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Notes on Intuitionistic Fuzzy Ideals of a Hemiring

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Abstract. In this paper, we made an attempt to study the algebraic nature of intuitionistic fuzzy ideals of a hemiring.

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INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring ($R ; + ; .$). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras ($R ; + ; .$) share the same properties as a ring except that ($R ; +$) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a.(b+c) = a. b+a. c and (b+c).a = b.a+c.a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1.a =a = a.1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. A subset I of a hemiring R is called a left ideal of R if I is closed under addition and RI $\subseteq$ I. A subset I of a hemiring R is called a right ideal of R if I is closed under addition and IR $\subseteq$ I. Both left and right ideal I of a hemiring R is called a ideal of R. After the introduction of fuzzy sets by L.A.Zadeh[10], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subsets (IFS) was introduced by K.T.Atanassov[4], as a generalization of the notion of fuzzy set. The notion of Fuzzy left *h*-ideals in hemirings with respect to a *s*-norm was introduced in [2].In this paper, we introduce the some Theorems in intuitionistic fuzzy ideal of a hemiring.

1.PRELIMINARIES

1.1Definition: Let X be a non-empty set. A fuzzy subset A of X is a function $A : X \to [0, 1]$.

1.2 Definition: An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form $A = \{ < x, \mu_A(x) , \nu_A(x) > / x \in X \}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to$

[0,1] define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \le \mu_A(x) + \nu_A(x) \le 1$.

1.3 Definition: Let R be a hemiring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic fuzzy ideal(IFI) of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \ge \min\{ \mu_A(x), \mu_A(y) \}$,

(ii) $\mu_A(xy) \ge \max\{ \mu_A(x), \mu_A(y) \}$,

(iii) $\nu_A(x + y) \le \max\{ \nu_A(x), \nu_A(y) \}$,

(iv) $\nu_A(xy) \le \min\{ \nu_A(x), \nu_A(y) \}$, for all x and y in R.

1.4 Definition: Let A and B be intuitionistic fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as $AxB = \{\langle (x, y), \mu_{AxB}(x,y), \nu_{AxB}(x,y) \rangle$ / for all x in G and y in H $\}$, where $\mu_{AxB}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $\nu_{AxB}(x, y) = \max\{ \nu_A(x), \nu_B(y) \}$.

1.5 Definition: Let A be an intuitionistic fuzzy subset in a set S, the strongest intuitionistic fuzzy relation on S, that is an intuitionistic fuzzy relation on A is V given by $\mu_V(x,y)= \min\{\mu_A(x), \mu_A(y) \}$ and $\nu_V(x, y) = \max\{\nu_A(x), \nu_A(y) \}$, for all x and y in S.

2. PROPERTIES OF INTUITIONISTIC FUZZY IDEAL OF A HEMIRING R.

2.1 Theorem: Let (R, +, .) be a hemiring. Intersection of any two intuitionistic fuzzy ideal of a hemiring R is an intuitionistic fuzzy ideal of R.

Proof: Let A and B be any two intuitionistic fuzzy ideal of a hemiring R and let x and y in R. Let $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in R \}$ and $B = \{ (x, \mu_B(x), \nu_B(x)) / x \in R \}$ and also let $C= A \cap B =\{(x, \mu_C(x), \nu_C(x)) / x \in R \}$, where $\min \{ \mu_A(x), \mu_B(x) \} = \mu_C(x)$ and $\max\{\nu_A(x), \nu_B(x)\}=\nu_C(x)$. Now, $\mu_C(x+y) = \min\{\mu_A(x+y), \mu_B(x+y)\} \ge \min\{\min\{\mu_A(x), \mu_A(y) \}, \min\{\mu_B(x), \mu_B(y)\}\} = \min\{\min\{\mu_A(x), \mu_B(x)\},$

$\min\{\mu_A(y), \mu_B(y)\}\} = \min\{\mu_C(x), \mu_C(y)\}$. Therefore, $\mu_C(x+y) \geq \min\{\mu_C(x), \mu_C(y)\}$, for all x and y in R. And, $\mu_C(xy) = \min\{\mu_A(xy), \mu_B(xy)\} \geq \min\{\max\{\mu_A(x), \mu_A(y)\}, \max\{\mu_B(x), \mu_B(y)\}\} \geq \max\{\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}\} = \max\{\mu_C(x), \mu_C(y)\}$. Therefore, $\mu_C(xy) \geq \max\{\mu_C(x), \mu_C(y)\}$, for all x and y in R. Also, $\nu_C(x+y) = \max\{\nu_A(x+y), \nu_B(x+y)\} \leq \max\{\max\{\nu_A(x), \nu_A(y)\}, \max\{\nu_B(x), \nu_B(y)\}\} = \max\{\max\{\nu_A(x), \nu_B(x)\}, \max\{\nu_A(y), \nu_B(y)\}\} = \max\{\nu_C(x), \nu_C(y)\}$. Therefore, $\nu_C(x+y) \leq \max\{\nu_C(x), \nu_C(y)\}$, for all x and y in R. And, $\nu_C(xy) = \max\{\nu_A(xy), \nu_B(xy)\} \leq \max\{\min\{\nu_A(x), \nu_A(y)\}, \min\{\nu_B(x), \nu_B(y)\}\} \leq \min\{\max\{\nu_A(x), \nu_B(x)\}, \max\{\nu_A(y), \nu_B(y)\}\} = \min\{\nu_C(x), \nu_C(y)\}$. Therefore, $\nu_C(xy) \leq \min\{\nu_C(x), \nu_C(y)\}$, for all x and y in R. Therefore, C is an intuitionistic fuzzy ideal of a hemiring R. Hence, intersection of any two intuitionistic fuzzy ideal of a hemiring R is an intuitionistic fuzzy ideal of R.

2.2 Theorem: Let (R, +, .) be a hemiring. The intersection of a family of intuitionistic fuzzy ideals of R is an intuitionistic fuzzy ideal of R.

Proof: Let $\{V_i : i \in I\}$ be a family of intuitionistic fuzzy ideals of a hemiring R and let $A = \bigcap_{i \in I} V_i$. Let x and y in R. Then, $\mu_A(x+y) = \inf_{i \in I} \mu_{V_i}(x+y) \geq \inf_{i \in I} \min\{\mu_{V_i}(x), \mu_{V_i}(y)\} = \min\{\inf_{i \in I} \mu_{V_i}(x), \inf_{i \in I} \mu_{V_i}(y)\} = \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. And, $\mu_A(xy) = \inf_{i \in I} \mu_{V_i}(xy) \geq \inf_{i \in I} \max\{\mu_{V_i}(x), \mu_{V_i}(y)\} \geq \max\{\inf_{i \in I} \mu_{V_i}(x), \inf_{i \in I} \mu_{V_i}(y)\} = \max\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Also, $\nu_A(x+y) = \sup_{i \in I} \nu_{V_i}(x+y) \leq \sup_{i \in I} \max\{\nu_{V_i}(x), \nu_{V_i}(y)\} = \max\{\sup_{i \in I} \nu_{V_i}(x), \sup_{i \in I} \nu_{V_i}(y)\} = \max\{\nu_A(x), \nu_A(y)\}$. Therefore, $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. And, $\nu_A(xy) = \sup_{i \in I} \nu_{V_i}(xy) \leq \sup_{i \in I} \min\{\nu_{V_i}(x), \nu_{V_i}(y)\} \leq \min\{\sup_{i \in I} \nu_{V_i}(x), \sup_{i \in I} \nu_{V_i}(y)\} = \min\{\nu_A(x), \nu_A(y)\}$. Therefore, $\nu_A(xy) \leq \min\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. That is, A is an intuitionistic fuzzy ideal of a hemiring R. Hence, the intersection of a family of intuitionistic fuzzy ideals of R is an intuitionistic fuzzy ideal of R.

2.3 Theorem: If A and B are any two intuitionistic fuzzy ideal of the hemirings R_1 and R_2 respectively, then AxB is an intuitionistic fuzzy ideal of R_1xR_2.

Proof: Let A and B be two intuitionistic fuzzy ideal of the hemirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in R_1xR_2. Now, $\mu_{AxB} [(x_1, y_1) + (x_2, y_2)] = \mu_{AxB} (x_1 + x_2, y_1 + y_2)= \min\{ \mu_A(x_1 + x_2), \mu_B(y_1 + y_2) \}\geq \min\{\min\{ \mu_A(x_1), \mu_A(x_2) \}, \min\{ \mu_B(y_1), \mu_B(y_2) \}\}= \min\{\min \{\mu_A(x_1), \mu_B(y_1) \}, \min\{\mu_A(x_2), \mu_B(y_2) \}\}= \min\{\mu_{AxB} (x_1, y_1), \mu_{AxB} (x_2, y_2)\}$. Therefore, $\mu_{AxB} [(x_1, y_1) + (x_2, y_2)] \geq \min\{\mu_{AxB} (x_1, y_1), \mu_{AxB} (x_2, y_2) \}$. Also, $\mu_{AxB} [(x_1,y_1)(x_2,y_2)]=\mu_{AxB}(x_1x_2,y_1y_2)=\min\{\mu_A(x_1x_2), \mu_B(y_1y_2)\}\geq \min \{ \max\{ \mu_A(x_1), \mu_A(x_2) \}, \max\{ \mu_B(y_1), \mu_B(y_2) \} \} \geq \max\{ \min \{ \mu_A(x_1), \mu_B(y_1) \}, \min\{ \mu_A(x_2), \mu_B(y_2) \}\} = \max\{\mu_{AxB} (x_1, y_1), \mu_{AxB} (x_2, y_2) \}$.Therefore, $\mu_{AxB} [(x_1, y_1) (x_2, y_2)] \geq \max\{\mu_{AxB} (x_1, y_1), \mu_{AxB} (x_2, y_2) \}$.And,$\nu_{AxB} [(x_1, y_1)+(x_2, y_2)] = \nu_{AxB} (x_1+ x_2, y_1 + y_2) = \max\{ \nu_A(x_1+ x_2), \nu_B(y_1+ y_2) \}\leq \max\{\max\{ \nu_A(x_1), \nu_A(x_2) \}, \max\{\nu_B(y_1), \nu_B(y_2)\}\} = \max\{\max \{\nu_A(x_1),\nu_B(y_1) \}, \max\{ \nu_A(x_2), \nu_B(y_2) \} \}= \max\{ \nu_{AxB} (x_1, y_1), \nu_{AxB} (x_2, y_2)\}$. Therefore, $\nu_{AxB} [(x_1, y_1) +(x_2, y_2)] \leq \max\{\nu_{AxB} (x_1, y_1), \nu_{AxB} (x_2, y_2) \}$. Also, $\nu_{AxB} [(x_1, y_1)(x_2, y_2)] = \nu_{AxB} (x_1x_2, y_1y_2) = \max \{ \nu_A(x_1x_2), \nu_B(y_1y_2) \}\leq \max\{ \min\{\nu_A(x_1), \nu_A(x_2) \}, \min\{ \nu_B(y_1), \nu_B(y_2) \} \}\leq\min\{\max\{\nu_A(x_1), \nu_B(y_1) \}, \max\{\nu_A(x_2), \nu_B(y_2) \} \}= \min\{ \nu_{AxB} (x_1, y_1), \nu_{AxB} (x_2, y_2) \}$. Therefore, $\nu_{AxB} [(x_1, y_1)(x_2, y_2)] \leq \min\{\nu_{AxB} (x_1, y_1), \nu_{AxB} (x_2, y_2) \}$. Hence AxB is an intuitionistic fuzzy ideal of hemiring of R_1xR_2.

2.4 Theorem: Let A and B be intuitionistic fuzzy ideals of the hemirings R_1 and R_2 respectively. Suppose that e and e^l are the identity element of R_1 and R_2 respectively. If AxB is an intuitionistic fuzzy ideal of R_1xR_2, then at least one of the following two statements must hold.

(i) $\mu_B(e^l) \geq \mu_A(x)$ and $\nu_B(e^l) \leq\nu_A(x)$, for all x in R_1,

(ii) $\mu_A(e) \geq \mu_B(y)$ and $\nu_A(e) \leq \nu_B(y)$, for all y in R_2.

Proof: Let AxB be an intuitionistic fuzzy ideal of R_1xR_2. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in R_1 and b in R_2

such that $\mu_A(a) > \mu_B(e^l)$, $\nu_A(a) < \nu_B(e^l)$ and $\mu_B(b) > \mu_A(e)$, $\nu_B(b) < \nu_A(e)$. We have, $\mu_{AxB}(a, b) = \min\{\mu_A(a), \mu_B(b)\} > \min\{\mu_B(e^l), \mu_A(e)\} = \min\{\mu_A(e), \mu_B(e^l)\} = \mu_{AxB}(e, e^l)$. And, $\nu_{AxB}(a, b) = \max\{\nu_A(a), \nu_B(b)\} < \max\{\nu_B(e^l), \nu_A(e)\} = \max\{\nu_A(e), \nu_B(e^l)\} = \nu_{AxB}(e, e^l)$. Thus AxB is not an intuitionistic fuzzy ideal of R_1xR_2. Hence either $\mu_B(e^l) \geq \mu_A(x)$ and $\nu_B(e^l) \leq \nu_A(x)$, for all x in R_1 or $\mu_A(e) \geq \mu_B(y)$ and $\nu_A(e) \leq \nu_B(y)$, for all y in R_2.

2.5 Theorem: Let A and B be two intuitionistic fuzzy subsets of the hemirings R_1 and R_2 respectively and AxB is an intuitionistic fuzzy ideal of R_1xR_2. Then the following are true:
(i) if $\mu_A(x) \leq \mu_B(e^l)$ and $\nu_A(x) \geq \nu_B(e^l)$, then A is an intuitionistic fuzzy ideal of R_1.
(ii)if $\mu_B(x) \leq \mu_A(e)$ and $\nu_B(x) \geq \nu_A(e)$, then B is an intuitionistic fuzzy ideal of R_2.
(iii)either A is an intuitionistic fuzzy ideal of R_1 or B is an intuitionistic fuzzy ideal of R_2.

Proof: Let AxB be an intuitionistic fuzzy ideal of R_1xR_2 and x and y in $R_1$and e^l in R_2. Then (x, e^l) and (y, e^l) are in R_1xR_2. Now, using the property that $\mu_A(x) \leq \mu_B(e^l)$ and $\nu_A(x) \geq \nu_B(e^l)$, for all x in R_1. We get, $\mu_A(x+y) = \min\{\mu_A(x+y), \mu_B(e^l+e^l)\} = \mu_{AxB}((x+y), (e^l+e^l)) = \mu_{AxB}[(x, e^l) + (y, e^l)] \geq \min\{\mu_{AxB}(x, e^l), \mu_{AxB}(y, e^l)\} = \min\{\min\{\mu_A(x), \mu_B(e^l)\}, \min\{\mu_A(y), \mu_B(e^l)\}\} = \min\{\mu_A(x), \mu_A(y)\} \geq \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R_1. Also, $\mu_A(xy) = \min\{\mu_A(xy), \mu_B(e^le^l)\} = \mu_{AxB}((xy), (e^le^l)) = \mu_{AxB}[(x, e^l)(y, e^l)] \geq \max\{\mu_{AxB}(x, e^l), \mu_{AxB}(y, e^l)\} = \max\{\min\{\mu_A(x), \mu_B(e^l)\}, \min\{\mu_A(y), \mu_B(e^l)\}\} = \max\{\mu_A(x), \mu_A(y)\}$.Therefore, $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\}$, for all x and y in R_1.And, $\nu_A(x+y) = \max\{\nu_A(x+y), \nu_B(e^l+e^l)\} = \nu_{AxB}((x+y), (e^l+e^l)) = \nu_{AxB}[(x, e^l) + (y, e^l)] \leq \max\{\nu_{AxB}(x, e^l), \nu_{AxB}(y, e^l)\} = \max\{\max\{\nu_A(x), \nu_B(e^l)\}, \max\{\nu_A(y), \nu_B(e^l)\}\} = \max\{\nu_A(x), \nu_A(y)\}$. Therefore, $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R_1.Also,$\nu_A(xy) = \max\{\nu_A(xy), \nu_B(e^le^l)\} = \nu_{AxB}((xy),(e^le^l)) = \nu_{AxB}[(x, e^l)(y, e^l)] \leq \min\{\nu_{AxB}(x, e^l), \nu_{AxB}(y, e^l)\} = \min\{\max\{\nu_A(x), \nu_B(e^l)\}, \max\{\nu_A(y), \nu_B(e^l)\}\} = \min\{\nu_A(x), \nu_A(y)\}$.Therefore, $\nu_A(xy) \leq \min\{\nu_A(x), \nu_A(y)\}$, for all x and y in R_1. Hence A is an intuitionistic fuzzy ideal of R_1. Thus (i) is proved. Now, using the property that $\mu_B(x) \leq \mu_A(e)$ and $\nu_B(x) \geq \nu_A(e)$, for all x in R_2, let x and y in R_2 and e

in R_1. Then (e, x) and (e, y) are in R_1xR_2.We get,$\mu_B(x+y) = \min\{\mu_B(x+y), \mu_A(e+e)\} = \min\{\mu_A(e+e), \mu_B(x+y)\} = \mu_{AxB}((e+e),(x+y)) = \mu_{AxB}[(e, x) +(e,y)] \geq \min\{\mu_{AxB}(e, x), \mu_{AxB}(e, y)\} = \min\{\min\{\mu_A(e), \mu_B(x)\}, \min\{\mu_A(e), \mu_B(y)\} = \min\{\mu_B(x), \mu_B(y)\} \geq \min\{\mu_B(x), \mu_B(y)\}$.Therefore, $\mu_B(x+y) \geq \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R_2. Also, $\mu_B(xy) = \min\{\mu_B(xy), \mu_A(ee)\} = \min\{\mu_A(ee), \mu_B(xy)\} = \mu_{AxB}((ee), (xy)) = \mu_{AxB}[(e, x)(e, y)] \geq \max\{\mu_{AxB}(e, x), \mu_{AxB}(e, y)\} = \max\{\min\{\mu_A(e), \mu_B(x)\}, \min\{\mu_A(e), \mu_B(y)\}\} = \max\{\mu_B(x), \mu_B(y)\}$. Therefore, $\mu_B(xy) \geq \max\{\mu_B(x), \mu_B(y)\}$, for all x and y in R_2. And, $\nu_B(x+y) = \max\{\nu_B(x+y), \nu_A(e+e)\} = \max\{\nu_A(e+e), \nu_B(x+y)\} = \nu_{AxB}((e+e),(x+y)) = \nu_{AxB}[(e, x) + (e,y)] \leq \max\{\nu_{AxB}(e, x), \nu_{AxB}(e,y)\} = \max\{\max\{\nu_A(e), \nu_B(x)\}, \max\{\nu_A(e), \nu_B(y)\}\} = \max\{\nu_B(x), \nu_B(y)\} \leq \max\{\nu_B(x), \nu_B(y)\}$. Therefore, $\nu_B(x+y) \leq \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R_2. Also, $\nu_B(xy) = \max\{\nu_B(xy), \nu_A(ee)\} = \max\{\nu_A(ee), \nu_B(xy)\} = \nu_{AxB}((ee), (xy)) = \nu_{AxB}[(e, x)(e, y)] \leq \min\{\nu_{AxB}(e, x), \nu_{AxB}(e, y)\} = \min\{\max\{\nu_A(e), \nu_B(x)\}, \max\{\nu_A(e), \nu_B(y)\}\} = \min\{\nu_B(x), \nu_B(y)\}$. Therefore, $\nu_B(xy) \leq \min\{\nu_B(x), \nu_B(y)\}$, for all x and y in R_2. Hence B is an intuitionistic fuzzy ideal of a hemiring R_2.Thus (ii) is proved.
(iii) is clear.

2.6 Theorem: Let A be an intuitionistic fuzzy subset of a hemiring R and V be the strongest intuitionistic fuzzy relation of R. Then A is an intuitionistic fuzzy ideal of R if and only if V is an intuitionistic fuzzy ideal of RxR.

Proof: Suppose that A is an intuitionistic fuzzy ideal of a hemiring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR. We have, $\mu_V(x+y) = \mu_V[(x_1, x_2)+(y_1, y_2)] = \mu_V(x_1+y_1, x_2+y_2) = \min\{\mu_A(x_1+y_1), \mu_A(x_2+y_2)\} \geq \min\{\min\{\mu_A(x_1), \mu_A(y_1)\}, \min\{\mu_A(x_2), \mu_A(y_2)\}\} = \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\} = \min\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\} = \min\{\mu_V(x), \mu_V(y)\}$. Therefore, $\mu_V(x+y) \geq \min\{\mu_V(x), \mu_V(y)\}$, for all x and y in RxR. And, $\mu_V(xy) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(x_1y_1, x_2y_2) = \min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} \geq \min\{\max\{\mu_A(x_1), \mu_A(y_1)\}, \max\{\mu_A(x_2), \mu_A(y_2)\}\} = \max\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\} = \max\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\} = \max\{\mu_V(x), \mu_V(y)\}$. Therefore, $\mu_V(xy) \geq \max\{\mu_V(x), \mu_V(y)\}$, for all x and y in RxR.

Also we have, $\nu_V(x+y) = \nu_V[(x_1, x_2) +(y_1, y_2)]= \nu_V(x_1+ y_1, x_2+ y_2) = \max\{\nu_A(x_1+ y_1), \nu_A(x_2+ y_2)\} \leq \max\{\max\{\nu_A(x_1), \nu_A(y_1)\}, \max\{\nu_A(x_2), \nu_A(y_2)\}\} = \max\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}\} = \max\{\nu_V(x_1, x_2), \nu_V(y_1, y_2)\}= \max\{\nu_V(x), \nu_V(y)\}$. Therefore, $\nu_V(x+y)\leq \max\{\nu_V(x), \nu_V(y)\}$, for all x and y in RxR. And, $\nu_V(xy) = \nu_V[(x_1, x_2)(y_1, y_2)]= \nu_V(x_1y_1, x_2y_2)= \max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} \leq \max\{\min\{\nu_A(x_1), \nu_A(y_1)\}, \min\{\nu_A(x_2), \nu_A(y_2)\}\} = \min\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}\}= \min\{\nu_V(x_1, x_2), \nu_V(y_1, y_2)\}= \min\{\nu_V(x), \nu_V(y)\}$. Therefore, $\nu_V(xy) \leq \min\{\nu_V(x), \nu_V(y)\}$, for all x and y in RxR. This proves that V is an intuitionistic fuzzy ideal of RxR. Conversely assume that V is an intuitionistic fuzzy ideal of RxR, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR, we have $\min\{\mu_A(x_1+ y_1), \mu_A(x_2+y_2)\}= \mu_V(x_1+y_1, x_2+y_2) = \mu_V[(x_1, x_2) + (y_1, y_2)] = \mu_V(x+y) \geq \min\{\mu_V(x), \mu_V(y)\} = \min\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\}= \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\}$.If we put $x_2 = y_2 = 0$, we get,$\mu_A(x_1+ y_1) \geq \min\{\mu_A(x_1), \mu_A(y_1)\}$, for all x_1 and y_1 in R. And, $\min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} = \mu_V(x_1y_1, x_2y_2) = \mu_V[(x_1, x_2)(y_1, y_2)]= \mu_V(xy) \geq \max\{\mu_V(x), \mu_V(y)\}= \max\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\} = \max\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\}$. If we put $x_2 = y_2 = 0$, we get $\mu_A(x_1y_1) \geq \max\{\mu_A(x_1), \mu_A(y_1)\}$, for all x_1 and y_1 in R. Also we have, $\max\{\nu_A(x_1+ y_1), \nu_A(x_2+ y_2)\} = \nu_V(x_1+ y_1, x_2+y_2) = \nu_V[(x_1, x_2) + (y_1, y_2)] = \nu_V(x + y) \leq \max\{\nu_V(x), \nu_V(y)\}= \max\{\nu_V(x_1, x_2), \nu_V(y_1, y_2)\}= \max\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}\}$. If we put $x_2 = y_2 = 0$, we get, $\nu_A(x_1+ y_1) \leq \max\{\nu_A(x_1), \nu_A(y_1)\}$, for all x_1 and y_1 in R. And, $\max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\}= \nu_V(x_1y_1, x_2y_2) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(xy) \leq \min\{\nu_V(x), \nu_V(y)\} = \min\{\nu_V(x_1, x_2), \nu_V(y_1, y_2)\}= \min\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}\}$. If we put $x_2 = y_2 = 0$, we get, $\nu_A(x_1y_1) \leq \min\{\nu_A(x_1), \nu_A(y_1)\}$, for all x_1 and y_1 in R. Therefore, A is an intuitionistic fuzzy ideal of R.

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