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Creep Transition in Non Homogeneous Thick-Walled Circular Cylinder under Internal and External Pressure

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Abstract

Creep stresses have been obtained for thick-walled circular cylinder under internal and external pressure using transition theory which is based on the concept of generalized principal strain measure. Results have been analyzed and discussed numerically as well as graphically. From the analysis, we can conclude that circular cylinder with external pressure and with nonlinear measure (N = 5) is on the safer side of the design as circumferential stresses are less for cylinder with above mentioned parameters as compared to other cases.

Mathematics Subject Classification: 74C20, 74C99

Keywords: Creep, Cylinder, Pressure, Non-homogeneous, Thermal

1 Introduction

Thick – Cylinders are very common component employed in numerous applications such as pressure vessels (*i.e.* hydraulic cylinders, gun barrels, pipes, boiler sand fuel tanks), accumulator shells, nuclear reactors, and military applications, pressure vessel

for industrial gases or a media transportation of high-pressurized fluids and piping of nuclear reactors. Creep analyses of thick walled cylinders subjected to various types of loading have been carried out by many investigators for steady as well as non-steady state creep. For example Creep analysis of thick-walled orthotropic cylinder made of isotropic material subjected to internal pressure has been presented by Weir, 1957; King and Mackie 1967 and Pai 1967. Gupta and Sharma [5] obtained solution for a thick-walled internally pressurized cylinder by using Seth's transition theory.
In the present work, we propose an approach of transition theory to determining the thermal stresses for non-homogeneous thick walled cylinder under internal and external pressure using the generalized principal strain measure theory [3-4].

2 Governing Equations

Consider a non-homogeneous thick-walled circular cylinder of internal and external radii a and b respectively, subjected to internal pressure p_1 and external pressure p_2 on the inner surface $r = a$ and outer surface $r = b$ respectively. The non-homogeneity in the cylinder is due to variation of compressibility $C = C_0\ r^{-k}$; where $a \le r \le b$; C_0 and k are constants.
In cylindrical polar co-ordinates the displacements are given by [3-4],

$$u = r(1-\beta)\ ; \quad v = 0 \quad \text{and} \quad w = d\ z \tag{1}$$

where β is a function of $r = \sqrt{x^2 + y^2}$ and d is a constant.
The generalized components of strain are

$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right]; e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right]; e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right];\ e_{r\theta} = e_{\theta z} = e_{zr} = 0 \tag{2}$$

where n is the measure and $\beta' = \frac{d\beta}{dr}$.
The stress strain relation for elastic isotropic material is

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij}\ ; \qquad (i, j = 1,2,3) \tag{3}$$

where $I_1 = e_{kk}$; T_{ij}, e_{ij} are stress and strain tensors respectively, λ, μ are Lame's constant, δ_{ij} is Kronecker delta.
Equations of equilibrium are all satisfied except

$$\frac{d}{dr}(T_{rr}) + \frac{(T_{rr} - T_{\theta\theta})}{r} = 0 \tag{4}$$

Using equation (4) in equation (5), one get a non linear differential equation in β as

$$nP\beta(P+1)^{n-1}\frac{dP}{d\beta}=r\left(\frac{\mu'}{\mu}-\frac{C'}{C}\right)[\{(3-2C)-(1-C)(1-d)^n\}\frac{1}{\beta^n}-(1-C)-(P+1)^n]+$$

$$+C[1-(P+1)^n]+r\,C'[1-\{2-(1-d)^n\}\frac{1}{\beta^n}]-nP[(1-C)+(P+1)^n]$$

where $r\beta'=\beta\,P$ and $C=\dfrac{2\mu}{(\lambda+2\mu)}$

The transition point of β in equation (6) are $P\to-1$ and $P\to\pm\infty$.

The boundary conditions are

$$T_{rr}=-p_1 \text{ at } r=a;\; T_{rr}=-p_2 \text{ at } r=b. \quad (6)$$

The resultant axial force in the cylinder is given by

$$L=2\pi\int_a^b rT_{zz}dr \quad (7)$$

3 Solution through The Principal Stress Difference:

The transition function R [3-4] is defined as

$$R\;=T_{rr}-T_{\theta\theta}=\frac{2\mu\beta^n}{n}\left[1-(P+1)^n\right] \quad (8)$$

Taking the logarithmic differentiation of equation (8) w.r.t. r and taking asymptotic value of β as $P\to-1$ and integrating, we get

$$R=A\frac{\mu^2}{C\,r^{2n}}\exp f \quad (9)$$

where A is the constant of integration and the value of f is

$$f=\int\left\{(n-1)C\,r^{-1}+C'\beta^{-n}(2-(1-d)^n)-C\mu'\mu^{-1}-(\mu'\mu^{-1}-C'C)\left[(3-2C)-(1-C)(1-d)^n\beta^{-n}\right]\right\}dr$$

From equations (8) and (9) using equation (5), we get

$$T_{rr}=-p_2+A\int_r^b Fdr;\; T_{\theta\theta}=-p_2+A\left[\int_r^b Fdr-rF\right];\; T_{zz}=\left(\frac{1-C}{2-C}\right)(T_{rr}+T_{\theta\theta})+2\mu\left(\frac{3-2C}{2-C}\right)e_{zz}$$

where $A=\dfrac{p_2-p_1}{\int_a^b Fdr}$; $F=\left(\mu^2/C\;r^{2n+1}\right)\exp f$ asymptotic value of β as $P\to-1$ is D/r, D being a constant.

Applying the closed end condition (8) in equation (10), we get

$$e_{zz}=\left[\frac{L}{2\pi}-\int_a^b\frac{r(1-C)(T_{rr}+T_{\theta\theta})}{(2-C)}dr\right]/\lambda\int_a^b\frac{rC(3-2C)}{(1-C)(2-C)}dr \quad (11)$$

As the non homogeneity in the cylinder is due to variable compressibility C, the creep stresses in a non homogeneous cylinder under internal and external pressure have been obtained using the following non-dimensional components

$$R_0 = \frac{a}{b};\ R = \frac{r}{b};\ \sigma_r = \frac{T_{rr}}{E};\ \sigma_\theta = \frac{T_{\theta\theta}}{E};\ \sigma_z = \frac{T_{zz}}{E};\ P_1 = \frac{p_1}{E};\ P_2 = \frac{p_2}{E}\ \text{as}$$

$$\sigma_r = -P_2 - \frac{(P_2 - P_1)\int_R^{R_0} F_1\, dr}{\int_{R_0}^1 F_1\, dr};\ \sigma_\theta = \sigma_r - RA_2F_1\ ;\ \sigma_z = \left(\frac{1 - C_0R^{-k}b^{-k}}{2 - C_0R^{-k}b^{-k}}\right)(\sigma_\theta + \sigma_r) + e_{zz}$$

$$\text{where } e_{zz} = 2\left\{\frac{L}{2\pi} - \int_{R_0}^1 Rb\left(\frac{1 - C_0R^{-k}b^{-k}}{2 - C_0R^{-k}b^{-k}}\right)(\sigma_\theta + \sigma_r)\, dR\right\} / \left(1 - R_0^2\right) \tag{12}$$

$$A_2 = \frac{P_2 - P_1}{\int_{R_0}^1 bF_1\, dr};\ F_1 = \frac{E^2}{4C_0}\left(\frac{2 - C_0R^{-k}b^{-k}}{3 - 2C_0R^{-k}b^{-k}}\right)^2 b^{k-2n-1}R^{k-2n-1}\exp f_1\ ;$$

$$f_1 = \frac{-(n-1)C_0R^{-k}b^{-k}}{k} - \frac{2kC_0R^{n-k}b^{n-k}}{D^n(n-k)} + \frac{kC_0}{D^n}\int\left(\frac{3 - 2C_0R^{-k}b^{-k}}{1 - C_0R^{-k}b^{-k}}\right)R^{n-k-1}b^{n-k}dR + \log\left(1 - C_0R^{-k}b^{-k}\right)$$

4 Numerical Discussion

The definite integrals in equations (12) have been evaluated by using Simpson's rule by taking D = 1. Curves have been drawn for radial and circumferential stresses for measures N = 1 and 5 with respect to the radii ratio R for different internal and external pressure as shown in figures 1 to 3 for k = -5, -3, -1 respectively. In classical theory, the measure N is equal to (1/n). From figure 1, it has been observed that for circular cylinder under external pressure, stresses are of compressible nature and are maximum at internal surface. Also compressible circumferential stress is maximum for highly compressible circular cylinder as compared to less compressible cylinder. Also it has been noted that with the change in measure from linear to nonlinear, circumferential stress goes on decreasing. With the increase in external pressure, for linear measure, circumferential stress increases. Again highly compressible cylinder is having high stress as compared to less compressible cylinder. Again, with the increase in measure, stress is going on decreasing. From figure 2, it has been observed that for cylinder without external pressure, circumferential stress again is of compressible nature and increases with the increase in measure. From the figure 3, it has been observed that for the cylinder whose external pressure has been increased and is more than that of internal pressure, circumferential stresses are maximum at the internal surface and are high as compared to the cylinder whose external pressure is zero. Also highly compressible cylinder having high stress as compared to less compressible cylinder. Also circumferential stresses are going on increasing for the cylinder whose internal pressure has been increased without external pressure. It has been noted that the increase in external pressure also increases circumferential stresses significantly.

Conclusion

From the above analysis we can conclude that circular cylinder with external pressure (without internal pressure or less internal pressure as compared to external pressure) and with nonlinear measure (N = 5) is on the safer side of the design because circumferential stresses are less for cylinder with above mentioned parameters as compared to other cases.

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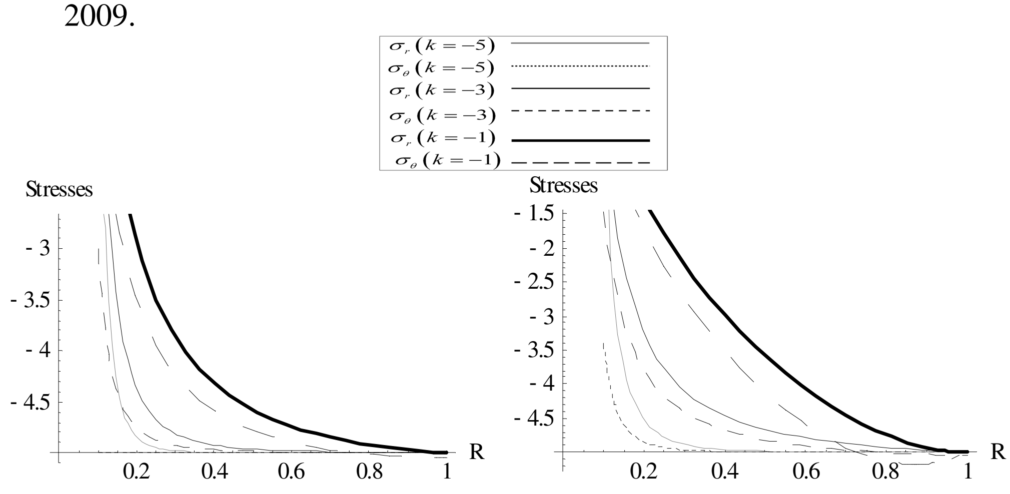

Figure 1. Creep Stresses for a thick-walled circular cylinder under internal pressure $(p_1 = 0)$ and external pressure $(p_2 = 5)$ for measure N = 1, 5 resp.

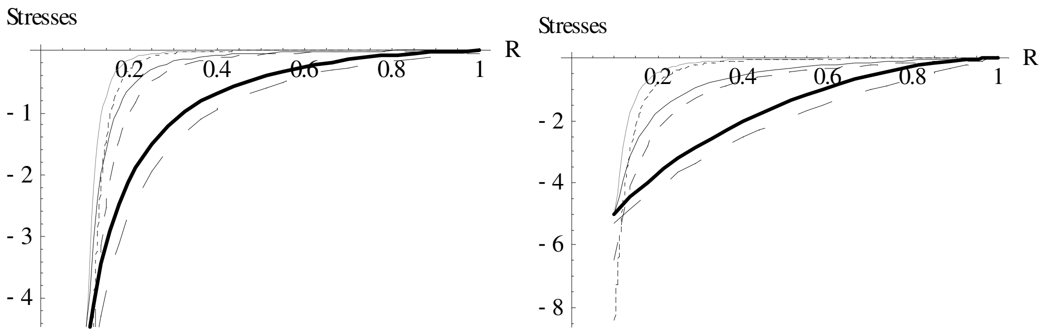

Figure 2. Creep Stresses for a thick-walled circular cylinder under internal pressure $\left(p_1 = 5\right)$ and external pressure $\left(p_2 = 0\right)$ for measure N = 1, 5 resp.

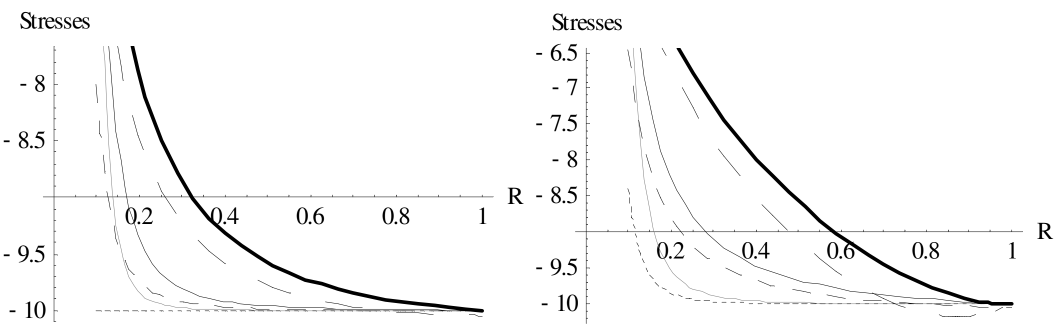

Figure 3. Creep Stresses for a thick-walled circular cylinder under internal pressure $\left(p_1 = 5\right)$ and external pressure $\left(p_2 = 10\right)$ for measure N = 1, 3, 5 resp.

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