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Social Structure and Agency as Time-scaled Dynamical System The Case of Scientific Capital

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Abstract

This article proposes a mathematical model of capital that further develops Pierre Bourdieu's concept of the scientific field. The model describes the joint evolution of scientific capital, the habitus of scientists, and scientific practice. The model is implemented as a random dynamic system with the help of self-consistent equations in the form of a Lorenz – Haken system. We show that if the characteristic relaxation time of the scientific capital is far greater than the relaxation times of the habitus of scientists and scientific activity, then the distribution of scientific capital is determined by the correlation of the parameters of external action on the scientific field with the intensity of fluctuations of scientific habitus and scientific activity.

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1 Introduction

Social structure is an aggregate of relations between elements of a whole that exists in the social world, where relations are invariant under various transformations [30, 36]. Social structure "is theoretical" [38, p. 284] and cannot be discovered by the researcher who remains in the domain of exclusively empirical descriptions. An understanding of structure can develop only within the framework of a sociological theory that allows the identification of a pattern of stable regularities in a series of isolated observations for their subsequent consolidation into a whole [16].

Social structure is given "axiomatically" to mean the sum total of all causal relations [41]. These relations may be interpreted as a statistical trend, a probabilistic mechanism for generating the events of the social world [19]. Agents are "always already" integrated into social structures. Social structure conditions action, both as its "limiting condition" or its "frame" [8], and as "a material cause of human activity" [32, p. 265].

"Social structure" *analytically*, i.e. *a priori*, opposes action [1]. However, a "structure" is the aggregate of stable relations in the object of research that determines its integrity and self-identity. In other words, social structure is that which ensures the persistence of the basic properties under various external and internal transformations, and is the basic characteristic of the object of sociological study, its invariant. On the other hand, "action" most generally is an event in the social world constructed by the sociologist [2]. However, an event produces change. Therefore we can argue that action changes the social world.

Thus — following "realist social theory" — we accept that social structures are necessary conditions and prerequisites for any action. However, in order that actions may be realized, social structures must be interiorized, effectively learned, and appropriated [13]. "Habitus" serves as a medium connecting stable social structures (which change over the long term) and unstable actions (which change in short periods), and itself changes in the mid-term. [3]. That agents have habitus means that there should be predispositions inherent in experience for regularly perceiving and assessing the social conditions of life, and likewise they should also be inclined to act in particular ways under such conditions [13]. One can argue that habitus integrates the past with the present, ensuring its continuation into the future, thus setting the ground for succession and order [12].

Thus, in constituting sociological discourse through the dichotomy of structure and agency, the sociologist begins to construct his conceptions on the basis of the dichotomy of permanency and change. Of course, any social structure, being a dynamic and historical entity, inevitably changes. Accordingly, we can relate social structure to an invariant of the social world only in the sense that the typical period of time required for changing it is much longer than the typical period of time needed to change actions. We see this proposition as the starting point of our study.

2 Brief Background

When dealing with complex sociological problems, it may be very important to build a simplified mathematical model because it helps to determine at what level of abstraction it would be acceptable to stop in a particular case. Also, there are so-called basic models — extremely simple but at the same time very deep mathematical constructions which render the description of the object of sociological study with a measure of accuracy. In models of this kind a mathematical structure in some manner corresponds to a pattern of social phenomena. This mathematical structure is then analyzed using mathematical tools in order to formulate sociologically meaningful statements. As a basic model, we will use a closed system of differential equations which under certain conditions can describe specific features of the temporal dynamics of the social world that are of interest to the sociologist. In many cases, an isolated (autonomous) object of sociological research studied with the help of the finite-dimensional determined mathematical model can be represented in the form of a dynamical system (DS), i.e. as a social process for which the following is explicitly determined:

1. *State* x — element ("point") of some real finite-dimensional linear space. The state expresses the aggregate of sociological quantities. These quantities and their relations taken together determine all the other variables significant for characterizing the object of sociological study.

2. *Phase space* — finite-dimensional smooth manifold $M \subset \mathbb{R}^d$, the elements of which ("points") are interpreted as the allowable states of the subject under consideration.

3. *Rule* (or *phase mapping*) $G := \{g^t(\cdot) \mid t \in \mathbb{T}\}$, which determines the evolution of the initial state x over the course of time $\mathbb{T} \subset \mathbb{R}^1$. (We take by definition $x(t) := g^t(x_0)$, $g^t(x) := g(x,t)$, and $g^0(x) := x$.) In these terms, if $(g^t(\cdot) \in G)\colon g^t(x) \in M$ — is the state at the moment $t \in \mathbb{T}$ during the initial state x, then the diffeomorphism $g^t(\cdot)$ (i.e. mutually univocal and smooth by both arguments x and t mapping $((t,x) \in \mathbb{T} \times M)\colon (t,x) \mapsto g^t(x)$ of direct product $\mathbb{T} \times M$ into manifold M with smooth inverse mapping)

$$(\forall g^t(\cdot) \in G)(\forall t \in \mathbb{T})(\forall x \in M)\colon g^t(x)\colon \mathbb{T} \times M \to M,$$

of the phase space converts each state $x \in M$ into a new state $g^t(x) \in M$. Note that the diffeomorphism $g^t(\cdot)$ possesses the group property:

$$g^0(\cdot) = \mathrm{id}_M,\ (\forall s, t \in \mathbb{T})\colon g^t(\cdot) \circ g^s(\cdot) = g^{t+s}(\cdot),\ g^{-t}(\cdot) = (g^t)^{-1}(\cdot),$$

where the composition symbol, $\circ$, means $g^t(\cdot) \circ g^s(\cdot) = g^t(\cdot)(g^s(\cdot))$.

Thus in the most basic case, a sociological *autonomous invertible DS* — is the triple $(M, \mathbb{T}, G)$, where the phase mapping $G \ni g^t(\cdot)\colon \mathbb{T} \times M \to M$ is the cartesian product $\mathbb{T} \times M$ in M that satisfies the following three axioms:

1. $(\forall x \in M)\colon g^o(x) = x$;
2. $(\forall x \in M)(\forall s, t \in \mathbb{T})\colon g^s(g^t(x)) = g^{t+s}(x)$;
3. $(x, t) \mapsto g^t(x)$ is a diffeomorphism.

We will call the pair (M, G) *phase flow*, assigned by the single-parameter group of diffeomorphisms of the set M. The smooth mapping $g(x, \cdot)\colon \mathbb{T} \to M$ for a given set $x \in M$ is interpreted as the *motion* of the point x affected by the flow (M, G):

$$(\forall t \in \mathbb{T})(\forall x \in M)\colon g(x, \cdot)\colon \mathbb{T} \to M,\ g(x, t) := g^t(x).$$

But many sociological models are non-autonomous and are described by non-autonomous ordinary differential equations (ODEs) of the form

$$(\forall (t, x) \in \mathbb{T} \times M) \left(f \in C^n \left(\mathbb{T} \times M; \mathbb{R}^{d+1}\right)\right) : \frac{dx}{dt} = f(t, x), \tag{1}$$

where $C^n\left(\mathbb{T} \times M; \mathbb{R}^{d+1}\right)$ is the space of the function $f\colon \mathbb{T} \times M \to \mathbb{R}^{d+1}$ such that f are n times continuously differentiable norm-bounded functions. Here t is the independent variable, and the vector $x(t)$ represents the state of the object of research at time t. Let the initial condition of the system of ODEs (1) at fixed time t_0 be x_0:

$$(t_0 \in \mathbb{T})(x_0 \in M)\colon x(t_0) = x_0. \tag{2}$$

Assuming the global existence and uniqueness of solutions for the ODEs (1) to initial value problems, there exists a solution $\varphi(t, t_0, x_0)\colon \mathbb{T} \to M$ of (1) (with $\varphi(t_0, t_0, x_0) = x_0$) as a smooth map from different points in $\mathbb{T}$ into different points in the d-dimensional phase space M. The smooth mapping $f(t, x)$ is called the *vector field*. A graph

$$\Gamma_x^t = \{(t, \varphi(t, t_0, x_0)) \mid \forall (t, \varphi(t, t_0, x_0)) \in \mathbb{T} \times M\} \subset \mathbb{T} \times M$$

of the solution of (1) in the extended phase space $\mathbb{T}\times M$ is known as an *integral curve*. On an integral curve, the vector-valued function $f(\tau, x)$ specifies the *phase velocity* vector $v(\tau, x)$ at every point $(\tau, x) \in \mathbb{T} \times M$:

$$(\forall \tau \in \mathbb{T})(\forall x \in M)\colon v(\tau, x) = \left.\frac{d\varphi(t, t_0, x_0)}{dt}\right|_{t=\tau} = f(\tau, \varphi(\tau, t_0, x_0)).$$

A geometric interpretation of a vector field $f(\cdot, \cdot)$ is that it is a set of phase velocity vectors $v(\cdot, \cdot)$ on different integral curves.

Primarily a projection of a solution $\varphi(t, t_0, x_0)$ of (1) onto the phase space, M is referred to as a *phase curve* (or an *trajectory*) of the ODE (1) through the point $x = x_0$:

$$\gamma_x = \{\varphi(t, t_0, x_0) \mid \forall\varphi(t, t_0, x_0) \in M\} \subset M.$$

In other words, the solution could be thought of as a point that moves along a phase curve, occupying different positions at different time points.

We say that the vector field $f(\cdot, \cdot)$ generates a *process* (hereafter in this section we use constructions adduced in [28, 40]) $g_s^t(x)\colon \mathbb{T} \times \mathbb{T} \times M \to M$, where

$$(\forall (t, s, x) \in \mathbb{T} \times \mathbb{T} \times M)\colon g_s^t(x) := (t, s, x) \mapsto g(t, s, x) \in M$$

is a diffeomorphism, and $g_s^t(x)$ satisfies (1) and (2) in the sense that

$$(\forall (t, x) \in \mathbb{T} \times M)\colon \left.\frac{d}{dt}\left(g_s^t(x)\right)\right|_{s=\tau}^{t=t} = f(t, g_s^t(g_\tau^s(x_0))).$$

We note that the process $g_s^t(x)$ (in its domain of definition) satisfies the initial value and evolution properties

- $(\forall s \in \mathbb{T})(x_0 \in M)\colon g_s^s(x_0) = x_0$;
- $(\forall \tau, s, t \in \mathbb{T} \mid \tau \leq s \leq t)(\forall x \in M)\colon g_\tau^t(x) = g_s^t(g_\tau^s(x))$.

For each process $g_s^t(x)$, there are defined phase curves γ_x. The simplest phase curve is a *fixed point* (equilibrium point, stationary point), where the integral curve is a single point: $\Gamma_x^t = \{x(t)\}$, or

$$(\forall (t, s) \in \mathbb{T} \times \mathbb{T})(x(t) \in M)\colon x(t) = g_s^t(x).$$

Let the phase space M and the *base space* (or the parameter space) P be smooth manifolds. A *non-autonomous DS* is a pair of smooth mappings $(g^t(\cdot), h^t(\cdot, \cdot))$ with the following properties:

- The base flow $g^t(\cdot)\colon \mathbb{T} \times P \to P$ satisfies the group property

  - $(\forall p \in P)\colon g^0(p) = p$;
  - $(\forall p \in P)(\forall s, t \in \mathbb{T})\colon g^s(g^t(p)) = g^{t+s}(p)$;

  and mapping $(t, p) \mapsto g^t(p)$ is smooth.

- The cocycle mapping $h^t(\cdot,\cdot)\colon \mathbb{T} \times P \times M \to M$ satisfies the cocycle property

  - $(\forall (p, x) \in P \times M)\colon h^0(p, x) = x$,
  - $(\forall s, t \in \mathbb{T})(\forall (p, x) \in P \times M)\colon h^{t+s}(p, x) = h^t(g^s(p), h^s(p, x))$,

  and mapping $(t, p, x) \mapsto h^t(p, x)$ is smooth.

We will call the *skew product flow* the pseudo-scalar product of vectors

$$\psi^t(p, x) := \big(g^t(p), h^t(p, x)\big),$$

where the mapping $\psi^t(\cdot,\cdot)\colon \mathbb{T}\times P\times M \to P\times M$ is a diffeomorphism. The non-autonomous invertible DS $(g^t(\cdot), h^t(\cdot,\cdot))$ generate skew-product flow $\psi^t(p, x)$.

3 Dynamical System "Social Structure – Agency": Preliminaries

The most fundamental social regularities are invariant to time-domain translations $t \to t+t_0$, so they lead to autonomous ODEs. However, non-autonomous ODEs are constantly used in sociology as mathematical models of various social processes. This happens because the exclusion of unknown mappings from autonomous dynamical systems leads to systems of lesser order, however they are non-autonomous ones. We will demonstrate this using the example of habitus as an element of autonomous dynamical system "Social Structure – Habitus – Agency".

Consider the autonomous DS

$$\begin{aligned}
\frac{dx_1}{dt} &= f_1(x_1, x_2, x_3),\\
\frac{dx_2}{dt} &= f_2(x_1, x_2, x_3),\\
\frac{dx_3}{dt} &= f_3(x_1, x_2, x_3).
\end{aligned} \tag{3}$$

Let us suppose that in this system vector x_1 describes a collection of sociological quantities which operationalize a certain social structure, vector x_2 represents a set of quantities which operationalize the habitus of the agents under consideration, while vector x_3 describes a set of quantities which expresses the agencies of these agents. Let us also suppose that if we solve the second equation of system (3), we will be able to express "habitus" x_2 in an explicit form via "social structure" x_1, "agency" x_3, the independent variable "time" t, and the initial value x_2° of the vector-valued function $x_2(t)$: $x_2(t) = \widetilde{f}(t, x_1, x_3, x_2^{\circ})$. In "pure" theory this assumption of course would be impossible (see e.g. [20]), however within the limits of an asymptotic model it is acceptable. Substituting the function $x_2(t)$ into the first and third equations of system (3), we arrive at the following non-autonomous system:

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(x_1, \widetilde{f}(t, x_1, x_3, x_2^{\circ}), x_3), \\ \frac{dx_3}{dt} &= f_3(x_1, \widetilde{f}(t, x_1, x_3, x_2^{\circ}), x_3). \end{aligned}$$

An interesting case is created when the non-autonomous DS, which serves as a model of the sociological system "Social Structure – Agency", is set by a system of ODEs determined in a certain manifold:

$$(i = 1, \dots, m) \colon \frac{d\xi_i}{d\zeta} = \Phi_i(\zeta, \xi) \tag{4}$$

where ξ_i, $\Phi_i(\cdot, \cdot)$ — d_i-dimensional vectors, $\xi := (\xi_1, \dots, \xi_m)$ — explicit state-variables (i. e. the dimensional sociological quantities which describe the state of the dynamic system under consideration), ζ — the independent variable (time), $m \geq 2$. In mathematical terms (we shall use the symbol $[\cdot]$ for denoting the dimension of a certain state-variable [43, p. 6]) this assumption can be written in the following form

$$[\zeta] = Z; \ (i = 1, \dots, m) \colon [\xi_i] = X_i,$$

where Z and X_i are proper units of measurement. Let us select units of measurement, by making $X_i = \max\{\xi_i\}$, and $Z = \max\{T_i\}$, where T_i is the time during which the phase variable ξ_i reaches its maximum value X_i. For better accuracy, $Z = T_1$. This means that for the system (4) the inequality $(\forall t \in \mathbb{T}) \colon t \leq T_1$ is true. Furthermore, ξ_1 is the slowest of the phase variables under consideration, i. e. it expresses social structure. According to the general provisions of dimensional theory [43], we have $\zeta = T_1 t$, $\xi_i = X_i x_i$ and after several transformations we can rewrite (4) into a dimensionless Caushi form:

$$\begin{gathered} (\forall (t, x) \in \mathbb{T} \times M) \left(f_1(\cdot, \cdot) \in C^2 \left(\mathbb{T} \times M; \mathbb{R}^{d_1} \right) \right) : \\ \frac{dx_1}{dt} = f_1(t, x), \ x_1(0) = x_1^{\circ} \in C^1 \left(\mathbb{T} \times M; \mathbb{R}^{d_1} \right), \\ (i = 2, \dots, m)(\forall (t, x) \in \mathbb{T} \times M) \left(f_i(\cdot, \cdot) \in C^2 \left(\mathbb{T} \times M; \mathbb{R}^{d_i} \right) \right) : \\ \frac{T_i}{T_1} \frac{dx_i}{dt} = f_i(t, x), \ x_i(0) = x_i^{\circ} \in C^1 \left(\mathbb{T} \times M; \mathbb{R}^{d_i} \right), \end{gathered} \tag{5}$$

where x_1, $f_1(\cdot,\cdot)$, x_1^{o} — d_1-dimensional vectors, x_i, $f_i(\cdot,\cdot)$, x_i^{o} — d_i- dimensional vectors, $x := (x_1, \dots, x_m)$ — explicit state-variables (sociological quantities describing the state of the DS under consideration), $d = d_1 + \ldots + d_m$, t — the independent variable (time), $m \geq 2$. Let us introduce the following notations: vector x_1 designates the simulated social structure, while vectors x_i designate i-agencies, respectively.

Next, let us assume that $g_s^t\{1\}$, $g_s^t\{i\}$ $(i = 2, \dots, m)$ are processes engendered by the vector fields $f_1(\cdot,\cdot)$ and $f_i(\cdot,\cdot)$, respectively. Then the mapping

$$(i = 2, \dots, m)\colon \eta_i\colon \left(M_i, \mathbb{T}, g_s^t\{i\}\right) \to \left(M_1, \mathbb{T}, g_s^t\{1\}\right)$$

is a structure-preserving map, i.e. homomorphism of the process $g_s^t\{i\}$ over $g_s^t\{1\}$, i.e.

$$(\forall (t \times s \times x_i) \in \mathbb{T} \times \mathbb{T} \times M_i) : \eta_i\left(g_s^t\{i\}(x_i)\right) = g_s^t\{1\}(\eta(x_i)).$$

Note that the manifold M can be represented as direct sum

$$M = \bigoplus_{i=1}^{m} M_i, \tag{6}$$

where M_1 is the phase space of the social structure, and the phase state of the ith agency under a fixed state of the social structure $x_1 \in M_1$ is represented by the manifold points $M_i|_{x_1} = \eta^{-1}(x_1)$.

The triple $(M, \mathbb{T}, g_s^t\{i\})$ is a homomorphic image of the triple $(M, \mathbb{T}, g_s^t\{1\})$, but there are substantial differences between them. The fact is that changes in social structure normally take a long time to happen and occur slowly, so in the timescale typical to agency it can be considered as almost permanent. Accordingly, we can see x_1 as a slow-phase variable, and x_i — as a fast-phase variable. This means that the ratio of increments Δx_1 and Δx_i during a brief period of time Δt is much less than one:

$$(i = 2, \dots, m)\colon \frac{\Delta x_1}{\Delta x_i} \ll 1.$$

Since the rate of change of x_i is significantly larger than the rate of change of x_1, after the transformations and selection of appropriate scales we have

$$\begin{aligned}(i = 2, \dots, m)(k_i \in \mathbb{Z})(0 = k_1 < k_2 < \ldots < k_m)\left(\frac{1}{\nu^{k_i}} \gg 1\right) &: \\ \frac{T}{T_i} = \frac{1}{\nu^{k_i}}&,\end{aligned} \tag{7}$$

where ν^{k_i} are d_i-dimensional vectors, and k_i — specially-selected whole numbers. Under these assumptions, we get

$$\begin{gathered}(i=2,\dots,m)\,(k_i \in \mathbb{Z})(0=k_1<k_2<\ldots<k_m):\\ \frac{dx_i}{dt}=\frac{1}{\nu^{k_i}}f_i(t,x),\ x_i(0)=x_i^{\circ}\left(\frac{1}{\nu^{k_i}}\right).\end{gathered} \tag{8}$$

By dividing the left and right sections of the equation (8) by $1/\nu^{k_i}$ and setting $\varepsilon^{k_i}=\nu^{k_i}$, we arrive at a system which is practically equivalent to (5)

$$\begin{gathered}(\forall i=1,\dots,m)(0<\varepsilon \in \mathbb{E} \subset \mathbb{R}^1)\left(f_i(\cdot,\cdot,\cdot) \in C^2\left(\mathbb{T}\times\mathbb{E}\times M;\mathbb{R}^{d_i+2}\right)\right)\\ (k_i \in \mathbb{Z})(0=k_1<k_2<\ldots<k_m):\\ \varepsilon^{k_i}\frac{dx_i}{dt}=f_i(t,x,\varepsilon),\ x_i(0)=x_i^{\circ}(\varepsilon)\in C^1\left(\mathbb{T}\times\mathbb{E}\times M;\mathbb{R}^{d_i+2}\right),\end{gathered} \tag{9}$$

where $\varepsilon^{k_i} \ll 1$ are small parameters. An advanced self-sufficient theory of that kind of system was originally developed by Tikhonov [47, 48], who is also credited with its main result [49]. A discussion of the general aspects of singular perturbation methods for ODEs can be found in, e.g., [18, 51]. Further results obtained using the approach developed by the Tikhonov school are presented in [35, 31, 50]. The system (9) is nothing but an expansion of the initial DS (5) by powers of the small parameter ε under a derivative.

Obviously, permanent ε^{k_i} *de facto* determine rates of change of the appropriate variables x_i. Indeed, if $\varepsilon \to 0$, the derivatives $x_i(t)$ will be very large:

$$\frac{dx_i(t)}{dt}=O\left(\frac{1}{\varepsilon^{k_i}}\right).$$

Therefore variables x_i, which express agencies will change rapidly. One can easily see that system (9) can be presented as

$$\begin{aligned}\frac{1}{T_1}\frac{dx_1}{dt}&=f_1(t,x),\\ (\forall i=2,\dots,m)\colon \frac{1}{T_i}\frac{dx_i}{dt}&=f_i(t,x),\end{aligned}$$

where $T_1=1$, $T_i=1/\varepsilon^{k_i}$ — are constants representing specific change periods of the relevant sociological quantities. Clearly, equations for operations with time constants ε^{k_i}, given that the inequality $\varepsilon^{k_i} \ll 1$ is true, reflect very fast social processes. Note that the equation for social structure which includes the time constant T_1 describes a very slow social process (compared with the time periods T_i). Regarding the social structure equation, one can say that during the time periods T_i, which are at least an order of magnitude less than the typical period T_1, the initial value x_1 doesn't have time to change significantly. Of course, for that to be true the DS must have a stable fixed point. (Approaches to the problem of stability of non-autonomous ODEs are discussed in [6].) Therefore in equations for agencies, the slow variable of the social structure can be replaced with constant values. And that means that the phase space of the social structure M_1 can be interpreted as parameter space P. In turn, the social structure process $g_s^t\{1\} \colon \mathbb{T} \times M_1 \to M_1$ can be presented as the base flow $\tilde{g}^t(\cdot)\colon\ (t, s, x_1^0) \mapsto (t, g_s^t\{1\}\,(x_1^0))$. Next, we can set cocycle $h^t\,(x_1, x_i) : \mathbb{T} \times M_1 \times M_i \mapsto M_i$ over $\tilde{g}^t\,(x)$, i.e. one has

- initial value condition: $h^0\,(x_1, x_i) = x_i$;
- cocycle property: $h^{t+s}\,(x_1, x_i) = h^t\,(\tilde{g}^t\,(x_1)\,, h^s\,(x_1, x_i))$.

In these terms, interaction between the social structure x_1 and the agency x_i can be described using skewed product flow

$$(i = 2, \ldots, m)\colon \psi^t(x_1, x_i) = \left(\tilde{g}^t\,(x_1)\,, h^t\big(x_1, x_i\big)\right)_{M_1 \times M_i}.$$

To a certain extent we can argue metaphorically that $\tilde{g}^t\,(x_1)$ describes social structure as an autonomous DS. However, at the same time the construct $\psi^t(x_1, x_i)$ describes the deviations of base flow from autonomous motion, which are conditioned by the agencies' effect over the social structure.

Substantively, (9) can be represented as a two-tier system. Although the division of variables happens only infinitesimally and not in the actual phase space M, it can be represented as the direct sum of the sub-spaces (6): d_1-dimensional phase space M_1, belonging to the social structure x_1, and $\bar{d}$- dimensional phase space M_a of agencies x_a ($x_a := (x_2, \ldots, x_m)$), where $\bar{d} = \sum_{i=2}^{m} d_i$. If we set $\varepsilon = 0$ and leave $x_i(0)$, we get from (9) the degenerate system of ODEs

$$\begin{aligned} \frac{d\widehat{x}_1}{dt} &= f_1\,(t, \widehat{x}, 0)\,,\ \widehat{x}_1(0) = x_1^{\mathrm{o}}(0), \\ (\forall i = 2, \ldots, m)\colon\ f_i\,(t, \widehat{x}, 0) &= 0. \end{aligned} \tag{10}$$

The solution to the above system, if it exists, is called the quasi-steady state [37, 45] approximation to (9). How close the solution of system (9) can be be to the solution of system (10) is provided by Tikhonov's theorem [49] which we will discuss below. Meanwhile let us take a look at the system (10) in zero-order approximation.

Suppose that in a domain of interest a system of algebraic or transcendental equations

$$(\forall i = 2, \ldots, m)\colon f_i\,(t, \widehat{x}, 0) = 0 \tag{11}$$

has $l \geq 1$ distinct real "isolated" roots for $(t, x_1) \in \mathbb{T} \times M_1$

$$(\forall i = 2, \dots, m)(k = 1, \dots, l)\colon \widehat{x}_i = \varphi_k(t, \widehat{x}_1) \in C^0(\mathbb{T} \times M_1; M_i), \tag{12}$$

such that

$$((t, \widehat{x}_1) \in \mathbb{T} \times M_1)(\exists \delta > 0)(|\widehat{x}_i - \varphi_k(t, \widehat{x}_1)| \in]0, \delta[)\colon f_i(t, x_1, 0) = 0. \tag{13}$$

Now we can substitute (12) to (10):

$$\frac{d\widehat{x}_1}{dt} = f_1(t, \widehat{x}_1, \varphi_k(t, \widehat{x}_1), 0), \ \widehat{x}_1(0) = x_1^{\mathrm{o}}(0). \tag{14}$$

The assumption that (12) – (13) is a natural one because in the opposite case it would be difficult to give any specific meaning to the system (14).

The root $\varphi_k(t, \widehat{x}_1)$ in the extended phase space $\mathbb{T} \times M$ yields the $d_i + 1$-dimensional hypersurface S, so that all phase curves of the degenerate system (10) lie on that surface. A hypersurface S (whose existence is guaranteed by the center manifold theory [15]) which asymptotically attracts all phase curves is usually referred to as the slow (invariant) manifold. Let us try to estimate under what conditions the motion of the DS "Social Structure – Agency" will occur in the neighbourhood of the order ε of the slow manifold S. (A detailed geometric theory of invariant manifold is presented in [21, 25, 29, 52].) Let ϱ_i $(i = 1, \dots, m)$ be the distances from S to a point $(t, x, 0)$ in the extended phase space $\mathbb{T} \times M$. If we set ϱ_i of the order ε^{k_i} $(i = 1, \dots, m)$, we get

$$(\forall i = 1, \dots, m)\colon f_i(t, x, 0) - f_i(t, \widehat{x}, 0) = \sum_{i=1}^{m} \frac{\partial f_i(t, x, 0)}{\partial x_i} \varrho_i + \dots,$$

so keeping in mind the equation $f_i(t, \widehat{x}, 0) = 0$ $(i = 2, \dots, m)$, we can record it as

$$(\forall i = 2, \dots, m)\colon \frac{dx_i}{dt} = \frac{1}{\varepsilon^{k_i}} \left(\frac{\partial f_1(t, x, 0)}{\partial x_1} \varrho_1 + \sum_{i=2}^{m} \frac{\partial f_i(t, x, 0)}{\partial x_i} \varrho_i + \dots \right). \tag{15}$$

Next, for each point $(t, x, 0)$ in the domain of the DS, let us set the phase speed vector

$$(\forall i = 2, \dots, m)\colon v(t, x, 0) = \left(f_1(t, x, 0), \frac{1}{\varepsilon^{k_i}} f_i(t, x, 0) \right) = \left(\frac{dx_1}{dt}, \frac{dx_i}{dt} \right).$$

Using (15), one can clearly see that only in the neighbourhood ε of hypersurface S, when distances ϱ_i $(i = 1, \dots, m)$ are of the order of ε^{k_i}, that the phase speed value will be limited under $\varepsilon \to 0$. Or rather, if the first component of the phase speed vector has a finite value, the second (under the same assumptions) would be infinitely large. Accordingly, in the system (9) it would be possible to differentiate slow from fast flows. Fast motions occur far away from S almost parallel to the extended phase subspace $\mathbb{T} \times M_a$, while slow motions only occur in the ε-neighbourhood $\mathbb{T} \times S$. That is why this neighbourhood is an area of slow motion which is approximately described by the system (10). The fact that fast motions sharply fade in the area adjacent to the hypersurface S allows one to analyse both motion components to a greater extent individually.

Thus outside a small neighbourhood of the hypersurface S and given a small ε, phase curves approach the hyperplanes $x_1 = \text{const}$. Motion along these phase curves becomes fast. This is an area of quick movement which can be approximated by the following system

$$(\forall i = 2, \ldots, m)\colon \tau_i = (t - t_0)\, \varepsilon^{-k_i},\ \frac{dx_i}{d\tau_i} = f_i\,(t, x, 0)\,, \tag{16}$$

which emerges from (9), if we move to short time periods τ_i and then set $\varepsilon = 0$. In (16) slow variables x_1 and t are considered parameters and the transition to τ_i in the right section is not made because $(\forall t \in \mathbb{T})\colon\ \tau_i \xrightarrow{\varepsilon \to 0} \infty$. Clearly, the root $\widehat{x}_i = \varphi_k\,(t, \widehat{x}_1)$ of the equation (11) sets fixed points of the system (3).

To summarise: we have analysed a point (t, x) located a finite distance from the slow manifold S. Within the limits of sensible assumption, one can argue that the social structure vector x_1 remains almost unchanged while the agency vectors x_i change quickly, almost instantly. Therefore movement along the phase curves of DS (9) turns out to be similar to movement in accordance with (16). Clearly the fixed points of the system (16) are determined by the roots of the equation $f_i\,(t, \widehat{x}, 0) = 0$. If the system (16) has stable fixed points, then the agency phase point x_i will move on to one of them. After that the social structure x_1 and agencies x_i will change in the neighbourhood of the hypersurface S with comparable speeds. The change in social structure x_1 will occur slowly due to the degenerate system (10). The motions on the slow manifold S evolve slowly. Furthermore, the hypersurface S attracts all motions that originate in its neighborhood. The sociological understanding of social structure hinges on the existence of a slow manifold S characterizing the long-term process dynamics: when the motions reach this manifold, knowing social structure suffices to approximate the full DS state. The resulting reduction in complexity can be significant for explaining the evolution of the system "Social Structure – Agency".

We will reproduce here neither the most general nor the most exact formulations of Tikhonov's theorem as it is cumbersome to prove (please refer to [31, p. 159–164]); we will simply explain its meaning.

Let us call subset $A \subseteq M$ domain attracting fixed point $\widehat{x}_i = \varphi_k\,(t, \widehat{x}_1)$ of system (16), provided that the following is true:

- A is a compact set;

- $(\forall i = 2, \dots, m)(\forall t \in \mathbb{T})(\forall \widehat{x}_i \in A)\colon \dfrac{d\widehat{x}_i}{d\tau_i} = f_i\,(t, \widehat{x}, 0)$;
- solutions of system (16) which have $x_i^{\mathrm{o}} \in A$ as starting points, under $\tau_i \to \infty$ will approach $\varphi_k(t, x_1)$.

Let us consider system (9). The "essential" of Tikhonov's theorem: *Consider*:

1*. *Equations* (11) *have real isolated roots*

$$(\forall i = 2, \dots, m)\colon \widehat{x}_i = \varphi_k\,(t, \widehat{x}_1) \subseteq M_1.$$

This condition allows us to exclude the fast variables which depict agency from system (10), which guarantees the relevancy of representing (10) using only social structure variables and the adequacy of the Cauchy problem (13).

2*. *Initial conditions* $(\forall i = 2, \dots, m)\colon x_i^{\mathrm{o}}$ *belong to the domain of attracting fixed point of system* (16): $x_i^{\mathrm{o}} \in A$.
If this condition remains true, after a short period of time the phase curve of the system (9) will reach the manifold S:

$$f_2(\widehat{x}, t, 0) = 0, \dots, f_m(\widehat{x}, t, 0) = 0.$$

3*. *Fixed points* $(\forall i = 2, \dots, m)\colon \widehat{x}_i = \varphi_k\,(t, \widehat{x}_1)$ *of system* (16) *are asymptotically stable according to Lyapunov* (for rigorous definitions and sufficient conditions for asymptotic stability according to Lyapunov, see [33]) *in the domain*

$$\begin{aligned}&(\forall i = 2, \dots, m)\colon\\ &D\colon \left\{\forall (t, x_1) \in \mathbb{T} \times M_1 \,|\, \exists \widehat{x}_i = \varphi_k\,(t, \widehat{x}_1) : f_i\,(t, \widehat{x}, 0) = 0\right\}.\end{aligned}$$

According to condition 3*, for agencies x_i domain D doesn't have a zero point, which excludes the infinitely small time domain where $\|x_i - \widehat{x}_i\|$ assumes large values. This condition guarantees that the phase curve of the system (9) remains within a small neighbourhood on the surface S, which allows us to argue that the solutions of systems (9) and (10) are close to each other.

Then for sufficiently small ε *system* (9) *has a unique solution*

$$(\forall i = 1, \dots, m)\colon \widehat{x}_i(t, \varepsilon),$$

such that the following limiting equalities hold:

$$(\forall t \in D)(\forall i = 1, \dots, m)\colon \lim_{\varepsilon \to 0} x_i(t, \varepsilon) = \widehat{x}_i(t). \tag{17}$$

From (17) it follows directly that

$$(\forall t \in D)(\forall i = 1, \dots, m) \colon x_i(t, \varepsilon) = \widehat{x}_i(t) + O(\varepsilon).$$

Of course, Tikhonov's theorem only sets the direction for studying the asymptotics of the solution to system (9), because the problem of constructing asymptotic decomposition for the solution and so forth remains unsolved. However, we were only interested in a qualitative result, which resulted in the following: over a short period of time fast variables which describe agencies relax into a steady state, and thus can be expressed via slow variables which express social structure.

Any dynamical system (including those which are the objects of sociological studies) is organised hierarchically: the different parameters of its state have unequal values. We are arguing that Tikhonov's theorem can be interpreted such that the transformation of a social structure, which happens over a prolonged period of time, determines the transformation of agency, which happens in a short period of time. In other words, quickly changing agency can be expressed via slowly changing social structure, and at the same time the number of degrees of freedom of the DS will reduce (see also [22]). It turns out that an asymptotic solution to the problem lies in the neighbourhood of a certain marginal state of agency which, among other things, is a function of social structure.

The possibility to present agency (interpreted as a fast sociological quantity) as a function of social structure (understood as a slowly changing invariant of state) clarifies the distinction between structure and agency. This distinction provides an asymptotic solution method which can be used to make sociological descriptions of DS in which fast and slow subsystems can be identified. In regard to sociological study, it means that we must first consider the changes of agency (of a fast dynamical subsystem) with a fixed social structure (of a slow dynamical subsystem), and then take the changes of the latter into account.

The most important condition of Tikhonov's theorem is the condition $3^\star$ of attraction by manifold S of phase curves of the initial DS (9). This suggests that social structure can be, among other things, interpreted as an area of the phase space of the DS (which simulates a certain subject in the social world) and towards which phase curves approach after transition processes fade. That is, social structure can be interpreted as a perfect final state which the DS, representing a social process, approaches in the short term. In more general terms, social structure can be represented as the aggregate of internal and external conditions that determines the choice of an object of sustainable development in the social world in the near term.

Thus the sociological results of applying Tikhonov's theorem to the DS "Social Structure – Agency" amount to the following:

- application of Tikhonov's theorem helps to analyse the relationship between social structure and agency;
- application of Tikhonov's theorem helps to better understand the sociological content of this relationship, and to clarify the concept of "social structure";
- mathematical constructs allow us to identify the sociological basis of very closely approximated empirical arguments regarding social structure as a regular cause of agency.

Without a doubt, mathematical formalism cannot replace sociological theory and Tikhonov's theorem is no exception. Still, the conceptual application of mathematics to a sociological study implies that mathematical constructs and statements no longer merely play the role of trivial illustrations to messages delivered in everyday language: they are starting to express their own specific meaning. That is, mathematics can be not only a particularly economical form of delivering sociological discourse; it is able to formulate messages inherent only to itself.

4 The Model of the Dynamics of the Scientific Capital

In this section we will make an effort to create a phenomenological model of the DS "Social Structure – Habitus – Agency". We will examine scientific capital (SC) as a social structure, understood as the aggregate of stable relations in a given scientific field that form the conditions and prerequisites for scientific agency (SA). According to P. Bourdieu [9, 10], SC is an accumulated history (transferred through time in various forms) that could be beneficial to the scientific field in different ways. Let H denote habitus. From sociological theory, it follows that the evolution of the DS "SC – H – SA" is formed as a result of the competition between the positive feedback of SC and H and the negative feedback of SA. We will form ODEs for SC, H, and SA based on these considerations.

To begin, let us assume that changes in the value near its "equilibrium" meaning x^e can be approximated by the formula

$$x = x^e \left(1 - \exp\left(\frac{t}{T}\right)\right), \tag{18}$$

where T is typical relaxation time. Then in the assumption $T \gg t$ it is possible to express (18) in the form of the following ODE:

$$\frac{dx}{dt} = -\frac{x}{T}.$$

So, assuming the dissipative character of the relaxation of SC x_1, H x_2, and SA x_3, we can represent the DS "SC – H – SA" to a "null approximation" through their values, all with the help of ODEs characterized by their speed of change in value of SC, H, and SA. The given ODEs contain dissipative terms whose values are inversely proportional to the corresponding relaxation times:

$$\frac{dx_1}{dt} = -\frac{x_1}{T_1}, \tag{19}$$

$$\frac{dx_2}{dt} = -\frac{x_2^s - x_2}{T_2}, \tag{20}$$

$$\frac{dx_3}{dt} = -\frac{x_3}{T_3}, \tag{21}$$

where T_1,T_2,T_3 is the relaxation time of SC, H, and SA, respectively, and $x_2^s \neq 0$ is the value of H outside of the scientific field. The nature of H is such that most of it is formed not within the scientific field, but rather within its surrounding social space [9]. It is this circumstance that reflects the element x_2^s, which plays the role of parameter of external influence in the DS "SC – H – SA".

The equation for the relaxation of SC (19) needs to be supplemented with the value $c_1 x_3$ ($c_1 = \text{const}_1$), which describes the reaction of SC to changes in SA:

$$T_1 \frac{dx_1}{dt} = -x_1 + c_1 x_3. \tag{22}$$

In the stationary state $\frac{dx_1}{dt} = 0$ the equation (22) turns into the equation $x_1 = c_1 x_3$, immediately establishing a linear dependence of SA on SC.

Further, we must adjust the expression (20), which fixes the dynamics of H, introducing the value $-c_2 x_1 x_3$ ($c_2 = \text{const}_2$) which expresses the negative feedback that occurs between H on the one hand, and SC and SA on the other. In this way we arrive at the equation

$$T_2 \frac{dx_2}{dt} = x_2^s - x_2 - c_2 x_1 x_3. \tag{23}$$

Finally, our description of the relaxation of SA should include a summand that characterizes its change, which is induced by SC. Because SA is also due to H that depends in turn on SC, then it is reasonable to supplement the equation (21) with the value $c_3 x_1 x_2$ ($c_3 = \text{const}_3$):

$$T_3 \frac{dx_3}{dt} = -x_3 + c_3 x_1 x_2. \tag{24}$$

A distinctive feature of the equations (22)–(24) is that they provide a self-consistent picture of the dynamics of SC, H, and SA. Each of these equations contains a dissipative summand that describes the process of relaxation. The non-linear summands contained in the right half of the equations (23) and (24) play a crucial role. The second of these reflects positive feedback that leads to a change in SA due to the interaction between SC and H. This relationship is partly offset by negative feedback between SC and SA in the equation (23): as long as the change in H destabilizes the DS "SC – H – SA", then SC and SA oppose it. With the removal of x_2^s the negative feedback of SC and SA with H brings about a decrease of H. On the other hand, the positive relationship of SC and H with SA leads to a growth in SA.

In non-dimensional form the equations (22)–(24) reduce to the well-studied Lorenz equations [46, 34, 14] (in Haken's form [23]):

$$\frac{dy_1}{d\tau} = -y_1 + y_3, \tag{25}$$

$$\varepsilon_1 \frac{dy_2}{d\tau} = y_2^s - y_2 - y_1 y_3, \tag{26}$$

$$\varepsilon_2 \frac{dy_3}{d\tau} = -y_3 + y_1 y_2, \tag{27}$$

where differentiation is conducted according to non-dimensional time $\tau = tT^{-1}$, the parameters $\varepsilon_1 = T_2 T_1^{-1}$ and $\varepsilon_2 = T_3 T_1^{-1}$ are introduced, as well as the dimensionless values

$$y_1 = (c_2 c_3)^{\frac{1}{2}} x_1, \; y_2 = (c_1 c_3) x_2, \; y_3 = (c_1^2 c_2 c_3)^{\frac{1}{2}} x_3.$$

It is easy to verify that given $T_1 \gg T_2 \gg T_3$ we have fulfilled the conditions of Tikhonov's theorem and get

$$\frac{d\widehat{y}_1}{d\tau} = -\widehat{y}_1 + \frac{y_2^s \widehat{y}_1}{1+\widehat{y}_1^2}, \; \widehat{y}_2 = \frac{y_2^s}{1+\widehat{y}_1^2}, \; \widehat{y}_3 = \frac{y_2^s \widehat{y}_1}{1+\widehat{y}_1^2}. \tag{28}$$

The in H and SA follow from the changes of SC given (28).

In the more complicated case when $T_1 > T_2 \gg T_3$ we change to the next DS:

$$\frac{d\widehat{y}_1}{d\tau} = -\widehat{y}_1(1-\widehat{y}_2), \tag{29}$$

$$\varepsilon_1 \frac{d\widehat{y}_2}{d\tau} = y_2^s - \widehat{y}_2(1+\widehat{y}_1^2).$$

The behavior of the DS (29) is given the parameters $y_2^s = (c_1 c_3) x_2^s$ and $\varepsilon_1 = T_2 T_1^{-1}$. The first parameter captures the degree of deviation of H from its stationary value while the second correlates the distinctive relaxation times of H and SS. The DS (29) possesses two singular points: $O_1(0, y_2^s)$ and $O_2((y_2^s -$

$1)^{\frac{1}{2}}, 1)$. The Lyapunov exponents of these points are defined by the formulas

$$\lambda_{O_1} = \frac{\varepsilon_1(y_2^s - 1) - 1}{2\varepsilon_1}\left(1 \pm \left(1 + \frac{4\varepsilon_1(y_2^s - 1)}{(\varepsilon_1(y_2^s - 1) - 1)^2}\right)^{\frac{1}{2}}\right),$$
$$\lambda_{O_2} = -\frac{y_2^s}{2\varepsilon_1}\left(1 \pm \left(1 - \frac{8\varepsilon_1(y_2^s - 1)}{(y_2^s)^2}\right)^{\frac{1}{2}}\right).$$

If $y_2^s < 1$, then point O_1 is a stable node, and point O_2 do not exist. Therefore the DS (29) evolves into a stationary state corresponding to point O_1. If it should happen that $y_2^s > 1$, then point O_1 turns into a saddle and produces the point O_2. When $\varepsilon_1 \leq (y_2^s)^2(8(y_2^s - 1))^{-1}$, then point O_2 becomes a stable node. If $\varepsilon_1 > (y_2^s)^2(8(y_2^s - 1))^{-1}$, then point O_2 is transformed into a stable focus.

At the limit $T_1 \gg T_2$ the decrease in the parameter ε_1 gives rise to the segment MO_1O_2 in the phase space of the DS (29), towards which all trajectories will eventually converge. Regardless of the initial conditions of the limit $\varepsilon_1 \to 0$ the DS (29) quickly returns to MO_1O_2, the location of which is not at all related to the structural features of DS (29), and which subsequently moves slowly along this trajectory. In this way the universal regularities of the scientific field are realized regardless of national peculiarity. It is the universal character of the evolution of DS "SC−H−SA" in the premise $T_1 \gg T_2 \gg T_3$ that makes possible both the historical analogies between various national fields of science and the appellation "international experience" in terms of the organization and management of science.

However, empirically relevant y_1, y_2, y_3 can only be adequately expressed with the help of random variables. In order to describe the evolution of the system "SC−H−SA" as a stochastic process using a dynamic approach, we need some new concepts.

A *metric dynamical system* (MDS) [5] $(\Omega, \theta) \equiv (\Omega, \mathcal{F}, \Pr, (\theta_t)_{t\in\mathbb{T}})$ with the time $\mathbb{T}$ is a complete probability space $(\Omega, \mathcal{F}, \Pr)$ (that serves as a model for the noise) with a flow $(\theta_t)_{t\in\mathbb{T}}$ $((\forall t \in \mathbb{T})\colon \theta_t\colon \Omega \to \Omega)$ such that

1. it is one-parameter group:

$$\theta_0 = \mathrm{id}_\Omega,\ (\forall t, s \in \mathbb{T})\colon \theta_t \circ \theta_s = \theta_{t+s},$$

   where $\circ$ means composition of mappings;

2. $(t, \omega) \mapsto \theta_t\omega$ is measurable;

3. $(\forall t \in \mathbb{T})\colon \theta_t \Pr = \Pr$, i.e. $(\forall B \in \mathcal{F})(\forall t \in \mathbb{T})\colon\ \Pr(\theta_t B) = \Pr(B)$.

Let X be a separable complete metric space with the Borel σ-algebra $\mathcal{B} = \mathcal{B}(X)$ generated by open sets of X. A *random dynamical system* (RDS) (see details in [4]) on X over the MDS (Ω, θ) with time $\mathbb{T}$ and phase space X is a pair (θ, ϕ) consisting of a MDS (Ω, θ) and a cocycle ϕ over θ (that acts as a model of the system perturbed by noise) of continuous mappings of X with time $\mathbb{T}$, i.e. measurable mapping $\theta\colon \mathbb{T} \times \Omega \times X \mapsto X$, $(t, \omega, x) \mapsto \phi(t, \omega, x)$, such that

1. the mapping $(\forall t \geq 0)(\forall \omega \in \Omega)\colon x \mapsto \phi(t, \omega, x) \equiv \phi(t, \omega)x$ is continuous for fixed ω;

2. the mappings $\phi(t, \omega) := \phi(t, \omega, \cdot)$ satisfy the cocycle property:

$$(\forall t, s \in \mathbb{T})(\forall \omega \in \Omega)\colon \phi(0, \omega) = \mathrm{id}_X,\ \phi(t + s, \omega) = \phi(t, \theta_s \omega) \circ \phi(s, \omega).$$

Let $x(t) \equiv x(t, \omega)\colon \mathbb{T} \times \Omega \to (\mathbb{R}^1_+, \mathcal{B}^1)$ be a stochastic process. Consider a filtration $(\mathcal{F}_t)_{t\geq 0}$ such that $x(t)$ is adapted to it, and Wiener process $\{w(t), t \in \mathbb{T}\}$ with respect to $(\mathcal{F}_t)_{t\geq 0}$. By definition (see details in [24]), a *stochastic differential equation* (SDE) is an equation of the form

$$dx(t) = a(t, x(t))dt + \sigma(t, x(t))dw(t), \tag{30}$$

with $x_0 = \xi$ (where ξ is an $\mathcal{B}$-measurable variable), $a(\cdot,\cdot)\colon \mathbb{T} \times \mathbb{R}^1 \to \mathbb{R}^1$, $\sigma(\cdot,\cdot)\colon \mathbb{T} \times \mathbb{R}^1 \to \mathbb{R}^1$ are measurable functions and $a(\cdot,\cdot) \in L^1(\mathbb{T} \times \Omega, \mathcal{P}, \mathrm{mes} \times \mathrm{Pr})$, $\sigma(\cdot,\cdot) \in L^2(\mathbb{T} \times \Omega, \mathcal{P}, \mathrm{mes} \times \mathrm{Pr})$ (here $\mathcal{P}$ is the predictable σ-algebra on $\mathbb{R}^1_+ \times \Omega$, and mes is the Lebesgue measure), satisfying a Lipschitz condition. Then SDE (30) generates an RDS [4].

Thus the stochastic nature of the evolution of DS "SC − H − SA", interpreted as an RDS, can be represented as if y_1, y_2, y_3 experienced fluctuations, the effect of which is represented by white noise $w(t)$ with intensities $\sigma_1^2, \sigma_2^2, \sigma_3^2$ measured in units of $(c_2 c_3)^{-1}$, respectively. The transition from DS "SC − H − SA" to RDS "SC − H − SA", which is characterized by the presence of stochastic sources resulting from the influence of random factors, means the transition from ODEs (25)–(27) to the following SDEs:

$$\begin{aligned}
\frac{dy_1}{d\tau} &= -y_1 + y_2 + \sigma_1 w(\tau),\\
\varepsilon_1 \frac{dy_2}{d\tau} &= y_2^s - y_2 - y_1 y_3 + \sigma_2 w(\tau),\\
\varepsilon_2 \frac{dy_3}{d\tau} &= -y_3 + y_1 y_2 + \sigma_3 w(\tau),
\end{aligned}$$

which are represented by a stochastic Lorenz system [5, 27, 42]. This RDS naturally combines the properties of two classes of systems: deterministic systems subject to the random influences of the outside environment, and systems with stochastic dynamics. The first of these classes occur in the absence of fluctuations $(\sigma_1, \sigma_2, \sigma_3 = 0)$. The inclusion of fluctuations $(\sigma_1, \sigma_2, \sigma_3 \neq 0)$ guarantees stochastic behavior.

As is known, the advanced geometric singular perturbation theory for SDEs [7], and proven theorem of Kabanov and Pergamenshchikov [39] comprise a stochastic version of Tikhonov's theorem (see details in [26]). Relying upon it and using the property of additivity of variances of Gaussian random variables, we arrive at the following degenerate system:

$$\frac{d\widehat{y}_1}{d\tau} = -\widehat{y}_1 + \frac{y_2^s \widehat{y}_1}{1+\widehat{y}_1^2} + \left(\sigma_1 + \frac{\sigma_3 + \sigma_2 \widehat{y}_1^2}{\left(1+\widehat{y}_1^2\right)^2}\right)^{\frac{1}{2}} w(\tau), \tag{31}$$

$$\widehat{y}_2 = \frac{y_2^s}{1+\widehat{y}_1^2} + \frac{\left(\sigma_2 + \sigma_3 \widehat{y}_1^2\right)^{\frac{1}{2}}}{1+\widehat{y}_1^2} w(\tau), \tag{32}$$

$$\widehat{y}_3 = \frac{y_2^s \widehat{y}_1}{1+\widehat{y}_1^2} + \frac{\left(\sigma_3 + \sigma_2 \widehat{y}_1^2\right)^{\frac{1}{2}}}{1+\widehat{y}_1^2} w(\tau). \tag{33}$$

As is known [24], the classical Itô type SDE is given by the corresponding forward Kolmogorov equation. Let us rewrite the equation (31) in the form

$$\frac{d\widehat{y}_1}{d\tau} = b(\widehat{y}_1) + a(\widehat{y}_1) w(\tau). \tag{34}$$

If $p(\tau, \widehat{y}_1)$ is the probability density for solutions to the SDE (34), then density will satisfy the following forward Kolmogorov equation:

$$\frac{\partial p(\tau, \widehat{y}_1)}{\partial \tau} = \frac{1}{2} \frac{\partial^2}{\partial \widehat{y}_1^2} \left(a^2(\widehat{y}_1) p(\tau, \widehat{y}_1)\right) - \frac{\partial}{\partial \widehat{y}_1} \left(b(\widehat{y}_1) p(\tau, \widehat{y}_1)\right),$$

and the invariant density $p(\widehat{y}_1)$, if it exists, is [17]

$$p(\widehat{y}_1) = \frac{C}{b(\widehat{y}_1)} \exp\left(2 \int^{\widehat{y}_1} \frac{a^2(u)}{b(u)} du\right), \tag{35}$$

where the normalization constant C is defined from the conditions of normalization $p(\widehat{y}_1)$

$$C \int_D \frac{1}{b(\widehat{y}_1)} \exp\left(2 \int^{\widehat{y}_1} \frac{a^2(u)}{b(u)} du\right) d\widehat{y}_1 = 1.$$

In this way, the view $p(\widehat{y}_1)$ does not depend on the intensity of fluctuation of SC σ_1 and is defined by the parameter of exterior influence y_2^s and by the intensity of fluctuations of σ_2, σ_3 H and SA, respectively.

5 An Empirical Application

The article of the second author [44] provides results for the definition of scientific capital 3450 of Russian researchers, based on data collected in the course of a HSE study "Monitoring Careers of Doctorate Holders" undertaken in the framework of the OECD / UNESCO Institute of Statistics / Eurostat project "Careers of Doctorate Holders". Statistical tests using the Kolmogorov – Smirnov test ($z = 0.531$, $p = 0.940$) showed that the distribution of scientific capital corresponds to gamma distribution with the parameters Gamma(1.642; 2.457^{-10}). It is obvious that gamma distribution represents a special case (35). Therefore we can confirm that empirical data does not contradict the stationary asymptotics described by the above model RDS "SC – H – SA".

6 Conclusion and Discussion

One of the peculiarities of the model used in sociology for studying the system "SC – H – SA" is its conscious employment of the hierarchy of distinctive times of change of SC and SA. It finds its mathematical validity in the method of singular disturbances applied to ODEs and to SDEs. The asymptotic behavior of the DS or RDS with dedicated fast and slow components is characterized by the fact that SA, which possesses a short relaxation time, has time to relax to stationary values caused by SC, which corresopends to the maximum time scale. As a result, the evolution of the system "SC – H – SA" is determined by changes in SC, while SA can be excluded from consideration. Such analysis based on the Tikhonov's theorem or on the Kabanov and Pergamenshchikov's theorem helps to understand the SC as a perfect final state, to which SA approaches in the short term.

From a phenomenological point of a view, the relaxation mode of behavior of the DS "SC – H – SA" is realized if the typical relaxation time of SC is much greater than the relaxation times of H and SA. The commensurability of individual relaxation times between SC and H results in a reactive mode of behavior. If the relaxation times of SC, H, and SA are commensurate, then the DS "SC – H – SA" can transform into a stochastic mode of behavior, characterized by a strange attractor.

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