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Rewarding Results or Rewarding Efforts?

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Abstract

The main objective of teaching is to make sure that the students learn the required material. From this viewpoint, it seems reasonable to reward students when they learn this material. In other words, it seems reasonable to assign the student's rewards (such as grades) based on their level of knowledge. However, experiments show that, contrary to this seemingly intuitive conclusion, we achieve much better results if we base rewards not only on the students' results, but also on the student's efforts. In this paper, we use the known fact about optimal control to explain this seemingly counterintuitive phenomenon.

1 Formulation of the Problem

Normally, grades reflect results. The main objective of teaching is to make sure that the students learn the material. From this viewpoint, it seems reasonable to assign grades based on the results, based on how well the students learned – and this is how we normally grade.

This does not mean that effort is not taken into account: in the borderline cases, when a grade is between A ("excellent") and B ("good"), or between B ("good") and C ("satisfactory"), we often give extra points for extra effort. However, these extra points form a small portion of the overall grade.

Seemingly counter-intuitive empirical observation. At first glance, it seems reasonable to reward results not efforts. However, contrary to this seemingly reasonable conclusion, experiments have shown that if we give more weight to *efforts* when deciding on the reward, students achieve better *results*; see, e.g., [1, 2, 3] and references therein.

What we do in this paper. In this paper, we provide an explanation for this seemingly counter-intuitive phenomenon.

2 Our Explanation

Our main idea: teaching can be viewed as a particular case of the general engineering control problem. The main objective of teaching is to optimally change the students' state of the mind. From the general engineering viewpoint, the problem of (optimally) changing the system's state is called the *control* problem. Thus, the problem of optimal teaching can be viewed as a particular case of a general problem of optimal control.

What is known about optimal control. Different control problems have different objectives. In the simplest case, the objective is to keep the value of a certain quantity at its desired level. For example, it is desirable to control the electric grid so that the voltage remains the same irrespective of the load. When we control a chemical plant, we want to make sure that we maintain the temperature at which the corresponding chemical reactions are the most efficient. An autopilot tries to preserve the same direction and speed of the plane, etc.

In such cases, what we want is to maintain the desired value x_0 of the corresponding quantity x. At first glance, it seems like the best way to do it is to react to deviations from x, i.e., to set up a system that engages the control force every time the actual value x deviates from the desired value x_0 – and the larger the deviation $q \stackrel{\text{def}}{=} x - x_0$, the larger the corresponding control force.

However, both mathematical analysis of the problem and control practice show that much better results are achieved when we use a proportional-integral-derivative (PID) controller, i.e., a controller whose control value depends not only on the deviation q, but also on the time derivative $\dot{q}$ and on the integral $\int^t q(s)\,ds$.

This explain the need to reward efforts. When we apply this general control conclusion to the teaching situation, we conclude that the best teaching is achieved when the reward (i.e., control) depends not only on the results q, but also on the first derivative $\dot{q}$ of q – i.e., on efforts. This is exactly what is empirically observed.

What we did. Thus, we explained the seemingly counter-intuitive empirical fact as a natural consequence of optimal control theory.

How this is useful. The fact that general control theory ideas lead to recommendations that are consistent with the educational experience makes us believe that a general use of control theory results can lead to better education.

3 Additional Result

How to interpret the integral part of the optimal control? In the above text, we used the proportional (P) and the derivative (D) components of the optimal control. To make the control really optimal, we also need to take into account the integral (I) part.

This part actually has a very good education interpretation: namely, it corresponds to the difference between the traditional Russian grading system and the American system. In Russia, the grade for the class used to be determined exclusively by the grade on the final exam, i.e., by the amount of knowledge gained by the student. In contrast, in the US, the grade on the final exam constitutes only a part of the final grade, other points come from averaging the grades obtained at all the previous class assignments, quizzes, and tests. Each such previous grade reflects the student's previous state of knowledge $q(s)$, $s < t$; thus, averaging over all such grades means, in effect, that we integrate $q(s)$ over such past time moments.

The fact that an integral term is important for optimal control indicates that the accumulating-grade American system is potentially better than the traditional Russian system – in which the grade depends only on the final knowledge.

References

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