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Dynamical Analysis of a Delayed Pest Management SEI System with Birth Pulse and Impulsive Harvesting at Different Moments

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Abstract: According to biological strategy for pest control, we consider a delayed pest management SEI model with birth pulse and impulsive harvesting at different moments? we get the conditions of the globally attractive infection-free boundary periodic solution of the system.

Keywords: delayed pest management SEI model, birth pulse, impulsive harvesting, globally stable

1. Introduction

Times delays of one type or another have been incorporated into biological models by author[1]. In general, delay-differential equations exhibit much more complicated dynamics than ordinary differential equations. Recently, impulsive equations with time delay have been studied[3], however, delay pest management epidemic models with impulsive effects at different moment, which add a constant have never been seen by now. Time delay and impulse are introduced into pest management epidemic disease models, which greatly enrich biologic background. Therefore, in present paper, based on biological control strategy in pest management, we consider the latent period for a pest management SEI model with impulsive control, which contains not only the process of periodic birth pulse but also periodic harvest pests by the specificity pesticide at different moment. Because the specificity pesticide does not affect the exposed, infectious pests, i.e., the specificity pesticide could only harvest the susceptible.

2. Global attractivity

In the following, we consider a delayed pest management SEI model with birth pulse and impulsive harvesting on susceptible at different moments

$$\begin{cases} \left.\begin{aligned} &\dot{S}(t) = A - \beta S(t)I(t) - bS(t), \\ &\dot{E}(t) = \beta S(t)I(t) - e^{-b\tau}\beta S(t-\tau)I(t-\tau) - bE(t), \\ &\dot{I}(t) = e^{-b\tau}\beta S(t-\tau)I(t-\tau) - bI(t) - \alpha I(t), \end{aligned}\right\} t \neq (n+l)T, t \neq (n+1)T, \\ \left.\begin{aligned} &\Delta S(t) = rS(t)(1-S(t)), \\ &\Delta E(t) = 0, \\ &\Delta I(t) = 0, \end{aligned}\right\} t = (n+l)T, n = 1, 2, \ldots \\ \left.\begin{aligned} &\Delta S(t) = -pS(t), \\ &\Delta E(t) = 0, \\ &\Delta I(t) = 0, \end{aligned}\right\} t = (n+1)T, n = 1, 2, \ldots \end{cases} \tag{2.1}$$

the initial condition for (2.1) is

$$(\eta_1(\varsigma), \eta_2(\varsigma), \eta_3(\varsigma)) \in C_+ = C([-\tau, 0], R_+^3, \eta_i(0) > 0), i = 1, 2, 3. \tag{2.2}$$

Where $S(t)$, $E(t)$ and $I(t)$ are densities of susceptible, exposed, infectious pests at time t, respectively. b is the natural death rate of the susceptible, exposed and infectious pests. A is population, which joins in the susceptible, and $A < b$. τ is the latent period of the disease, β is the contact rate, α is the death rate because of disease(the disease-related death rate). $S(t)$ in the absence of $I(t)$ grows logistically with an intrinsic birth rate constant r. The pulse birth and impulsive harvesting occurs every T period. $rS(t)(1-S(t))$ represents the birth pulse effort of pest population at $t = (n+l)T, 0 < l < 1, n \in Z^+$. $A, \beta, b, \tau, T, r, p$ are positive constants, in this paper, we always assume that $K = 1$, $0 \le p < 1$ represents the harvesting effort of the susceptible pests at $t = (n+l)T$, $n \in Z^+$. $\Delta S(t) = S(t^+) - S(t)$, $\Delta E(t) = E(t^+) - E(t)$, $\Delta I(t) = I(t^+) - I(t)$.

Since the equation for $E(t)$ of the system (2.1) is independent of other equations. We simplify system (2.1) and restrict our attention to the following system.

$$\begin{cases} \left.\begin{array}{l} \dot{S}(t) = A - \beta S(t)I(t) - bS(t), \\ \dot{I}(t) = e^{-b\tau}\beta S(t-\tau)I(t-\tau) - bI(t) - \alpha I(t), \end{array}\right\} t \neq (n+l)T, t \neq (n+1)T, \\ \left.\begin{array}{l} \Delta S(t) = rS(t)(1-S(t)), \\ \Delta I(t) = 0, \end{array}\right\} t = (n+l)T, n = 1, 2, \ldots, \\ \left.\begin{array}{l} \Delta S(t) = -pS(t), \\ \Delta I(t) = 0, \end{array}\right\} t = (n+1)T, n = 1, 2, \ldots. \end{cases}$$

$$(\eta_2(\varsigma), \eta_3(\varsigma)) \in C_+ = C([-\tau, 0], R_+^3, \eta_i(0) > 0), i = 2, 3. \tag{2.3}$$

We will firstly obtain the sufficient condition of the stability of infection-free periodic solution of system (2.1) with (2.2).

If the absence of $I(t)$ the system (2.3) reduces to

$$\begin{cases} \dot{S}(t) = A - bS(t), & t \neq (n+l)T, t \neq (n+1)T, \\ \Delta S(t) = rS(t)(1-S(t)), & t = (n+l)T, \\ \Delta S(t) = -pS(t), & t = (n+1)T, n \in Z^+. \end{cases} \tag{2.4}$$

By calculation, we can get the analytic solution of system (2.4) between pulses,

$S(t) =$

$$\begin{cases} \left(S(nT^+) - \lambda\right)e^{-b(t-n\tau)} + \lambda, & t \in [nT, (n+l)T), \\ \left[e^{-blT}(1+r-2\lambda r)\left(S(nT^+)-\lambda\right) - re^{-2blT}\left(S(nT^+)-\lambda\right)^2 + \lambda r(1-\lambda)\right]e^{-b(t-(n+l)\tau)} + \lambda, & t \in ((n+l)T, (n+1)T]. \end{cases}$$

Where $\lambda = \dfrac{A}{b} > 0.$ Considering the last two equations of system (2.4), we have the stroboscopic map of system (2.4)

$$S\left((n+1)T^+\right) = (1-p)\lambda + (1-p)e^{-b(1-l)T} \left[e^{-blT}(1+r-2\lambda r)\left(S(nT^+)-\lambda\right) - re^{-2blT}\left(S(nT^+)-\lambda\right)^2 + r\lambda(1-\lambda)\right]. \tag{2.5}$$

Let

$$F(S) = (1-p)\lambda + (1-p)e^{-b(1-l)T} \cdot \left[e^{-blT}(1+r-2\lambda r)\left(S(nT^+)-\lambda\right) - re^{-2blT}\left(S(nT^+)-\lambda\right)^2 + r\lambda(1-\lambda)\right],$$

If $\lambda < 1$ and $r > \dfrac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$,there exists only one positive fixed point denoted by

$A_1(S_1)$, the positive fixed point $A_1(S_1)$ is locally stable, if $r > \dfrac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$. Where

$$S_1 = S^* + \lambda$$

$$= \frac{(1-p)(1+r-2\lambda r)e^{-bT} - 1 + \sqrt{\left[(1-p)(1+r-2\lambda r)e^{-bT} - 1\right]^2}}{2(1-p)re^{-b(1+l)T}} +$$

Since

$$\frac{4r(1-p)e^{-b(1+l)T}\left[\lambda r(1-\lambda)(1-p)e^{-b(1-l)T} - \lambda p\right]}{2(1-p)re^{-b(1+l)T}} + \lambda$$

$$\left|\frac{dF(S)}{dS}\right|_{S=S_1} = 1 - \sqrt{\left[(1-p)(1+r-2\lambda r)e^{-bT} - 1\right]^2 + 4r(1-p)e^{-b(1+l)T}\left[\lambda r(1-\lambda)(1-p)e^{-b(1-l)T} - \lambda p\right]}$$
$$< 1,$$

and instable if $r < \dfrac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$.

Moreover, we can show $A_1(S_1)$ is globally stable, this is true when the following conditions are satisfied [3]:

(i) If $S_1 > S > 0$ then $S_1 > F(S) > S$; (ii) If $S > S_1$, then $S > F(S) > S_1$.

By calculation, we obtain

$$\left|\frac{dF}{dS}\right| = e^{-bT}(1-p)(1+r-2\lambda r) - 2(1-p)re^{-b(1+l)T}\left(S(nT^+) - \lambda\right) > 0.$$

This deduces $S_1 > F(S)$, when $S_1 > S > 0$, we know

$$F(S) - S = (1-p)e^{-b(1-l)T}\left[e^{-blT}(1+r-2\lambda r)\left(S(nT^+) - \lambda\right) - re^{-2blT}\left(S(nT^+) - \lambda\right)^2 + r\lambda(1-\lambda)\right]$$
$$- p\lambda - (S - \lambda) > 0,$$

This yield $F(S) > S$. So we get $S_1 > F(S) > S$, under the assumption $S_1 > S > 0$, otherwise

$S > F(S) > S_1$, if $S > S_1$. Thus the condition (i) and (ii) are satisfied.

Now we can deduce the positive equilibrium $A_1(S_1)$ of system (2.5) is globally stable, and the corresponding positive periodic $\tilde{S}(t)$ is the following, which is also globally stable:

$$\tilde{S}(t)=\begin{cases} S^{*}e^{-b(t-nT)}+\lambda, t\in\left[nT,(n+l)T\right), \\ [e^{-blT}(1+r-2\lambda r)S^{*}-re^{-2blT}S^{*2}+\lambda r(1-\lambda)]e^{-b(t-(n+l)T)}+\lambda, \\ \quad t\in\left((n+l)T,(n+1)T\right]. \end{cases} \tag{2.6}$$

While if $\lambda<\frac{1}{2}$, $\frac{p}{(1-\lambda)e^{blT}-p(1-\lambda)-\lambda p^{2}e^{-blT}}\le r<\frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$, eq (2.6) has two positive fixed points denoted by $A_2(S_2)$ and $A_2(S_3)$, where

$S_2=S^{*}+\lambda$

$$=\frac{(1-p)(1+r-2\lambda r)e^{-bT}-1+\sqrt{\left[(1-p)(1+r-2\lambda r)e^{-bT}-1\right]^{2}+4r(1-p)e^{-b(1+l)T}\left[\lambda r(1-\lambda)(1-p)e^{-b(1-l)T}-\lambda p\right]}}{2(1-p)re^{-b(1+l)T}}+\lambda,$$

$S_3=$

$$\frac{(1-p)(1+r-2\lambda r)e^{-bT}-1-\sqrt{\left[(1-p)(1+r-2\lambda r)e^{-bT}-1\right]^{2}+4r(1-p)e^{-b(1+l)T}\left[\lambda r(1-\lambda)(1-p)e^{-b(1-l)T}-\lambda p\right]}}{2(1-p)re^{-b(1+l)T}}+\lambda,$$

The positive fixed point $A_2(S_2)$ and $A_2(S_3)$ are constant instable. On the one hand, because

$\left|\frac{dF(S)}{dS}\right|_{S=S_2}<1$, we obtain that $r>\frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$, obviously, it is contradiction with the condition of $\frac{p}{(1-\lambda)e^{blT}-p(1-\lambda)-\lambda p^{2}e^{-blT}}\le r<\frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$. On the other hand,

$\left|\frac{dF(S)}{dS}\right|_{S=S_3}<1$, evidently, inequality is not set up constantly.

Concluding above results, we obtain the following proposition:

Theorem 4.1. System (2.5) has only one positive fixed point $A_1(S_1)$ when $\lambda<1$, $r>\frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$; when $\lambda<\frac{1}{2}$ and $\frac{p}{(1-\lambda)e^{blT}-p(1-\lambda)-\lambda p^{2}e^{-blT}}\le r<\frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$, eq (4.2) exist two positive equilibrium $A_2(S_2)$ and $A_2(S_3)$.

Theorem 4.2. Equilibrium $A_1(S_1)$ is locally stable if $r > \frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$ and it is globally stable, correspondingly, system (2.4) has a globally stable positive periodic solution $\tilde{S}(t)$, where

$$\tilde{S}(t)=\begin{cases} S^*e^{-b(t-nT)}+\lambda, & t\in[nT,(n+l)T), \\ [e^{-blT}(1+r-2\lambda r)S^*-re^{-2blT}S^{*2}+\lambda r(1-\lambda)]e^{-b(t-(n+l)T)}+\lambda, & t\in((n+l)T,(n+1)T]. \end{cases}$$

Theorem4.3. When $\lambda < \frac{1}{2}$ and $\frac{p}{(1-\lambda)e^{blT}-p(1-\lambda)-\lambda p^2e^{-blT}} \le r < \frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$, system (2.4) exist two positive fixed point $A_2(S_2)$ and $A_2(S_3)$, they are instability at any condition.

Theorem 4.4. If $r > \frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$ and $G_1 < 1$ hold, the infection-free periodic solution ($\tilde{S}(t)$, 0, 0) of system (2.1) with (2.2) is globally attractive, where

$$G_1=\frac{\beta e^{-b\tau}}{(b+\alpha)}\max\left\{S^*, e^{-blT}(1+r-2\lambda r)S^*-re^{-2blT}S^{*2}+\lambda r(1-\lambda)\right\}.$$

Proof: It is easy to see that the global attraction of infection-free periodic solution $(\tilde{S}(t),0,0)$ of system (2.1) with (2.2) is equivalent to the global attraction of infection-free periodic solution $(\tilde{S}(t),0,0)$ of system (2.3). So we only devote to system (2.3). Since $G_1 < 1$, so

$$e^{-b\tau} > \frac{\beta}{(b+\alpha)}\max\left\{S^*, e^{-blT}(1+r-2\lambda r)S^*-re^{-2blT}S^{*2}+\lambda r(1-\lambda)\right\}, \tag{2.7}$$

We can choose ε_0 sufficiently small such that

$$e^{-b\tau} > \frac{\beta}{(b+\alpha)}\left[\max\left\{S^*, e^{-blT}(1+r-2\lambda r)S^*-re^{-2blT}S^{*2}+\lambda r(1-\lambda)\right\}+\varepsilon_0\right]. \tag{2.8}$$

It follows from the first equation of system (2.3) that $\dot{S}(t) \le A-bS(t)$.

By $\lambda < 1$ and $r > \frac{pe^{b(1-l)T}}{(1-\lambda)(1-p)}$ and the comparison theorem of impulsive equation , Then there exists an integer $k_1 > k_{0,}\ t > k_1$, such that

$$S(t) < \tilde{S}(t) + \varepsilon_0 \le \max\left\{S^*, e^{-blT}(1+r-2\lambda r)S^* - re^{-2blT}S^{*2} + \lambda r(1-\lambda)\right\}$$
$$+\varepsilon_0 =: \delta, \quad nT < t \le (n+1)T, n > k_1.$$

From the second equation of the system (2.3), we have

$$\dot{I}(t) \le e^{-b\tau}\beta\delta I(t-\tau) - (b+\alpha)I(t), \quad t > nT+\tau, n > k_1.$$

Consider the following comparison differential equation $\dot{y}(t) = e^{-b\tau}\beta\delta y(t-\tau) - (b+\alpha)y(t), \quad t > nT+\tau, n > k_1.$

From (2.9), we have $e^{-b\tau}\beta\delta < b+\alpha,$ so we have $\lim_{t\to\infty} y(t) = 0.$

Let $(S(t), I(t))$ be the solution of system (2.3) and $I(\varsigma) = \eta_3(\varsigma)(\varsigma \in [-\tau, 0]),$ y(t) is the solution of system (2.4) with initial condition $y(\varsigma) = \eta_3(\varsigma)(\varsigma \in [-\tau, 0]).$ By the comparison theorem, we have $\lim_{t\to\infty} I(t) < \lim_{t\to\infty} y(t) = 0,$ incorporating into the positivity of $I(t),$ we have $\lim_{t\to\infty} I(t) = 0.$ Therefore, for any ε' (sufficiently small), there exists an integer $k_2 (k_2 T > k_1 T + \tau)$ such that $I(t) < \varepsilon'$ for all $t > k_2 T.$

From the first equation of the system (2.3), we have
$-(\beta\varepsilon' + b)S(t) + a \le \dot{S}(t) \le -bS(t) + a,$

then we have $y_1(t) \le S(t) \le y_2(t)$ and $y_1(t) \to \tilde{S}(t), y_2 \to \tilde{S}(t)$ as $t \to \infty.$ Where

$$\tilde{y}_1(t) = \begin{cases} y_1^* e^{-(\beta\varepsilon'+b)(t-nT)} + \lambda_1, & t \in [nT, (n+l)T), \\ [e^{-(\beta\varepsilon'+b)lT}(1+r-2\lambda_1 r)y^* - re^{-2(\beta\varepsilon'+b)lT}y_1^{*2} + \lambda_1 r(1-\lambda_1)]e^{-(\beta\varepsilon'+b)(t-(n+l)T)} + \lambda_1, & t \in ((n+l)T, (n+1)T]. \end{cases}$$

$$\tilde{y}_2(t) = \begin{cases} y_2^* e^{-b(t-nT)} + \lambda, & t \in [nT, (n+l)T), \\ [e^{-blT}(1+r-2\lambda r)y_2^* - re^{-2blT}y_2^{*2} + \lambda r(1-\lambda)]e^{-b(t-(n+l)T)} + \lambda, & t \in ((n+l)T, (n+1)T]. \end{cases}$$

Therefore, for any $\varepsilon_1, \varepsilon_2 > 0,$ there exists a integer $k_3, n > k_4$ such that

$$\tilde{y}_1(t) - \varepsilon_1 < S(t) < \tilde{y}_2(t) + \varepsilon_2.$$

Let $\varepsilon' \to 0,$ we have $\tilde{S}(t) - \varepsilon_1 < S(t) < \tilde{S}(t) + \varepsilon_2,$ for t large enough. Let

$\varepsilon_1, \varepsilon_2 \to 0,$ it implies $S(t) \to \tilde{S}(t)$ as $t \to \infty.$ This completes the proof.

References

[1] Q. Gan, R. Xu and P. Yang, Bifurcation analysis for a predator-prey system with prey dispersal and time delay, Int. J. Biomath. 1(2) (2008) 209-224.

[2] J. Jian and L. Chen, Global attractivity of a stage-structure variable coefficients predator-prey system with time delay and impulsive perturbations on predators, Int.[J]. Biomath.1 (2) (2008) 197-208.

[3] L. Chen, Biomathematical Models and Research (in Chinese) (Chinese Science and Technology Publishing House, Beijing, (1988), pp. 84.

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