Applied Mathematical Sciences, Vol. 6, 2012, no. 27, 1303 - 1324

A New Approach for Solving Fuzzy Critical Path Problem Using L-L Fuzzy Numbers

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Abstract

This paper presents Linear programming models to calculate fuzzy earliest and fuzzy latest events time in project scheduling problem with L-L fuzzy numbers as activity times. The solution methodology is to apply a combination of Zadeh's extension principle and linear programming model. The membership function of earliest and latest times of events are derived by calculating lower and upper bounds of earliest and latest times considering different α-cuts of fuzzy duration. The resulting approach avoids generating negative and infeasible solution. A numerical illustration is given to demonstrate superiority of proposed models over existing models in the literature.

Keywords: Linear programming; L-L fuzzy number; critical path; project scheduling

1. Introduction

Time management play more significant role in project management compares to scheduling control, resource management and cost management. Critical Path Method (CPM) is a powerful tool that is most useful in practice and is applied in the planning and control of complicated projects in real world applications [2]. To deal quantitatively with imprecise data, the Program Evaluation and Review Technique (PERT) [10, 17] and Monte Carlo simulation [15] based on the probability theory can be employed. Program Evaluation and Review Technique (PERT) uses different distributions to estimate the durations of activities [9]. In order to use probability distributions, the distributions are modeled based on the

assumption of observation of past performance of activities. When we have activities without past performance observation then PERT is not useful. There are some critiques of PERT [21]. The detailed critiques of PERT can be found in the work of Shipley et al. [21]. An alternative way to deal with imprecise data is to employ the concept of fuzziness [10], whereby the vague activity times can be represented by fuzzy sets. The comparison of the fuzzy approach and stochastic approach to the project scheduling can be found in Shipley et al. [21].

Gazdik [11] developed FNET which is a technique based on a combination of fuzzy sets and theory of graphs. An extension of FNET was proposed by Nasution [12] and Lorterapong and Moselhi [18]. Nasution [12] introduces and interactive fuzzy subtraction in the backward calculations to show that fuzzy numbers could be exploited further in fuzzy network. McCahon [20], Chang, Tsujimura and Tazawa[3], Yao and Lin[22] presented three methodologies to calculate the fuzzy completion time. Chen[6] also proposed an approach based on the extension principle and linear programming formulation. He used fuzzy numbers of L-R and L-L types for activities duration and derived the membership function of fuzzy total duration.

Several studies have investigated the case where activity times in a project are approximately known and more suitably represented by fuzzy sets rather than crisp numbers[8,18]. In particular, the problems of computing the intervals of possible values of the latest starting times and floats of activities with imprecise durations represented by fuzzy or interval numbers have attracted intensively attentions and many solutions methods have been proposed [19,7]. Most of them are straight forward extensions of deterministic CPM. They are mainly based on the CPM with formulas for the forward and backward recursions, in which the deterministic activity times are replaced with the fuzzy activity times. However, as noted by Zielinski [19], the backward recursion fails to compute the sets of possible values of the latest starting times and floats of activities. Moreover, for the same path, different definitions of the fuzzy critical path give different estimations of the degree of criticality.

Dubois et al. [7] proposed several heuristics for computing the sets of possible values of the latest starting times and floats of activities using rigorous formulization of fuzzy PERT. Zielinski [19] developed new polynomial algorithms for determining the intervals of the latest starting times in the general network. Chanas and Zielinski [5] discussed the complexity of criticality, Chanas and Zielinski [14] proposed a natural generalization of the criticality concept for project networks with interval and fuzzy activity times, in which two methods of calculating the degree of possible criticality and some results are provided. The advantage of this method is that it prevents fuzzy numbers from getting larger and also the result of subtraction of each fuzzy number from itself is crisp zero.

Chen and Haung[4] also proposed an approach using positive triangular fuzzy number. This method however does not support backward pass calculations in direct manner similar to that used in the forward pass. This is mainly due to the fact that fuzzy subtraction is not proportionate to the inverse of fuzzy addition. Therefore, this method is incapable of calculating project characteristics such as

the latest times. Abalfazl Zareei[1] et al proposed an approach for solving fuzzy critical path problem using analysis of events for LR fuzzy numbers. In this paper, we present linear programming models to calculate fuzzy earliest and fuzzy latest events time in project scheduling problem with L-L fuzzy numbers as activity times. In addition, we define a new measure of criticality for events and path criticality in project network. In section 2, we present fuzzy concepts, reference functions, and critical path using linear programming, Proposed method of Finding fuzzy earliest and latest times is presented in Section 3. In section 4, criticality degree and path criticality are defined. Numerical example is presented in section 5 to illustrate the linear programming models. In section 6, comparative analysis is presented.

2. Background information

2.1 Fuzzy concepts

Some necessary backgrounds and notions of fuzzy set theory [13] are reviewed.

Definition 1: A classical (crisp) set is normally defined as a collection of elements or objects x∈X which can be finite, countable, or over countable.

Definition 2: If X is a collection of objects denoted generically by x then *a* fuzzy set $\widetilde{A}$ in X is a set of order pairs: $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) / x \in X\}$, $\mu_{\widetilde{A}}(x)$ is called the membership function or grade of membership of x in $\widetilde{A}$ which maps X to the membership space M. When M contains only the two points 0 and 1, $\widetilde{A}$ is non fuzzy and $\mu_{\widetilde{A}}(x)$ is identical to the characteristic function of a non-fuzzy set. The range of the membership function is a subset of the non-negative real number whose supremum is finite.

Definition 3: The support of a fuzzy set $\widetilde{A}$, S ($\widetilde{A}$), is the crisp set of all x∈X such that $\mu_{\widetilde{A}}(x) > 0$.

Definition 4: The (crisp) set of elements that belong to the fuzzy set $\widetilde{A}$ at least to the degree α is called the α-level set : $A_\alpha = \{x \in X / \mu_{\widetilde{A}}(x) \geq \alpha\}$.

Definition 5: A fuzzy set $\widetilde{A}$ is convex if
$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)), x_1, x_2 \in X, \lambda \in [0,1]$.

Definition 6 : A fuzzy set $\widetilde{A}$ is normal if ∃ x_0∈R such that $\mu_{\widetilde{A}}(x_0)$=1.

Definition 7: Let X denote a universal set. Then the fuzzy subset $\widetilde{A}$ of X is defined by its membership function $\mu_{\widetilde{A}} : X \rightarrow [0, 1]$, which assigns a real number $\mu_{\widetilde{A}}(x)$ in the interval [0, 1], to each element $x \in X$, where the value of $\mu_{\widetilde{A}}(x)$ at x shows the grade of membership of x in $\widetilde{A}$. A fuzzy number is a convex normalized fuzzy set of the real line R whose membership function is piecewise continuous.

Definition 8: A fuzzy number $\widetilde{A}$ = (*a*,b,c,d) is a fuzzy set defined on the set of real numbers R characterized by means of a membership function $\mu_{\widetilde{A}}(x), \mu_{\widetilde{A}} : R \rightarrow [0,1]$:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & for \quad x \le a, \\ f_{\widetilde{A}}(x) & for \quad a \le x \le b, \\ 1 & for \quad b \le x \le c, \\ g_{\widetilde{A}}(x) & for \quad c \le x \le d, \\ 0 & for \quad x \ge d, \end{cases}$$

where $f_{\widetilde{A}}$ and $g_{\widetilde{A}}$ are continuous functions, $f_{\widetilde{A}}$ is increasing (from 0 to 1), $g_{\widetilde{A}}$ is decreasing (from 1 to 0).

Definition 9: A fuzzy set $\widetilde{A}$=($x^L_{\alpha=1}$, $x^U_{\alpha=1}$,ls,rs)$_{LR}$ is called a fuzzy number of L-R type with the subset [$x^L_{\alpha=1}$, $x^U_{\alpha=1}$] consisting of the real numbers with the highest chance of realization, and ls and rs being the left shape and right spreads that are non-negative real numbers, respectively,if its membership function $\mu_{\widetilde{A}}(x)$ has the form as follows: $\mu_{\widetilde{A}}(x) = \begin{cases} L\left(\dfrac{x^L_{\alpha=1} - x}{ls}\right) & for \;\; x \le x^L_{\alpha=1} \\ 1 & for \;\; x^L_{\alpha=1} \le x \le x^U_{\alpha=1}, \\ R\left(\dfrac{x^U_{\alpha=1} - x}{rs}\right) & for \;\; x \ge x^U_{\alpha=1} \end{cases}$

where L and R are continuous, non-increasing functions that define the left and right shapes of $\mu_{\widetilde{A}}(x)$, respectively; L(0)=R(0)=1.

Definition 10 : A fuzzy set $\widetilde{A}$=($x^L_{\alpha=1}$, $x^U_{\alpha=1}$,ls,rs)$_{LL}$ is called a fuzzy number of L-L type with the subset [$x^L_{\alpha=1}$, $x^U_{\alpha=1}$] consisting of the real numbers with the highest chance of realization, and ls and rs being the left shape and right spreads that are

non-negative real numbers, respectively, if its membership function $\mu_{\tilde{A}}(x)$ has

the form as follows: $\mu_{\tilde{A}}(x)=\begin{cases} L\left(\dfrac{x^L_{\alpha=1}-x}{ls}\right) & for\ \ x\le x^L_{\alpha=1} \\ 1 & for\ \ x^L_{\alpha=1}\le x\le x^U_{\alpha=1}, \\ L\left(\dfrac{x^U_{\alpha=1}-x}{rs}\right) & for\ \ x\ge x^U_{\alpha=1} \end{cases}$

where we assume $x^L_{\alpha=1}\le x^U_{\alpha=1}, x^L_{\alpha=1}>0$ and $x^U_{\alpha=1}>0$. L:[0,∞)→[0,1] is a reference function such that
i) L(0)=1,
ii) L is upper semi-continuous,
iii)L is non-increasing,
iv) $\lim_{r\to\infty}$ L(r)=0.

Definition 11 : One of the most basic concepts of fuzzy set theory which can be used to generalize crisp mathematical concepts to fuzzy sets is the extension principle.
Let X be a Cartesian product of Universes $X = X_1,\ldots X_r$, and $\tilde{A}_1,...,\tilde{A}_r$ be r fuzzy sets in $X_1, \ldots, X_r$, respectively. f is a mapping from X to a Universe Y, $y = f(x_1,\ldots,x_r)$. Then the extension principle of a fuzzy set $\tilde{B}$ in Y by
$\tilde{B}=\{(y,\mu_{\tilde{B}}(y))/\, y=f(x_1,...,x_r),(x_1,...,x_r)\in X\}$

where $\mu_{\tilde{B}}(y)=\left\{\begin{matrix}\sup\limits_{(x_1,...,x_r)\in f^{-1}(y)} \min\{\mu_{\tilde{A}_1}(x_1),...,\mu_{\tilde{A}}(x_r)\}, iff^{-1}(y)\neq\phi \\ 0, otherwise\end{matrix}\right\}$,

where f^{-1} is the inverse of f.
For r =1, the extension principle, of course, reduces to
$\tilde{B}=f(\tilde{A})=\{(y,\mu_{\tilde{B}}(y))/\, y=f(x), x\in X$

where $\mu_{\tilde{B}}(y)=\left\{\begin{matrix}\sup\limits_{x\in f^{-1}(y)} \mu_{\tilde{A}}(x),\ if\quad f^{-1}(y)\neq\phi \\ 0, otherwise\end{matrix}\right\}$

2.2 Reference functions

If the set $[t^L_{\alpha=1}, t^U_{\alpha=1}]$ is a singleton, that is $t^L_{\alpha=1}=t^U_{\alpha=1}=t_{\alpha=1}$, the fuzzy number is represented as $\tilde{T}=(t_{\alpha=1}, ls, rs)_{LR}$.

A function that defines the left and right shapes of a fuzzy number is called the reference function of this fuzzy number. Two special cases are triangular (the set $[t_{\alpha=1}^{L}, t_{\alpha=1}^{U}]$ is a singleton) and trapezoidal fuzzy number, for which L(x)=R(x)= $\max\{0,1-|x|\}$ are linear functions. Five commonly used linear and nonlinear reference functions with parameter p, denoted as RF_p, and are summarized as follows [19]:

Linear: $RF_p(x) = \max\{0,1-|x|\}$ (a)

Exponential: $RF_p(x) = e^{-px}, \quad p \geq 1,$ (b)

Power: $RF_p(x) = \max(0,1-x^p), \quad p \geq 1,$ (c)

Exponential power: $RF_p(x) = e^{-x^p}, p \geq 1,$ (d)

Rational: $RF_p(x) = 1/(1+x^p), \quad p \geq 1.$ (e)

The proposed approach involves the α-cuts of activity times being L-R fuzzy numbers $\widetilde{T}_{ij} = ((t_{ij})_{\alpha=1}^{L}, (t_{ij})_{\alpha=1}^{U}, ls_{ij}, rs_{ij})_{L_{ij}-R_{ij}} \quad \forall (i,j) \in A.$ if these two inverse functions L^{-1} *and* R^{-1} exists, clearly they can be represented as [19]

$$(T_{ij})_{\alpha} = [(T_{ij})_{\alpha}^{L}, (T_{ij})_{\alpha}^{U}] = [(t_{ij})_{\alpha=1}^{L} - L_{ij}^{-1}(\alpha).ls_{ij}, (t_{ij})_{\alpha=1}^{U} + R_{ij}^{-1}(\alpha).rs_{ij}], \quad \text{(f)}$$

All of the inverse function of the linear and non-linear reference functions listed in (a)-(e) exist and are as follows:

For linear: $RF_p^{-1}(\alpha) = 1-\alpha, \quad \alpha \in (0,1],$ (a')

For exponential: $RF_p^{-1}(\alpha) = -(\ln\alpha)/p, \alpha \in (0,1],$ (b')

For power: $RF_p^{-1}(\alpha) = \sqrt[p]{1-\alpha}, \alpha \in (0,1],$ (c')

For exponential power: $RF_p^{-1}(\alpha) = \sqrt[p]{-\ln\alpha}, \alpha \in (0,1],$ (d')

For rational: $RF_p^{-1}(\alpha) = \sqrt[p]{(1-\alpha)/\alpha}, \alpha \in (0,1].$ (e')

2.3 Critical path method using linear programming

Consider a project network S = <V, A, t> consisting of a finite set V of nodes and a set $A \subset V \times V$ of arcs with crisp activity times, which are determined by a function $t : A \rightarrow R^{+} \cup \{0\}$ and attached to the arcs. Denote t_{ij} as the time period of activity $(i, j) \in A$. The Linear programming formulation assumes that a unit flow enters the project at the start node and leaves at the finish node. Let x_{ij} be the decision variable denoting the amount of flow in $(i, j) \in A$. Since only one unit of flow could be in any arc at any one time, the variable x_{ij} must assume binary values (0 or 1) only. The CPM problem with n nodes is formulated as

$$D = \max \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} x_{ij}$$

$$\text{s.t} \quad \sum_{j=1}^{n} x_{1j} = 1 \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = \sum_{k=1}^{n} x_{ki}, i = 2,..., n-1$$

$$\sum_{k=1}^{n} x_{kn} = 1,$$

$$x_{ij} = 0 \text{ or } 1, (i, j) \in A.$$

The objective is to maximize the total duration time of the project network from node 1 to node n. The constraints are called the flow conservation equations and indicate the flow may be neither created nor destroyed in the project network. The critical path for this project network consists of a set of activities $(i, j) \in A$ from the start to the finish in which each activity in the path corresponds to the optimal decision variable $x_{ij}^* = 1$ in the optimal solution to Eq. (1). The total duration time needed to complete the project is given as the maximal objective value D of Eq.(1).

Consider a project network $S_f = \langle V, A, \widetilde{T} \rangle$ with fuzzy activity times. V and A are the same as in the crisp case except that the activity time are approximately known and defined by function $\widetilde{T} : A \to FN(R^+)$, where $FN(R^+)$ is the set of non-negative fuzzy numbers. Denote fuzzy number $\widetilde{T}_{ij}$ as the fuzzy duration time of activity $(i, j) \in A,$ and its membership function is $\mu_{\widetilde{T}_{ij}}(t_{ij})$. We then have

$$\widetilde{T}_{ij} = \left\{ (t_{ij}, \mu_{\widetilde{T}_{ij}}(t_{ij})) / t_{ij} \in S(\widetilde{T}_{ij}) \right\}, \quad (i, j) \in A,$$

where, $S(\widetilde{T}_{ij})$ is the support of $\widetilde{T}_{ij}$, which denotes the universe set of the activity time of activity $(i, j) \in A.$ Consequently, the fuzzy CPM problem is of the following form:

$$\begin{aligned}
\widetilde{D} = \max \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\widetilde{T}_{ij}x_{ij} \\
s.t. \quad & \sum_{j=1}^{n}x_{ij} = 1, \\
& \sum_{j=1}^{n}x_{ij} = \sum_{k=1}^{n}x_{ki}, \quad i = 2,...,n-1, \\
& \sum_{k=1}^{n}x_{kn} = 1, \\
& x_{ij} = 0 \ \ or \ \ 1, \quad (i,j) \in A.
\end{aligned} \tag{2}$$

Similar to Eq. (1), the objective is to maximize the total duration time of the project network from node 1 to node n except that the total duration times are fuzzy. That is, the optimal solution to Eq.(2) can be used to identify the critical path with longest path length in the set of all paths in $S_f = \langle V, A, \widetilde{T} \rangle$ denote as P_f. The critical path for the project network is called fuzzy critical path.

3. Proposed method of finding fuzzy earliest and latest times

Linear programming model for calculation of earliest events time in a project network with n nodes is defined as follows:

$$\begin{aligned}
& Z_1 = \min(E_1 + E_2 + ... + E_n) \\
& \quad subject\ to\ E_j \geq E_i + T_{ij} \\
& \quad E_i, E_j \geq 0 \ \ \forall (i,j) \in A.
\end{aligned} \tag{3}$$

Linear programming model for calculation of latest times and the objective changes to find the maximum quantity of events time:

$$\begin{aligned}
& Z_2 = \max(L_1 + L_2 + ... + L_n) \\
& \quad subject\ to\ L_j \geq L_i + T_{ij} \\
& \quad L_i, L_j \geq 0 \ \ \forall (i,j) \in A.
\end{aligned} \tag{4}$$

Denote the fuzzy activity durations of activity (i,j) as $\widetilde{T}_{ij}$ with membership function $\mu_{\widetilde{T}_{ij}}(t_{ij})$ and consider A as the set of all activities in project network. Then we have:

$$\widetilde{T}_{ij} = \left\{ (t_{ij}, \mu_{\widetilde{T}_{ij}}(t_{ij})) / t_{ij} \in S(\widetilde{T}_{ij}) \right\}, \quad (i,j) \in A, \tag{5}$$

$S(\widetilde{T}_{ij})$ is the support of fuzzy number $\widetilde{T}_{ij}$, and consists of all the members of fuzzy set $\widetilde{T}_{ij}$ with membership degree greater than zero.

Using fuzzy number as activities duration, Eq.(3) will be rearranged as follows:

$$\begin{aligned} \widetilde{z}_1 &= \min(\widetilde{E}_1 + \widetilde{E}_2 + ... + \widetilde{E}_n) \\ &\textit{subject to } \ \widetilde{E}_j \geq \widetilde{E}_i + \widetilde{T}_{ij}. \end{aligned} \tag{6}$$

Similar to Eq. (3), the objective is finding the lowest value of earliest event times. The difference to Eq.(3) is that the activities duration are fuzzy.

Assume $\widetilde{E}_k$ as the earliest time of event $k(1 \leq k \leq n)$. In order to derive membership function of $\widetilde{E}_k$, we calculate the quantity of membership function in difference α-cut of $\widetilde{T}_{ij}$ is as follows[4]:

$$\begin{aligned} (\widetilde{T}_{ij}) &= [\inf_{t_{ij}} \{t_{ij} \in S(\widetilde{T}_{ij}) / \mu_{\widetilde{T}_{ij}} \geq \alpha\}, \sup_{t_{ij}} \{t_{ij} \in S(\widetilde{T}_{ij}) / \mu_{\widetilde{T}_{ij}} \geq \alpha\}] \\ &= [(\widetilde{T}_{ij})_\alpha^L, (\widetilde{T}_{ij})_\alpha^U] \end{aligned} \tag{7}$$

On the basis of Zadeh's extension principle, the membership function $\mu_{\widetilde{E}_k}(d)$ is defined as follows:

$$\mu_{\widetilde{E}_k} = \sup_{t_{ij} \in R, (ij) \in A} \min_{(ij) \in A} \{\mu_{\widetilde{E}_k}(d) / d = D(t)\}. \tag{8}$$

where t is the vector of activities.

We find the lower limit $(E_k)_\alpha^L$ and upper limit $(E_k)_\alpha^U$ of α-cut of $\mu_{\widetilde{E}_k}(d)$:

$$(\widetilde{E}_k)_\alpha^L = \min\{E_k(t) / (\widetilde{T}_{ij})_\alpha^L \leq t_{ij} \leq (\widetilde{T}_{ij})_\alpha^U \quad \forall (i, j) \in A\}, \tag{9}$$

$$(\widetilde{E}_k)_\alpha^U = \max\{E_k(t) / (\widetilde{T}_{ij})_\alpha^L \leq t_{ij} \leq (\widetilde{T}_{ij})_\alpha^U \quad \forall (i, j) \in A\}. \tag{10}$$

Occurrence time of events is relevant activity duration and any increase in activity duration causes increase in events occurrence time, therefore Eqs. (9) and (10) rearrange to the following forms:

$$(\widetilde{E}_k)_\alpha^L = \min\{E_k(t) / t_{ij} = (\widetilde{T}_{ij})_\alpha^L \quad \forall (i, j) \in A\}, \tag{11}$$

$$(\widetilde{E}_k)_\alpha^L = \max\{E_k(t) / t_{ij} = (\widetilde{T}_{ij})_\alpha^U \quad \forall (i, j) \in A\}. \tag{12}$$

Eq. (6) will be rearranged as follows to derive membership function of earliest times.

$$\begin{aligned} \widetilde{z}_{1\alpha}^L &= \min((\widetilde{E}_1)_\alpha^L + (\widetilde{E}_2)_\alpha^L + ... + (\widetilde{E}_n)_\alpha^L) \\ &\textit{subject to } \ (\widetilde{E}_j)_\alpha^L \geq (\widetilde{E}_i)_\alpha^L + (\widetilde{T}_{ij})_\alpha^L. \end{aligned} \tag{13}$$

$$\begin{aligned} \widetilde{z}_{1\alpha}^U &= \min((\widetilde{E}_1)_\alpha^U + (\widetilde{E}_2)_\alpha^U + ... + (\widetilde{E}_n)_\alpha^U) \\ &\textit{subject to } \ (\widetilde{E}_j)_\alpha^U \geq (\widetilde{E}_i)_\alpha^U + (\widetilde{T}_{ij})_\alpha^U. \end{aligned} \tag{14}$$

To find the membership function of latest times ($\mu_{\widetilde{L}_k}(d)$), we have the sequence as what we had for calculation of earliest times. First using fuzzy numbers as activity duration, Eq.(4) changes to following form:

$$\begin{aligned} &\tilde{z}_2 = \max(\tilde{L}_1 + \tilde{L}_2 + ... + \tilde{L}_n) \\ &\quad subject\ to\ \ \tilde{L}_j \geq \tilde{L}_i + \tilde{T}_{ij} \\ &\quad\quad \tilde{L}_n = \tilde{E}_n \\ &\quad\quad \tilde{L}_1 = \tilde{0}. \end{aligned} \tag{15}$$

Note that $\tilde{0}$ is a crisp zero $(\tilde{0} = (0,0,0)_{L-L})$.

Employing Zadeh's extension principle and different α-cuts of $\tilde{T}_{ij}$, rearrange (15) to the following forms:

$$\begin{aligned} &\tilde{z}_{2\alpha}^{\ L} = \max((\tilde{L}_1)_\alpha^L + (\tilde{L}_2)_\alpha^L + ... + (\tilde{L}_{n-1})_\alpha^L) \\ &\quad subject\ to\ \ (\tilde{L}_j)_\alpha^L \geq (\tilde{L}_i)_\alpha^L + (\tilde{T}_{ij})_\alpha^L \\ &\quad\quad (\tilde{L}_n)_\alpha^L = (\tilde{E}_n)_\alpha^L \\ &\quad\quad (\tilde{L}_1)_\alpha^L = 0. \end{aligned} \tag{16}$$

$$\begin{aligned} &\tilde{z}_{2\alpha}^{\ U} = \max((\tilde{L}_1)_\alpha^U + (\tilde{L}_2)_\alpha^U + ... + (\tilde{L}_{n-1})_\alpha^U) \\ &\quad s.t.(\tilde{L}_j)_\alpha^U \geq (\tilde{L}_i)_\alpha^U + (\tilde{T}_{ij})_\alpha^U \\ &\quad\quad (\tilde{L}_n)_\alpha^U = (\tilde{E}_n)_\alpha^U \\ &\quad\quad (\tilde{L}_n)_\alpha^U = 0. \end{aligned} \tag{17}$$

where (16) and (17) are solvable after the calculation of lower and upper limit of earliest time of node n.

Adding surplus variables S_k to constraints of (3) would rearrange them as follows:

$$\tilde{E}_j = \tilde{E}_i + \tilde{T}_{ij} + \tilde{S}_k. \tag{18}$$

We have known that the result of fuzzy addition and subtraction of L-L fuzzy numbers (as in our case), are LL fuzzy number [8]. The earliest and latest time of event 1 is crisp zero and it can be shown by $(\tilde{0} = (0,0,0)_{L-L})$. Eq.(18) shows the precedence relationship between activities which illustrates that any change in activities duration will affect occurrence times of events. Therefore, earliest time of events is L-L fuzzy number.

4. Criticality degree for events and path criticality in fuzzy project network

Upper Total float time with α = $(T_k)_\alpha^U = (L_k)_\alpha^U - (E_k)_\alpha^U$ and lower total float time with α = $(T_k)_\alpha^L = (L_k)_\alpha^L - (E_k)_\alpha^L$. To calculate the degree of criticality for events, the value of

$$T_i = \sum_{\alpha} \left[(T_k)_{\alpha}^{L} + (T_k)_{\alpha}^{U} \right], k = 1,2,...,n. \tag{19}$$

The criticality degree for event i can be obtained as below :

$$\text{CD (i)} = \frac{T_i}{Max\{T_i / i = 1,...,n\}} \tag{20}$$

In order to find critical path, critical events to be found, then critical path will be observed. The path criticality of the k^{th} path (P_k) , PC(k) : k=1,2,…,m can be defined as follows :

$$PC(k) = 1 - \text{Max } \{CD(i) / i \in P_k\} \tag{21}$$

where m is the number of paths in the fuzzy project network.
The path with PC(k) =minimum is the critical path.

5. Numerical example

To confirm the validity of proposed method an example which was studied by Chen [6] is experimented. Project network is shown in Fig. 1.activities duration ($\widetilde{T}_{ij}$) expressed as L-L fuzzy numbers given in Table I with its left shape ($\widetilde{L}_{ij}$) function.

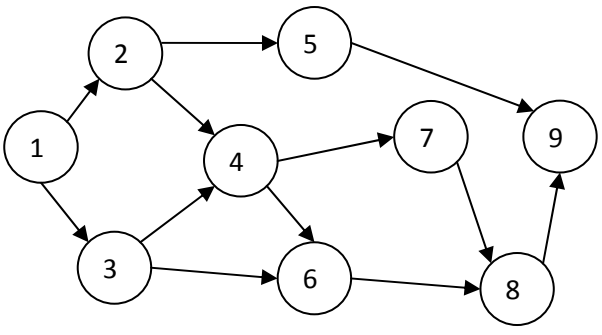

Fig . 1. The network structure of project.

According to Eqs.(13) and (16), the problem is formulated as follows :

$$\widetilde{Z}_{1\alpha}^{\,L} = \min((\widetilde{E}_1)_\alpha^L + (\widetilde{E}_2)_\alpha^L + \ldots + (\widetilde{E}_9)_\alpha^L)$$

$$\begin{aligned}
s.t. \quad & (\widetilde{E}_2)_\alpha^L \geq (\widetilde{E}_1)_\alpha^L + (1-\sqrt{1-\alpha}),\\
& (\widetilde{E}_3)_\alpha^L \geq (\widetilde{E}_1)_\alpha^L + 2,\\
& (\widetilde{E}_4)_\alpha^L \geq (\widetilde{E}_2)_\alpha^L,\\
& (\widetilde{E}_4)_\alpha^L \geq (\widetilde{E}_3)_\alpha^L,\\
& (\widetilde{E}_5)_\alpha^L \geq (\widetilde{E}_2)_\alpha^L + (2-\sqrt{1-\alpha}),\\
& (\widetilde{E}_6)_\alpha^L \geq (\widetilde{E}_3)_\alpha^L + 6,\\
& (\widetilde{E}_6)_\alpha^L \geq (\widetilde{E}_4)_\alpha^L + (5-\sqrt{1-\alpha}),\\
& (\widetilde{E}_7)_\alpha^L \geq (\widetilde{E}_4)_\alpha^L + (9-\sqrt{1-\alpha}),\\
& (\widetilde{E}_8)_\alpha^L \geq (\widetilde{E}_6)_\alpha^L + (4-2\sqrt{1-\alpha}),\\
& (\widetilde{E}_8)_\alpha^L \geq (\widetilde{E}_7)_\alpha^L + (3-2\sqrt{1-\alpha}),\\
& (\widetilde{E}_9)_\alpha^L \geq (\widetilde{E}_5)_\alpha^L + (8-2\sqrt{1-\alpha}),\\
& (\widetilde{E}_9)_\alpha^L \geq (\widetilde{E}_8)_\alpha^L + (6-2\sqrt{1-\alpha})
\end{aligned}$$

$$\widetilde{Z}_{2\alpha}^{\,L} = \min((\widetilde{L}_1)_\alpha^L + (\widetilde{L}_2)_\alpha^L + \ldots + (\widetilde{L}_9)_\alpha^L)$$

$$\begin{aligned}
s.t. \quad & (\widetilde{L}_2)_\alpha^L \geq (\widetilde{L}_1)_\alpha^L + (1-\sqrt{1-\alpha}),\\
& (\widetilde{L}_3)_\alpha^L \geq (\widetilde{L}_1)_\alpha^L + 2,\\
& (\widetilde{L}_4)_\alpha^L \geq (\widetilde{L}_2)_\alpha^L,\\
& (\widetilde{L}_4)_\alpha^L \geq (\widetilde{L}_3)_\alpha^L,\\
& (\widetilde{L}_5)_\alpha^L \geq (\widetilde{L}_2)_\alpha^L + (2-\sqrt{1-\alpha}),\\
& (\widetilde{L}_6)_\alpha^L \geq (\widetilde{L}_3)_\alpha^L + 6,\\
& (\widetilde{L}_6)_\alpha^L \geq (\widetilde{L}_4)_\alpha^L + (5-\sqrt{1-\alpha}),\\
& (\widetilde{L}_7)_\alpha^L \geq (\widetilde{L}_4)_\alpha^L + (9-\sqrt{1-\alpha}),\\
& (L_8)_\alpha^L \geq (\widetilde{L}_6)_\alpha^L + (4-2\sqrt{1-\alpha}),\\
& (\widetilde{L}_8)_\alpha^L \geq (\widetilde{L}_7)_\alpha^L + (3-2\sqrt{1-\alpha}),\\
& (\widetilde{L}_9)_\alpha^L \geq (\widetilde{L}_5)_\alpha^L + (8-2\sqrt{1-\alpha}),\\
& (\widetilde{L}_9)_\alpha^L \geq (\widetilde{L}_8)_\alpha^L + (6-2\sqrt{1-\alpha})\\
& (\widetilde{L}_9)_\alpha^L = (\widetilde{E}_9)_\alpha^L
\end{aligned}$$

Using the Eqs. (14) and (17) , the problem is formulated as follows :

$$\widetilde{Z}_{1\alpha}^{U} = \min((\widetilde{E}_1)_\alpha^U + (\widetilde{E}_2)_\alpha^U + ... + (\widetilde{E}_9)_\alpha^U)$$

$$s.t. \quad (\widetilde{E}_2)_\alpha^U \geq (\widetilde{E}_1)_\alpha^U + (1.5 + \sqrt{1-\alpha}),$$
$$(\widetilde{E}_3)_\alpha^U \geq (\widetilde{E}_1)_\alpha^U + (3 + 2\sqrt{1-\alpha}),$$
$$(\widetilde{E}_4)_\alpha^U \geq (\widetilde{E}_2)_\alpha^U,$$
$$(\widetilde{E}_4)_\alpha^U \geq (\widetilde{E}_3)_\alpha^U,$$
$$(\widetilde{E}_5)_\alpha^U \geq (\widetilde{E}_2)_\alpha^U + (3 + 2\sqrt{1-\alpha}),$$
$$(\widetilde{E}_6)_\alpha^U \geq (\widetilde{E}_3)_\alpha^U + (7 + 2\sqrt{1-\alpha}),$$
$$(\widetilde{E}_6)_\alpha^U \geq (\widetilde{E}_4)_\alpha^U + (5 - \sqrt{1-\alpha}),$$
$$(\widetilde{E}_7)_\alpha^U \geq (\widetilde{E}_4)_\alpha^U + (9 + \sqrt{1-\alpha}),$$
$$(\widetilde{E}_8)_\alpha^U \geq (\widetilde{E}_6)_\alpha^U + (4 + 2\sqrt{1-\alpha}),$$
$$(\widetilde{E}_8)_\alpha^U \geq (\widetilde{E}_7)_\alpha^U + 4,$$
$$(\widetilde{E}_9)_\alpha^U \geq (\widetilde{E}_5)_\alpha^U + (9 + 4\sqrt{1-\alpha}),$$
$$(\widetilde{E}_9)_\alpha^U \geq (\widetilde{E}_8)_\alpha^U + (9 + 3\sqrt{1-\alpha})$$

$$\widetilde{Z}_{2\alpha}^{U} = \max((\widetilde{L}_1)_\alpha^U + (\widetilde{L}_2)_\alpha^U + ... + (\widetilde{L}_9)_\alpha^U)$$

$$s.t. \quad (\widetilde{L}_2)_\alpha^U \geq (\widetilde{L}_1)_\alpha^U + (1.5 + \sqrt{1-\alpha}),$$
$$(\widetilde{L}_3)_\alpha^U \geq (\widetilde{L}_1)_\alpha^U + (3 + 2\sqrt{1-\alpha}),$$
$$(\widetilde{L}_4)_\alpha^U \geq (\widetilde{L}_2)_\alpha^U,$$
$$(\widetilde{L}_4)_\alpha^U \geq (\widetilde{L}_3)_\alpha^U,$$
$$(\widetilde{L}_5)_\alpha^U \geq (\widetilde{L}_2)_\alpha^U + (3 + 2\sqrt{1-\alpha}),$$
$$(\widetilde{L}_6)_\alpha^U \geq (\widetilde{L}_3)_\alpha^U + (7 + 2\sqrt{1-\alpha}),$$
$$(\widetilde{L}_6)_\alpha^U \geq (\widetilde{L}_4)_\alpha^U + (5 - \sqrt{1-\alpha}),$$
$$(\widetilde{L}_7)_\alpha^U \geq (\widetilde{L}_4)_\alpha^U + (9 + \sqrt{1-\alpha}),$$
$$(\widetilde{L}_8)_\alpha^U \geq (\widetilde{L}_6)_\alpha^U + (4 + 2\sqrt{1-\alpha}),$$
$$(\widetilde{L}_8)_\alpha^U \geq (\widetilde{L}_7)_\alpha^U + 4,$$
$$(\widetilde{L}_9)_\alpha^U \geq (\widetilde{L}_5)_\alpha^U + (9 + 4\sqrt{1-\alpha}),$$
$$(\widetilde{L}_9)_\alpha^U \geq (\widetilde{L}_8)_\alpha^U + (9 + 3\sqrt{1-\alpha})$$
$$(\widetilde{L}_9)_\alpha^U = (\widetilde{E}_9)_\alpha^U$$

The lower and upper bound of earliest times and latest times, in different α-cuts of L shape functions of activities duration are calculated and shown in Tables II, III, IV and V.

A Tora software [13] is used to solve the above linear programming models. To find the criticality degree of each event and path criticality , we calculate $T_{i\alpha}^{L}$ *and* $T_{i\alpha}^{U}$ for each α and tabulated in Tables VI and VII. Criticality degree of each event is calculated and presented in Table VIII. Shapes of earliest time of events 2, 3, 5,6 and 9 are shown in Figs.2 to 6. We represent all the figures using Scatter plot to show them.

Table I : The fuzzy activity times

$\widetilde{T}_{ij}$	$\widetilde{L}_{ij}(x)$	$\widetilde{L}_{ij}^{-1}(\alpha)$
$\widetilde{T}_{12}=(1,1.5,1,1)_{L_{12}-L_{12}}$	$L_{ij}(x)=\max(1-x^2,0)$	$L_{ij}^{-1}(\alpha)=1-\sqrt{1-\alpha}$
$\widetilde{T}_{13}=(2,3,0,2)_{L_{13}-L_{13}}$	$L_{ij}(x)=e^{-x}$	$L_{ij}^{-1}(\alpha)=2$
$\widetilde{T}_{24}=(0,0,0,0)_{L_{24}-L_{24}}$	$L_{ij}(x)=\max(1-x,0)$	$L_{ij}^{-1}(\alpha)=0$
$\widetilde{T}_{25}=(2,3,1,2)_{L_{12}-L_{12}}$	$L_{ij}(x)=\max(1-x^4,0)$	$L_{ij}^{-1}(\alpha)=1-\sqrt[4]{1-\alpha}$
$\widetilde{T}_{34}=(0,0,0,0)_{L_{34}-L_{34}}$	$L_{ij}(x)=\max(1-x,0)$	$L_{ij}^{-1}(\alpha)=0$
$\widetilde{T}_{36}=(1,1.5,1,1)_{L_{36}-L_{36}}$	$L_{ij}(x)=e^{-x^2}$	$L_{ij}^{-1}(\alpha)=6$
$\widetilde{T}_{46}=(5,5,1,1)_{L_{46}-L_{46}}$	$L_{ij}(x)=\max(1-x,0)$	$L_{ij}^{-1}(\alpha)=4+\alpha$
$\widetilde{T}_{36}=(1,1.5,1,1)_{L_{36}-L_{36}}$	$L_{ij}(x)=\max(1-x^4,0)$	$L_{ij}^{-1}(\alpha)=9-\sqrt[4]{1-\alpha}$
$\widetilde{T}_{12}=(1,1.5,1,1)_{L_{12}-L_{12}}$	$L_{ij}(x)=\max(1-x^4,0)$	$L_{ij}^{-1}(\alpha)=8-2\sqrt[4]{1-\alpha}$
$\widetilde{T}_{12}=(1,1.5,1,1)_{L_{12}-L_{12}}$	$L_{ij}(x)=\max(1-x^2,0)$	$L_{ij}^{-1}(\alpha)=4-2\sqrt{1-\alpha}$
$\widetilde{T}_{12}=(1,1.5,1,1)_{L_{12}-L_{12}}$	$L_{ij}(x)=\max(1-x,0)$	$L_{ij}^{-1}(\alpha)=1+2\alpha$
$\widetilde{T}_{12}=(1,1.5,1,1)_{L_{12}-L_{12}}$	$L_{ij}(x)=\max(1-x^2,0)$	$L_{ij}^{-1}(\alpha)=6-2\sqrt{1-\alpha}$

Table II Calculation result of lower bound of earliest times of left shape of $\widetilde{T}_{ij}$

α	$(E_1)_\alpha^L$	$(E_2)_\alpha^L$	$(E_3)_\alpha^L$	$(E_4)_\alpha^L$	$(E_5)_\alpha^L$	$(E_6)_\alpha^L$	$(E_7)_\alpha^L$	$(E_8)_\alpha^L$	$(E_9)_\alpha^L$
1	0.0000	1.0000	2.0000	2.0000	3.0000	8.0000	11.0000	14.0000	20.0000
0.9	0.0000	0.6838	2.0000	2.0000	2.3675	8.0000	10.6838	13.0513	18.4189
0.8	0.0000	0.5528	2.0000	2.0000	2.1056	8.0000	10.5528	12.6584	17.7639
0.7	0.0000	0.4523	2.0000	2.0000	1.9046	8.0000	10.4523	12.3568	17.2614
0.6	0.0000	0.3675	2.0000	2.0000	1.7351	8.0000	10.3675	12.1026	16.8377
0.5	0.0000	0.2929	2.0000	2.0000	1.5858	8.0000	10.2929	11.8787	16.4648
0.4	0.0000	0.2254	2.0000	2.0000	1.4508	8.0000	10.2254	11.6762	16.1270
0.3	0.0000	0.1633	2.0000	2.0000	1.3267	8.0000	10.1633	11.4900	15.8167
0.2	0.0000	0.1056	2.0000	2.0000	1.2112	8.0000	10.1056	11.3167	15.5279
0.1	0.0000	0.0513	2.0000	2.0000	1.1026	8.0000	10.0513	11.1539	15.2266

Table III Calculation result of upper bound of earliest times of left shape of $\widetilde{T}_{ij}$

α	$(E_1)_\alpha^U$	$(E_2)_\alpha^U$	$(E_3)_\alpha^U$	$(E_4)_\alpha^U$	$(E_5)_\alpha^U$	$(E_6)_\alpha^U$	$(E_7)_\alpha^U$	$(E_8)_\alpha^U$	$(E_9)_\alpha^U$
1	0.0000	1.5000	3.0000	3.0000	4.5000	10.0000	12.0000	16.0000	25.000
0.9	0.0000	1.8162	3.6325	3.6325	5.4487	11.2649	12.9487	16.9487	26.8974
0.8	0.0000	1.9472	3.8944	3.8944	5.8416	11.7889	13.3416	17.34164	27.6833
0.7	0.0000	2.0477	4.0955	4.0955	6.1432	12.1909	13.6432	17.6432	28.2863
0.6	0.0000	2.1325	4.2649	4.2649	6.3974	12.5298	13.8974	17.8974	28.7947
0.5	0.0000	2.2071	4.4142	4.4142	6.6213	12.8284	14.1213	18.2426	29.3639
0.4	0.0000	2.2746	4.5492	4.5492	6.8238	13.0984	14.3238	18.6476	29.9714
0.3	0.0000	2.3366	4.6733	4.6733	7.0099	13.3466	14.5099	19.0199	30.5299
0.2	0.0000	2.3944	4.7889	4.7889	7.1833	13.5777	14.6833	19.3666	31.0498
0.1	0.0000	2.4487	4.8974	4.8974	7.3461	13.7947	14.8461	19.6921	31.5382

Table IV Calculation result of lower bound of latest times of left shape of $\widetilde{T}_{ij}$

α	$(L_1)_\alpha^L$	$(L_2)_\alpha^L$	$(L_3)_\alpha^L$	$(L_4)_\alpha^L$	$(L_5)_\alpha^L$	$(L_6)_\alpha^L$	$(L_7)_\alpha^L$	$(L_8)_\alpha^L$	$(L_9)_\alpha^L$
1	0.0000	2.0000	2.0000	2.0000	12.0000	10.0000	11.0000	14.0000	20.0000
0.9	1.5811	3.5811	3.5811	3.5811	12.6324	11.2649	12.2649	14.6325	20.0000
0.8	2.2361	4.2361	4.2361	4.2361	12.8944	11.7889	12.7889	14.8944	20.0000
0.7	2.7386	4.7386	4.7386	4.7386	13.0955	12.1909	13.1909	15.0955	20.0000
0.6	3.1623	5.1623	5.1623	5.1623	13.2649	12.5298	13.5298	15.2649	20.0000
0.5	3.5355	5.5355	5.5355	5.5355	13.4142	12.8284	13.8284	15.4142	20.0000
0.4	3.8729	5.8729	5.8729	5.8729	13.5492	13.0984	14.0984	15.5492	20.0000
0.3	4.1833	6.1833	6.1833	6.1833	13.6733	13.3466	14.3466	15.6733	20.0000
0.2	4.3346	6.3346	6.3346	6.3346	13.8915	13.5777	13.3889	15.7889	20.0000
0.1	4.6714	6.6714	6.6714	6.6714	13.9480	13.7947	14.6974	15.8974	20.0000

Table V Calculation result of upper bound of latest times of left shape of $\widetilde{T}_{ij}$

α	$(L_1)_\alpha^U$	$(L_2)_\alpha^U$	$(L_3)_\alpha^U$	$(L_4)_\alpha^U$	$(L_5)_\alpha^U$	$(L_6)_\alpha^U$	$(L_7)_\alpha^U$	$(L_8)_\alpha^U$	$(L_9)_\alpha^U$
1	6.5382	9.5382	9.5382	9.5382	22.5382	18.5382	18.5382	22.5382	31.5382
0.9	4.6408	8.2732	8.2732	8.2732	21.2732	1.9570	17.5895	21.5895	31.5382
0.8	3.8549	7.7493	7.7493	7.7493	20.7493	16.3021	17.1965	21.1965	31.5382
0.7	3.2518	7.3473	7.3473	7.3473	20.3473	15.7995	16.8950	20.8950	31.5382
0.6	2.7434	7.0083	7.0083	7.0083	20.2083	15.3759	16.6408	20.6408	31.5382
0.5	2.1742	6.7097	6.5884	6.7097	19.7097	15.0026	16.4768	20.4768	31.5382
0.4	1.5668	6.4398	6.1159	6.4398	19.4398	14.6652	16.2144	20.2144	31.5382
0.3	1.0082	6.1915	5.6815	6.1951	19.1915	14.3549	16.0282	20.0282	31.5382
0.2	0.4883	5.9604	5.2772	5.9604	19.9604	14.0660	15.8549	19.8549	31.5382
0.1	0.0000	5.7434	4.8974	5.7434	18.7434	13.7947	15.6921	19.6921	31.5382

Table VI Calculations of $T_{i\alpha}^{L}$

α	$(T_1)_\alpha^L$	$(T_2)_\alpha^L$	$(T_3)_\alpha^L$	$(T_4)_\alpha^L$	$(T_5)_\alpha^L$	$(T_6)_\alpha^L$	$(T_7)_\alpha^L$	$(T_8)_\alpha^L$	$(T_9)_\alpha^L$
1	0	1	0	0	9	2	0	0	0
0.9	1.5811	2.8973	1.5811	1.5811	10.2649	3.2649	1.5811	1.5812	1.5811
0.8	2.2361	3.6833	2.2361	2.2361	10.7888	3.7889	2.2361	2.236	2.2361
0.7	2.7386	4.2863	2.7386	2.7386	11.1909	4.1909	2.7386	2.7387	2.7386
0.6	3.1623	4.7948	3.1623	3.1623	11.5298	4.5298	3.1623	3.1623	3.1623
0.5	3.5355	5.2426	3.5355	3.5355	11.8284	4.8284	3.5355	3.5355	3.5355
0.4	3.8729	5.6475	3.8729	3.8729	12.0984	5.0984	3.873	3.873	3.873
0.3	4.1833	6.02	4.1833	4.1833	12.3466	5.3466	4.1833	4.1833	4.1833
0.2	4.3346	6.229	4.3346	4.3346	12.6803	5.5777	3.2833	4.4722	4.4721
0.1	4.6714	6.6201	4.6714	4.6714	12.8454	5.7947	4.6714	4.7435	4.7734

Table VII Calculations of $T_{i\alpha}^{U}$

α	$(T_1)_\alpha^U$	$(T_2)_\alpha^U$	$(T_3)_\alpha^U$	$(T_4)_\alpha^U$	$(T_5)_\alpha^U$	$(T_6)_\alpha^U$	$(T_7)_\alpha^U$	$(T_8)_\alpha^U$	$(T_9)_\alpha^U$
1	6.5382	8.0382	6.5382	6.5382	18.0382	8.5382	6.5382	6.5382	6.5382
0.9	4.6408	6.457	4.6407	4.6407	15.8245	5.6921	4.6408	4.6408	4.6408
0.8	3.8549	5.8021	3.8549	3.8549	14.9077	4.5132	3.8549	3.8549	3.8549
0.7	3.2518	5.2996	3.2518	3.2518	14.2041	3.6086	3.2518	3.2518	3.2519
0.6	2.7434	4.8758	2.7434	2.7434	13.8109	2.8461	2.7434	2.7434	2.7435
0.5	2.1742	4.5026	2.1742	2.2955	13.0884	2.1742	2.3555	2.2342	2.1743
0.4	1.5668	4.1652	1.5667	1.8906	12.616	1.5668	1.8906	1.8906	1.5668
0.3	1.0082	3.8548	1.0082	1.5218	12.1816	1.0083	1.5183	1.5183	1.0083
0.2	0.4883	3.566	0.4883	1.1723	12.7771	0.4883	1.1716	1.1716	0.4884
0.1	0	3.2947	0	0.846	11.3973	0	0.846	0.846	0

Table VIII Events' degree of criticality

Event(i)	1	2	3	4	5	6	7	8	9
T_i	56.5824	93.2769	56.5823	59.0665	253.4213	74.8581	59.0175	59.2849	56.822
CD(i)	0.2233	0.3680	0.2233	0.2331	1	0.2954	0.2329	0.2339	0.2242

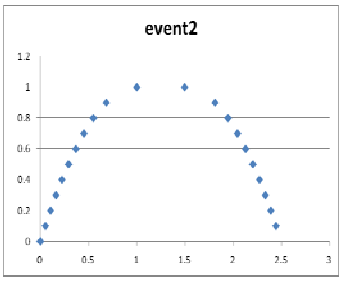

Fig.2

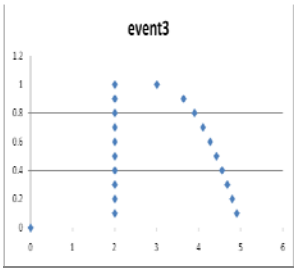

Fig.3

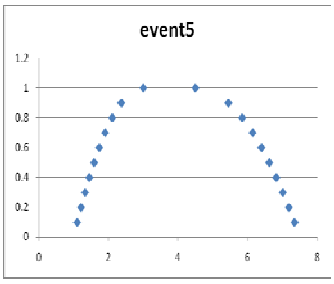

Fig.4.

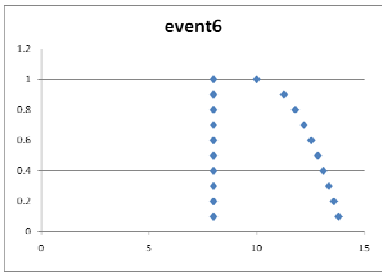

Fig. 5.

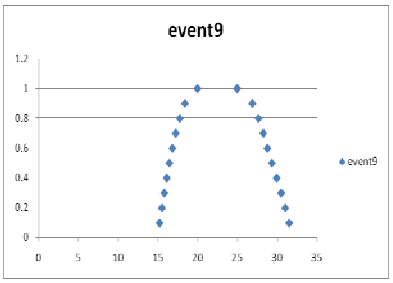

Fig. 6.

6. Comparative Analysis

In order to confirm the validity of propose method, the proposed method and Zareei et al [1] , Chen [6] are compared by finding the most critical paths based on activity times of Table I. As per table IX, Paths ranked in first and second place of proposed method is the same as Chen[6] and Zareei et al [1] and this shows the validity of proposed method. Paths of third , fourth , fifth and sixth place is the same as Zareei et al [1] and also the last path criticality is 0. Since, the Path 1-2-5-9 does not play any significant role in the fuzzy project network. Chen [6]and Zareei et al [1] considers path 1-3-4-7-8-9 as the critical path with relative degree of criticality 1 and 0.991 ,this path is not critical in α-cut range 0.2 and 0.8 for left shape of activity durations. In our approach we consider this path as the critical path with path degree of criticality 0.7661. Comparison path criticality is presented in Table IX.

Table IX The comparison of path criticality

Path	Proposed approach	A.Zareei et al. [1]	Chen[6]
1-3-4-7-8-9	0.7661	0.991	1
1-3-6-8-9	0.7046	0.9134	0.9656
1-2-4-7-8-9	0.632	0.8193	0.9212
1-3-4-6-8-9	0.7046	0.9134	0.8789
1-2-4-6-8-9	0.632	0.8193	0.8001
1-2-5-9	0	0.8193	0.5717

7. Conclusions

We have proposed and developed Linear programming models to calculate fuzzy earliest and fuzzy latest events time in project scheduling problem with L-L fuzzy numbers as activity times. By applying this approach no infeasible and negative solution is generated. The membership functions of earliest and latest times are generated by solving the proposed linear programming models in case of fuzzy critical path method problem. In this paper, Criticality degree for events and path criticality in fuzzy project network have defined.

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Received: October, 2011