j}$, $u_{i+1,j}$ and $\theta_{i-1,j}, \theta_{i,j}, \theta_{i+1,j}$ at the time level *j* in equation (6) and (7) explicitly. The equations (6) and (7), written at all interior mesh points form a tridiagonal system of linear algebraic equations, with the following initial and boundary conditions for velocity and temperature fields;

$$u_{i,1}=0,\quad i=1,2,\cdots,q+1$$
$$\left.\begin{aligned}&\left(\beta+\Delta\eta\right)u_{1,j}-\beta u_{2,j}=R_0\Delta\eta\sin\left(\left(j-1\right)\Delta\tau\right)\\&\left(\beta+\Delta\eta\right)u_{q+1,j}-\beta u_{q,j}=0\end{aligned}\right\},\ j=2,\cdots,p+1 \tag{8}$$

$$\theta_{i,1}=0,\quad i=1,2,\cdots,q+1$$
$$\left.\begin{aligned}&\left(\alpha+\Delta\eta\right)\theta_{1,j}-\alpha\theta_{2,j}=0\\&\left(\alpha+\Delta\eta\right)\theta_{q+1,j}-\alpha\theta_{q,j}=\Delta\eta\end{aligned}\right\},\ j=2,\cdots,p+1. \tag{9}$$

For numerical solution for the present problem, the computational domains $0<\tau<\infty$ and $0<\eta<1$ are divided into intervals with step sizes $\Delta\tau=0.002$ and $\Delta\eta=0.002$ for time τ and space η, respectively. In order to check the stability of the finite difference scheme, computations are carried out by changing step sizes slightly. It is seen that no significant change is observed in the results obtained by the numerical scheme. It is also known that implicit Crank-Nicolson scheme is convergent and stable for all values of λ. To check our numerical code, results for velocity field are compared with those obtained analytically by Chauhan and Kumar (2009). It is found that the numerical results are in good agreement with that of analytical ones.

4 Discussion

In this research, the velocity and temperature fields are investigated in a horizontal porous layer bounded by impermeable plates, when the lower plate oscillates in the direction of its length. Numerical results for velocity field are compared with the analytical results obtained by Chauhan and Kumar (2009) in table 1, to validate the numerical code. It is seen that numerical results are in good agreement with that of analytical ones. Figure 1 shows the variations of the velocity profiles in the channel for various values of the pertinent parameters. It is observed that the effect of the permeability, K is to increase the flow in the channel because the Darcy's resistance is inversely proportional to K. Thus as K increases, flow in the channel enhances. Further it is seen that by increasing the value of the slip parameter, β, flow decreases in the channel except in the region near the upper fixed plate where flow enhances by β.

Figures 2-3 depict the variations of the temperature profiles for various values of the parameters. On comparing various curves in the fig. 2, it is seen that the effect of the radiation parameter F or the permeability parameter K is to decrease the temperature at all points in the flow field of the channel. It is seen that the temperature field increases with the increase in the value of the Brinkman number Br because viscous forces generates more energy to enhance fluid temperature in the channel, and it is already known that Br accounts for the relevance of viscous heating. However the effect of the velocity slip parameter β is to reduce the temperature in the channel. Figure 3 shows that temperature near the oscillating lower plate increases by increasing the value of the temperature slip parameter α or the thermal conductivity ratio parameter ϕ_2, while temperature decreases near the upper fixed plate by α or ϕ_2.

The physical quantity of interest to the engineers is the non-dimensional rate of heat transfer. The effects of various pertinent parameters on the rate of heat transfer at the channel walls are depicted in figs. 4-7. It is found that the rate of heat transfer at the lower oscillating wall (Figs. 4-5), increases with the increase in the values of the Brinkman number Br, or viscosity ratio parameter ϕ_1, or time parameter τ, while it decreases with the increase of velocity slip parameter β, or temperature slip parameter α, or radiation parameter F, or the permeability parameter K. Figures 6-7 show that the rate of heat transfer decreases at the upper fixed wall of the channel with the increase in the Br value, becomes zero at certain critical Brinkman number Br^*, then changes sign and further increases in magnitude. It is seen that this critical Brinkman number Br^*, where the transfer of heat changes direction, increases with the increase in F or β or K values, while changes take place at earlier Brinkman number values by the parameters α or ϕ_1 or τ values.

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Table 1 Velocity vs y for $R = 2, R_0 = 1, \beta = 0.1, \varepsilon = 0.6, K = 0.1, \phi_1 = 1.25$

Y	t = 0.1		t = 0.3		t = 0.5	
	Numerical	Analytical	Numerical	Analytical	Numerical	Analytical
0.0	0.059890	0.059490	0.206107	0.205452	0.349420	0.348558
0.2	0.013440	0.013318	0.082770	0.082467	0.162090	0.161659
0.4	0.002190	0.002181	0.030649	0.030518	0.072295	0.072082
0.6	0.000247	0.000225	0.010324	0.010271	0.030702	0.030601
0.8	0.000018	0.000021	0.003103	0.003083	0.011993	0.011945
1	0.000001	0.000014	0.000701	0.000692	0.003227	0.003203

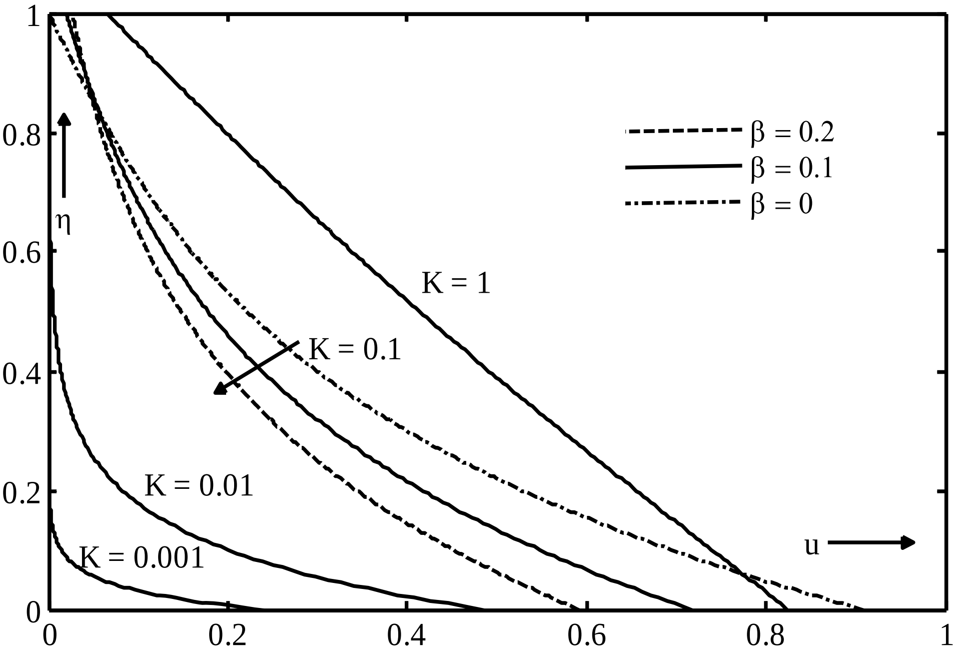

Fig. 1 Velocity vs *y* for $R = 2, R_0 = 1, \varepsilon = 0.6, \phi_1 = 1.25, \tau = 2$

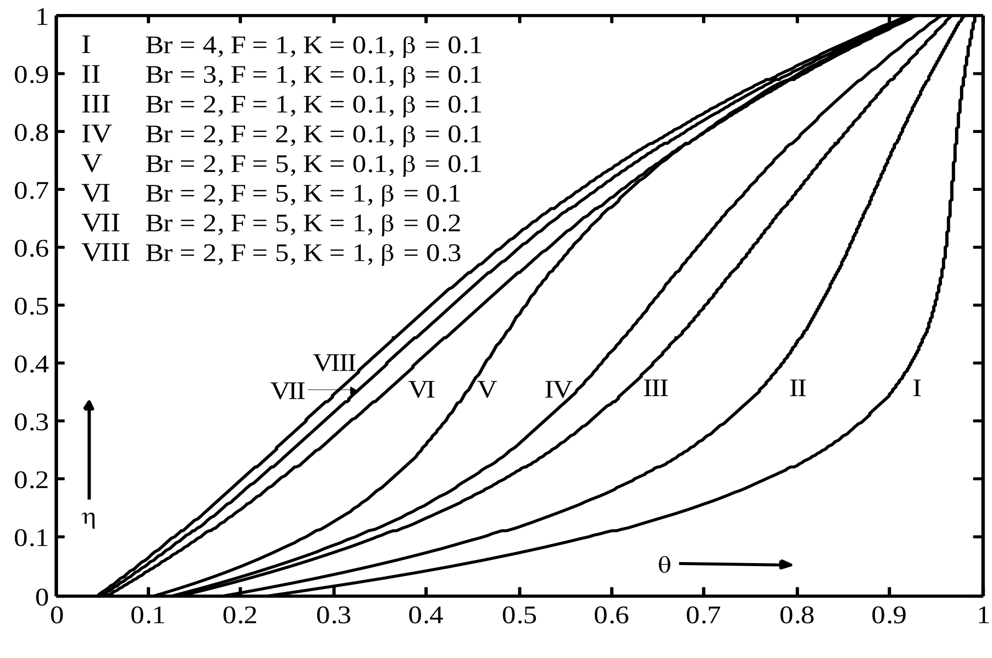

Fig. 2 Temperature vs *y* for $R = 2, R_0 = 1, \tau = 2, \varepsilon = 0.6,$
$\phi_1 = 1.25, Pe = 1, \ \phi_2 = 1.5, \alpha = 0.05$

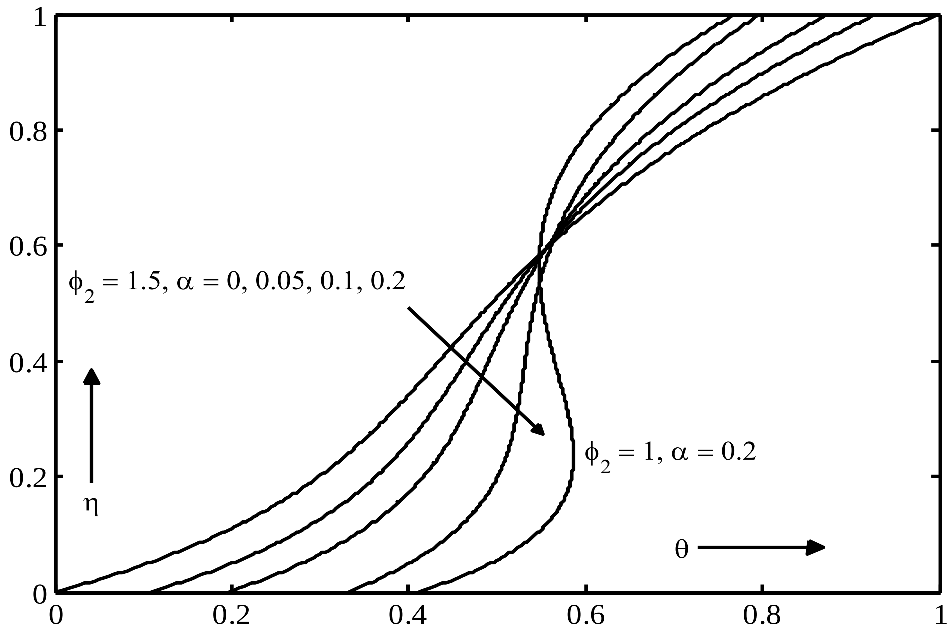

Fig. 3 Temperature vs *y* for $R = 2, R_0 = 1, \tau = 2, \beta = 0.1,$
$\varepsilon = 0.6, K = 0.1, \phi_1 = 1.25, Br = 2, Pe = 1, F = 5$

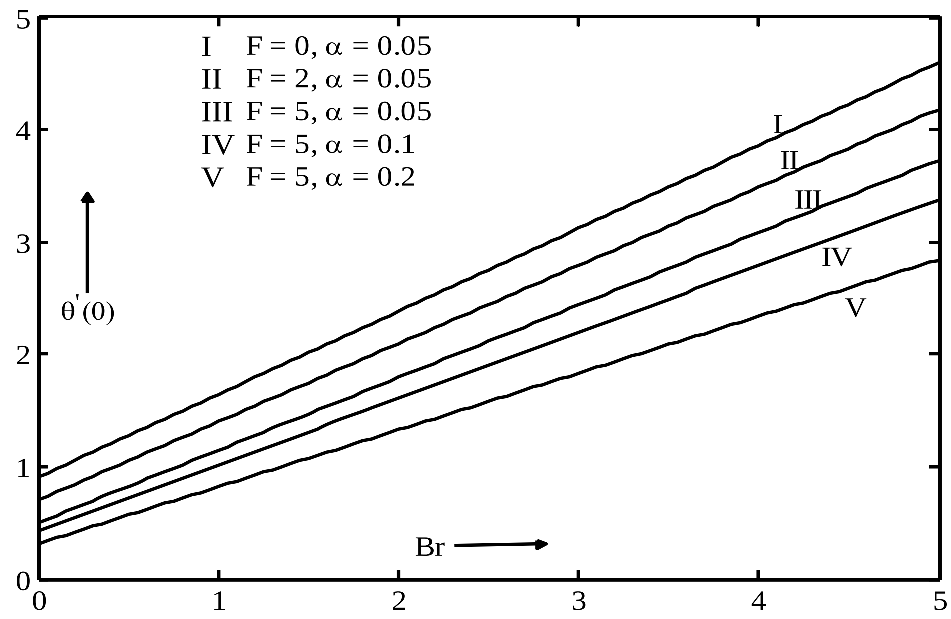

Fig. 4 Rate of heat transfer at the lower plate $\theta'(0)$ vs Br for
$\tau = 1, R = 2, R_0 = 1, \beta = 0.1, \varepsilon = 0.6, K = 0.1, \phi_1 = 1.25, Pe = 1, \phi_2 = 1.5$

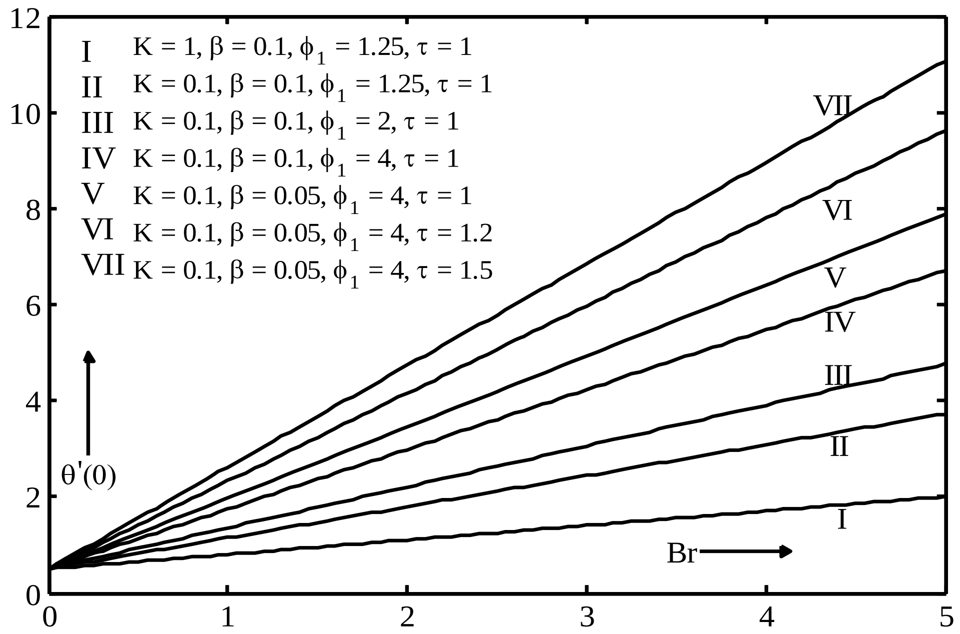

Fig. 5 Rate of heat transfer at the lower plate $\theta'(0)$ vs Br for $R = 2,\ R_0 = 1,\ \varepsilon = 0.6,\ \ Pe = 1,\ \phi_2 = 1.5,\ \alpha = 0.05$

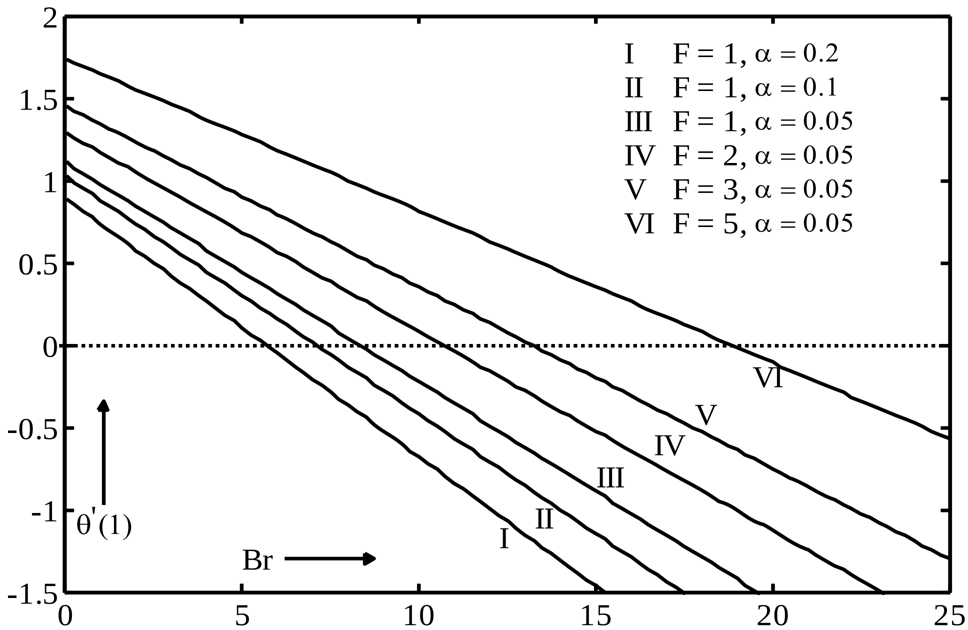

Fig. 6 Rate of heat transfer at the upper plate $\theta'(1)$ vs Br for $\tau = 1,\ R = 2,\ R_0 = 1,$ $\beta = 0.1,\ K = 0.1,\ \varepsilon = 0.6,\ \phi_1 = 1.25,\ Pe = 1,\ \phi_2 = 1.5$

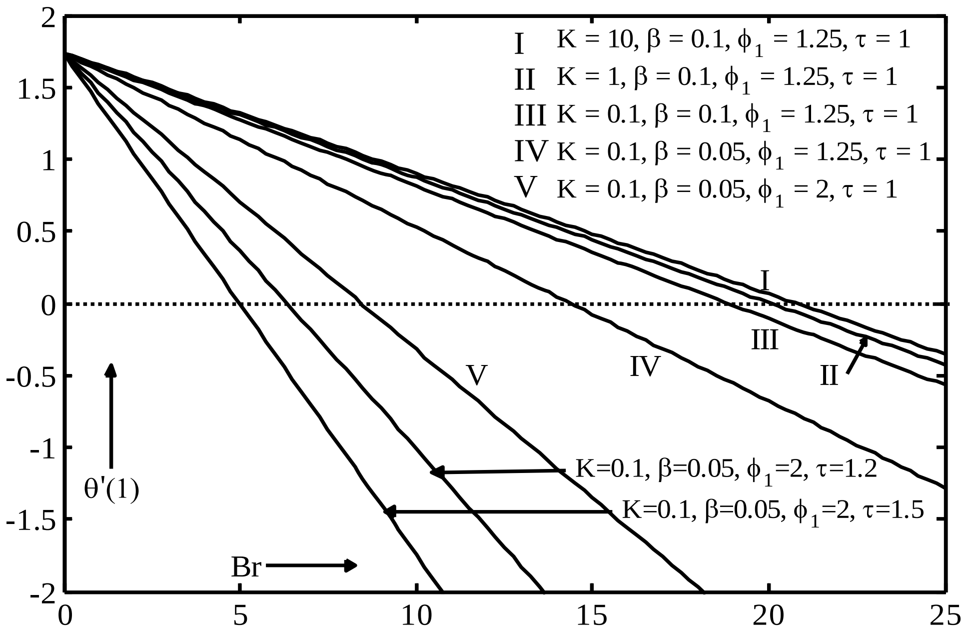

Fig. 7 Rate of heat transfer at the upper plate $\theta'(1)$ vs Br for

$R = 2, R_0 = 1, \varepsilon = 0.6, \ Pe = 1, F = 5, \phi_2 = 1.5, \alpha = 0.05$

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