Applied Mathematical Sciences, Vol. 5, 2011, no. 80, 3971 - 3979

An Extensive Study of Mathematical Wastewater Flow Model over Slime Layers

Mahmood K. Jasim

DMPS, College of Arts & Sciences, University of Nizwa, Oman
mahmoodkhalid@unizwa.edu.om

Afaf M. Weli

College of Pharmacy & Nursing, University of Nizwa, Oman
afaf@unizwa.edu.om

Abstract

A mathematical model for wastewater treatment using biofilm grown on a support media over slime layers was developed. An attempt has been made to set up a relationship between removal efficiency and dispersion characteristic of trickling filter. Analytic solutions have been obtained and analyzed.

Keywords: Mathematical Model, Microbiological System, Biotrickling Filters, Population dynamics, Biochemical Oxygen Demand

1. Introduction

Mathematical models have been available for a wide variety of biological systems, development and verification studies of models for trickling bed bioreactors remain sparse in the literature [1]. Trickling filter is a continuous reactor used for wastewater treatment. Many attempts have been made to incorporate substrate utilization kinetics into differential equations of dispersed flow models.

Fixed growth biological systems (Trickling filters) are those that contact wastewater with microbial growths attached to the surfaces of supporting media where the wastewater is sprayed over a bed of fixed media (crushed stone , plastic, ….etc).

As the wastewater flows over the slime layer (that coat the surface), organic matter and dissolved oxygen are extracted and metabolic end products such as carbon dioxide are released. Dissolved oxygen in the liquid is replenished by absorption from the air in the voids surrounding the filter media. Although very thin the biological layers is an aerobic at the bottom. Therefore, although biological filtration is commonly referred to as aerobic treatment, it is in fact a facultative system incorporating both aerobic and anaerobic activity, which is shown in figure below [1].

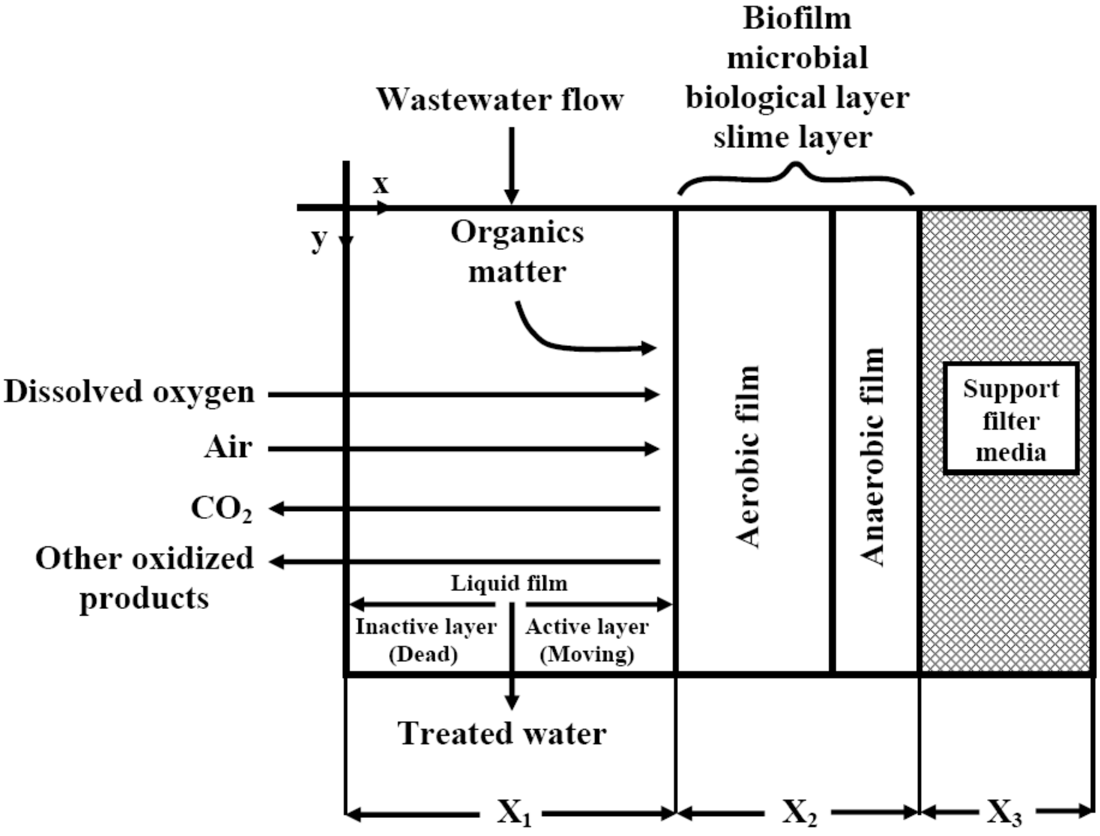

Fig. (1), illustrating the biological process in a filter bed (Trickling filter).

Biochemical oxygen demand (BOD) load on a trickling filter is calculated using the raw BOD, the primary effluent applied to the filter, with out regard to any BOD contribution in the re-circulated flow from the fine clarifier. Thus, BOD loading can be defined as the ratio between settled wastewater and volume of filter media. Where BOD loading equal to grams of BOD applied per cubic meter per day, settled BOD equal to raw wastewater BOD remaining after primary sedimentation grams per day. (gm/d) and volume of media equal to volume of stone in the filter cubic meters (m^3).

Hydraulic loading (Q_H: cubic meters per square meter per day) is the amount of liquid to the filter surface including both untreated wastewater (Q m^3/d) and recirculation flows (Q_R m^3/d), which is defined as:

$$Q_H = \frac{Q + Q_R}{A} \tag{1}$$

where A is the surface area of filters (m^2).

Recirculation ratio (R) is the ratio of re-circulated flow to the wastewater entering the treatment plant, which is defined by:

$$R = \frac{Q_R}{Q} \tag{2}$$

In the present article the optimal treatment system design incorporating with a trickling filter, anoxic and clarifier for nitrogen removal from domestic wastewater have been considered theoretically and analyzed with concern to reality.

2. Mathematical Models

Several mathematical equations have been developed for calculating BOD removal efficiency of biological filter based on such factors as depth of bed, kind of media, temperature , recirculation and organic loading [3,4,5,6,7,8,9,10,11].

The system is a symmetric trickle bed bioreactor of fixed biofilm thickness $\delta_i = x_2 - x_1$, with a liquid film having thickness δ_1, and gas surrounding it as shown in Figure 1. As a substrate enters the system with concentration ϕ_0, it transfers into the liquid by diffusion and convection. Then it transfers into the biofilm by diffusion due to the concentration gradient and is removed by the biomass [1]. So, extensive studies for two approaches (models) have been considered as follows:

I- Steady State Bio-Filtration Model

Steady state biofilm process models describing a soluble substrate and single species and well established. They have been applied successfully to various biofilm reactors. The fundamental equation governing transport of the substrate and carbon dioxide are dispersion, advection and transfer to the solids water phase. The change in concentration may be modeled:

$$\frac{\partial \phi}{\partial t} = \Delta \frac{\partial^2 \phi}{\partial x^2} - v \frac{\partial \phi}{\partial x} - \left(\frac{1}{n} - 1 \right) \left[k \left(k_n \phi - \phi_{as} \right) \right] \tag{3}$$

where ϕ : concentration in wastewater mgm/cm^3.

Δ: dispersion coefficient in water m^2/hr .

x : distance of travel in filter (m) .

t : time (hr) .

v : axial interstitial velocity of wastewater m/hr .

ϕ_{as} : concentration in solid water phase mg/cm^3 .

n : filter material porosity.

k : transfer rate constant (hr $^{-1}$)

k_n : equilibrium value for ratio of concentration in the solid water phase to air phase concentration.

The differential equations can be simplified to describe steady state conditions to biological filter. The solution were used to determine constants and for comparison with numerical solution under steady state condition (constant input concentrations and adsorptive equilibrium); Biological removal will occur as a result of biological degradation only.

$$\frac{\partial\phi}{\partial t}=\Delta\frac{\partial^2\phi}{\partial x^2}-v\frac{\partial\phi}{\partial x}-\left(\frac{1}{n}-1\right)\left[\frac{\partial\phi_{as}}{\partial t}+b_1\phi_{as}\right] \tag{4}$$

At equilibrium, the concentration in the air and solid water phases are proportional:

$$\phi_{as}=k_n\phi \tag{5}$$

So, $$\frac{\partial\phi_{as}}{\partial t}=k_n\frac{\partial\phi}{\partial t} \tag{6}$$

Substituting (5) and (6) in equation (4), we get

$$\frac{\partial\phi}{\partial t}=\Delta\frac{\partial^2\phi}{\partial x^2}-v\frac{\partial\phi}{\partial x}-\left(\frac{1}{n}-1\right)\left[k_n\frac{\partial\phi}{\partial t}+k_1k_n\phi\right] \tag{7}$$

The ratio of the masses of biomass in the two phases is:

$$k_m=\frac{\phi_{as}(1-n)}{\phi_n}=k_n\left(\frac{1}{n}-1\right) \tag{8}$$

Let us define the "retardation factor

$$R_f=\frac{\phi_A\quad total}{\phi_A\quad mobile\ \ phase}=1+k_m \tag{9}$$

Thus,

$$\frac{\partial\phi}{\partial t}=\Delta\frac{\partial^2\phi}{\partial x^2}-v\frac{\partial\phi}{\partial x}-\left[k_m\frac{\partial\phi}{\partial t}+k_1k_m\phi\right] \tag{10}$$

$$(1+k_m)\frac{\partial\phi}{\partial t}=\Delta\frac{\partial^2\phi}{\partial x^2}-v\frac{\partial\phi}{\partial x}-\left[k_m\frac{\partial\phi}{\partial t}+k_1k_m\phi\right] \tag{11}$$

Substituting the value of R_f :

$$\frac{\partial\phi}{\partial t}=\frac{\Delta}{R_f}\frac{\partial^2\phi}{\partial x^2}-\frac{v}{R_f}\frac{\partial\phi}{\partial x}-\frac{k_1k_m}{R_f}\phi \tag{12}$$

For steady state, and if the assumptions are made that dispersion is negligible, reduces to:

$$v\frac{d\phi}{dx}+k_1k_m\phi=0 \tag{13}$$

which is 1st order linear differential equation having the general solution:

$$\phi=\phi_i\, e^{\frac{-k_1k_mx}{v}} \tag{14}$$

where ϕ_i is the initial substrate concentration mg / cm^3

II- Biofilm model

Biofilm is very complex, both physically and microbiologically [12]. These dense layers of bacteria are characterized by their ability to adhere a solid medium. The conceptual model used in this model consists of parallel planes over which biofilm grow. Substrate concentration in the *x* direction decreases from liquid surface to biofilm surface due to mass – transfer inside the liquid layer. The mass flux N_x of substrate through the liquid film $(0<x<x_1)$ and biofilm$(x_1<x<x_2)$ at certain, arbitrary location along the axis of a bioreactor is given by [4,5,6]:

$$N_x=-\frac{\Delta}{1-f_x}\frac{\partial\phi}{\partial x} \tag{15}$$

where

N_x is the substrate flux

Δ is the molecular diffusivity of substrate in the liquid film

x is the length dimension normal to biofilm surface

ϕ_x is the substrate concentration at any location inside that liquid layer

The mass flux balance between the diffusion and reaction, using Monod kinetics, at any point within the biofilm by assuming f_x is much less than unity, leads to a non-dimensional differential equation [1],

$$-\Delta\frac{\partial^2\phi}{\partial x^2}+\gamma_iq_m\frac{\phi}{K_s+\phi}=0 \tag{16}$$

where γ_i is the biomass density

q_m is the maximum specific uptake rate

K_s is the Monod saturation constant

Using ϕ_0 as the mole density scale and x_2 as a length scale and then the governing system for substrate concentration within liquid film and biofilm is given by:

$$\frac{d^2\phi}{dx^2}-\psi_i{}^2\frac{\phi}{1+\beta\phi}=0 \tag{17}$$

where $\psi_i=\sqrt{\frac{\gamma_iq_m}{\Delta K_s}}$ which measure the relative importance of the reaction and diffusion rate.

$\beta=\frac{\phi_0}{K_s}=0.0373$ is the dimensionless initial mole density [1].

As a test of accuracy and validation for the case $\phi << K_s$, the first order reaction occurs and the diffusion reaction equation becomes linear which is defined as:

$$\frac{d^2\phi}{dx^2} - \psi_i{}^2 \phi = 0 \tag{18}$$

with boundary conditions $\phi(0) = 1 \ \& \ \phi'(1) = 0$

Equation (18), gives the general solution

$$\phi = \frac{1}{1+e^{2\psi_i}} e^{\psi_i x} + \frac{1}{1+e^{2\psi_i}} e^{-\psi_i x} \tag{19}$$

Thus, the solution so obtained by (19) is matching with John W Barton et al (1998).
Now, to get the most general exact solution equation (17) can be written as:

$$\frac{d^2\phi}{dx^2} = \psi_i{}^2 \frac{\phi}{1+\beta\phi} \tag{20}$$

Multiply equation (20) by $2\frac{d\phi}{dx}$, we get

$$2\frac{d\phi}{dx}\frac{d^2\phi}{dx^2} = \psi_i{}^2 \frac{\phi}{1+\beta\phi} 2\frac{d\phi}{dx} \tag{21}$$

Integrating with respect to *x*, we get

$$\left(\frac{d\phi}{dx}\right)^2 = \frac{2\psi_i{}^2}{\beta}\phi - \frac{2\psi_i{}^2}{\beta^2} ln\left|1+\beta\phi\right| \tag{22}$$

This is the most general exact solution and can be written as:

$$\phi = \int \sqrt{\frac{2\psi_i{}^2}{\beta}\phi - \frac{2\psi_i{}^2}{\beta^2} ln\left|1+\beta\phi\right|}\ dx$$

3. Sensitive Analysis to the Result

The use of fixed film biological treatment systems for the treatment of domestic wastewater is not a new concept. Beginning with demonstration by Alexander Muller in 1865, and due to the complexity of the mass transport mechanisms, a 2nd order differential equations was used to develop models that describe the BOD and $No_{3-}N$ concentrations through the depth of the vessel [8]. Accordingly the mathematical modeling of pre-anoxic is the best system for denitrification in fixed film domestic wastewater treatment system with out an additional carbon source.

The proposed trickling filter model which is based on the theoretical description and model simulation calculations was tested for accuracy by comparing model predictions with several data to perform a better performance. Figure 2 below shows the behavior of effectness of diffusivity on percent removal

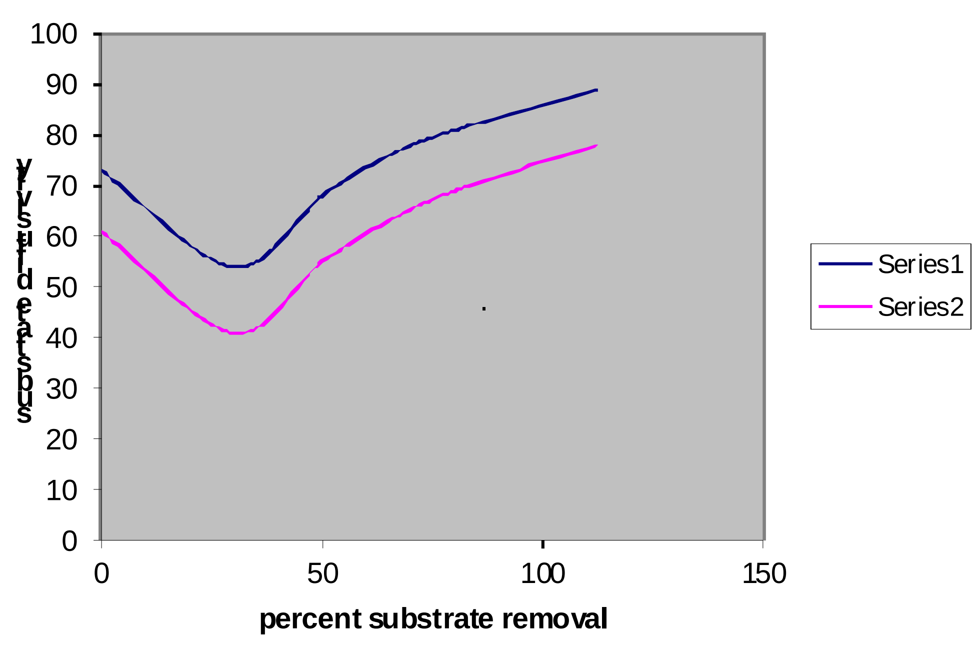
Fig. 2, Shows the effect of diffusivity on percent substrate removal

Summing up, the study provides a framework for BOD filter, derivations, modeling and the model so discussed is able to describe the dynamic biofilm growth. The model has been extensively verified using different approaches and shown that having the most general solution.

Furthermore, these models need to be tested with real data having more-profile data which to be used for the development of the models and will be considered by the authors.

Acknowledgments

The authors wish to express their sincere thanks to the referees, Editorial Board, as well as to Dr. Emil Minchev President of Hikari Ltd Managing Editor of Applied Mathematical Sciences Journal.

References

1- John W Barton et al, (1998), " Predictive Mathematical modeling of trickling bed biofilters for elucidating mass transfer and kinetic effects" ORNL/ CP-97102, CONF-980632.

2- Yaqiong Han, Weijiang Zhang and Jiao Xu, (2011), "A simplified mathematical model of multi species biofilm for simultaneous removal of sulfur dioxide and nitric oxide using a Biotrickling filter" African Journal of Microbiology research, Vol. 55, 541-550.

3- Mahmood K Jasim et al, (2006), "Mathematical model of fixed growth biological system" Published in College of Education Journal, Al-Mustansiryha University, Vol. 1.

4- Yilmaz Mulslu, (1990), "Use of dispersed flow models in design of biofilm reactors" 53, 297-314.

5- Yilmaz Mulslu (1995), "Kinetic of biofilm in unsaturated granular media", (1995), journal of environmental engineering. Vol. 121. No.1

6- Yilmaz Mulslu (1983) " Dispersion in the inclined plane model of trickling filter", water Res. 17(1) 105 – 115

7- Eckenfelder, W. W., Jr (1963) "Trickling filter design and performance", Transactions, American society of civil engineers, Vol. 128.

8- Jank, Bruce, E. and W. Ronald Drynam (1973) "Substrate removal mechanism of Trickling filters", journal environmental engineering division, proceedings American society of civil engineers, Vol. 99, EE3, 187.

9- Atkinson, Bernard and John. A. Howell, (1975) "slimeee holdup, influent BOD, and Mass transfer in trickling filters", journal environmental engineering division, proceeding American society of civil engineers, Vol. 101, EE4, 585.

10- Williamson, Kenneth and Perry L. McCarty, (1976) "A model of substrate utilization by bacterial films", journal water pollution, Vol. 48 ,1:9

11- Eunsung Kan, M A Deshusses, (2008), " Modeling of foamed emulsion bioreactor: Model development and experimental validation" Biotechnology and bioengineering Journal, Vol 99, No 5.

12- Leslie Grady, C P , Diagger, C T and Lim, H C, (1999)," Biological wastewater treatment", 2nd Ed., Marcel Dekker, Inc., New York.

Received: July, 2011