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On Symmetric Bi-Derivations of BCI-Algebras

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Abstract. The notion of left-right (resp. right-left) symmetric bi-derivation of BCI-algebras is introduced and some related properties are investigated.

Keywords: BCI-algebras, symmetric bi-derivation, trace, regular derivations

1. Introduction

Y. B. Jun and X. L. Xin applied the notion of derivation in ring and near ring theory to BCI-algebras [6]. And H. A. S. Abujabal and N. O. Al-Shehri investigated some fundamental properties and proved some results on derivations of BCI-agebras in [1]. The concept of symmetric bi-derivation was introduced by Gy. Maksa in [8] (see also [9]). J. Vukman proved some results concerning symmetric bi-derivation on prime and semi prime rings [11, 12]. Y. Çeven introduced symmetric bi-derivation in lattices and investigated some related properties [4]. In this paper the notion of left-right (resp. right-left) symmetric bi-derivation of BCI-algebras is introduced and some of its properties are investigated. Also the new concept, called componentwise regular (in particular d-regular) derivation, is introduced and proved some of its properties. By componentwise regular derivation, we give a characterization of BCK-algebras.

2. Preliminaries

Let X be a nonempty set with binary operation $*$ and a constant 0. $(X, *, 0)$ is called BCI-algebra if it satisfies the following axioms for all $x, y, z \in X$:

I $((x * y) * (x * z)) * (z * y) = 0$
II $(x * (x * y)) * y = 0$
III $x * x = 0$
IV $x * y = 0$ and $y * x = 0$ imply $x = y$.

In any BCI-algebra X, a binary operation $\leq$ on X is defined for all $x, y \in X$ $x \leq y$ if and only if $x * y = 0$. Then also $(X, \leq)$ is partially ordered set. A BCI-algebra X satisfying $0 \leq x$ for all $x \in X$, is called BCK-algebra.

Let X, Y, be BCI-algebras. An operation $*$ on the cartesian product $X \times Y$ of X, Y is defined as follows
$(x_1, y_1) * (x_2, y_2) = (x_1 * x_2, y_1 * y_2)$
$0 = (0, 0)$
Then $(X \times Y, *, 0)$ is a BCI-algebra, and it is called the product of X, Y [7].
A BCI-algebra X has the following properties for all $x, y, z \in X$:

(1) $x * 0 = 0$
(2) $(x * y) * z = (x * z) * y$
(3) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$
(4) $(x * z) * (y * z) \leq x * y$
(5) $x * (x * (x * y)) = (x * y)$
(6) $0 * (x * y) = (0 * x) * (0 * y)$
(7) $x * 0 = 0$ implies $x = 0$

For a BCI-algebra X, we denote $X_+ = \{x \in X | 0 \leq x\}$, the BCK-part of X and by $G(X) = \{x \in X | 0 * x = x\}$, the BCI-$G$ part of X. If $X_+ = \{0\}$, then X is called a p-semisimple BCI-algebra. In a p-semisimple BCI-algebra X, the following properties hold:

(8) $(x * z) * (y * z) = x * y$
(9) $0 * (0 * x) = x$
(10) $x * (0 * y) = y * (0 * x)$
(11) $x * y = 0$ implies $x = y$
(12) $x * a = x * b$ implies $a = b$
(13) $a * x = b * x$ implies $a = b$
(14) $a * (a * x) = x$

Let X be a p-semisimple algebra BCI-algebra, we define addition " + " as $x + y = x * (0 * y)$ for all $x, y \in X$. Then $(X, +)$ is an abelian group with identity 0 and $a - y = x * y$. Conversely, let $(X, +)$ be an abelian group with identity 0 and klet $x - y = x * y$. Then X is a p-semisimple BCI-algebra and $x + y = x * (0 * y)$ for all $x, y \in X$(see [5]).

For a BCI-algebra X we denote $x \wedge y = y * (y * x)$, in particular $0 * (0 * x) = a_x$ and $L_p(X) := \{a \in X | x * a = 0 \Longrightarrow x = a, \forall x \in X\}$. We call the elements of $L_p(X)$ p-atoms of X. For any $a \in X$, $V(a) := \{x \in X | \ a * x = 0\}$, which is called branch of X with respect to a. It follows that $x * y \in V(a * b)$ whenever $x \in V(a)$ and $y \in V(b)$ for all $a, b \in L_p(X)$ and all $x, y \in X$. Note that $L_p(X) = \{x \in X | a_x = x\}$, which is the p-semisimple part of X, and X is p-semisimple BCI-algebra if and only if $L_p(X) = X$. Note also that $a_x \in L_p(X)$, that is, $0 * (0 * a_x) = a_x$, which implies $a_x * y \in L_p(X)$ for all $y \in X$. It is clear that $G(X) \subset L_p(X)$, $x * (x * a) = a$ and $a * x \in L_p(X)$ for all $a \in L_p(X)$ and $x \in X$. For more details we refer to [2, 7, 13, 14].

Definition 2.1. ([10]) A BCI-algebra X is said to be commutative if $x \wedge y = x$ whenever $x \leq y$ for all $x, y \in X$.

Definition 2.2. ([3])A BCI-algebra X is said to be branchwise commutative if $x \wedge y = y \wedge x$ for all $x, y \in V(a), a \in L_p(X)$.

Definition 2.3. ([4]) A BCI-algebra X is commutative if and only if it is branchwise commutative.

3. SYMMETRIC BI-DERIVATIONS

Definition 3.1. Let X be a BCI-algebra. A mapping $D(.,.) : X \times X \to X$ is symmetric if $D(x, y) = D(y, x)$ holds for all pairs $x, y \in X$.

Definition 3.2. Let X be a BCI-algebra and $D(.,.) : X \times X \to X$ be a symmetric mapping. A mapping $d : X \to X$ defined by $d(x) = D(x, x)$ is called trace of D.

Definition 3.3. Let X be a BCI-algebra and $D(.,.) : X \times X \to X$ be a symmetric mapping. If D satisfies the identity $D(x * y, z) = D(x, z) * y \wedge x * D(y, z)$ for all $x, y, z \in X$, then D is called $left - right\ symmetric\ bi - derivation$ (briefly $(l, r)-symmetric\, bi-derivation$). If D satisfies the identity $D(x * y, z) = x * D(y, z) \wedge D(x, z) * y$ for all x, $y, z \in X$, then we say that D is $right - left\ symmetric\ bi - derivation$ (briefly $(r,\ l) - symmetric\ bi - derivation$). Moreover if d is both an $(r,\ l)-$ and a $(l,\ r) - symmetric\ bi - derivation$, it is said that D is $symmetric\ bi - derivation$.

Example 3.1. *Let $X = \{0, 1, 2\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2
0	0	0	2
1	1	0	2
2	2	2	0

Define a mapping $D(.,.) : X \times X \to X$ *by*

$$D(x,z) = \begin{cases} 0 & , \text{ if } (x,z) = (0,1) \text{ and } (x,z) = (1,0) \\ x * z & , \text{ otherwise} \end{cases}$$

Then it can be checked that D is both $(l,r) - symmetric\ bi - derivation$ and $(r,l) - symmetric\ bi - derivation.$

Example 3.2. *Let* $X = \{0,1,2,3\}$ *be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	2
3	3	3	3	0

Define a mapping $D(.,.) : X \times X \to X$ *by*

$$D(x,z) = \begin{cases} 2 & , \text{ if } (x,z) = (2,2) \\ 0 & , \text{ otherwise} \end{cases}$$

Then it can be checked that D is both $(l,r) - symmetric\ bi - derivation$ and $(r,l) - symmetric\ bi - derivation.$

Example 3.3. *Let* X *be a* $p - semisimple$ *BCI-Algebra. Define a mapping* $D(.,.) : X \times X \to X$ $D(x,z) = x + z$, *for all* $x, z \in X$. *For all* $x, y, z \in X$

$$\begin{aligned} D(x * y, z) &= (x * y) + z = (x * y) * (0 * z) \\ &= (x * (0 * z)) * y = (x + z) * y \end{aligned}$$

On the other hand

$$\begin{aligned} D(x,z) * y \wedge x * D(y,z) &= (x * D(y,z) =) * ((x * D(y,z) * (D(x,z) * y)) \\ &= D(x,z) * y = (x + z) * y \end{aligned}$$

So D is $(l,r) - symmetricbi - derivation.$ But for the following $p - semisimple$ BCI-Algebra $X = \{0,1,2\}$ given Cayley table as

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

$$D(1*2,2) = (1*2)+2 = (1*2)*(0*2) = 2*1$$

but

$1 * D(2,2) \wedge D(1,2) * 2 = 1 * D(2,2) = 1 * (2(0*2) = 1 * (2*1) = 1*1 = 0$

So it is not $(r,l) - symmetric\ bi - derivation.$

Proposition 3.4. *Let X be a BCI-algebra and $D(.,.) : X \times X \to X$ be a symmetric mapping. Then*

(i) *If D is a $(l,r) - symmetric\ bi - derivation$, then $D(x,z) = D(x,z) \wedge x$ for all $x,z \in X$*

(ii) *If D is a $(r,l) - symmetric\ bi - derivation$, then $D(x,z) = x \wedge D(x,z)$ for all $x,z \in X$ if and only if $D(0,z) = 0$ for all $z \in X$.*

Proof. (i) Let D be a $(l,r)-symmetric\,bi-derivation.$ Then for all $x,z \in X$

$$\begin{aligned} D(x,z) &= D(x*0,z) = D(x,z)*0 \wedge x * D(0,z) \\ &= (x*D(0,z))*((x*D(0,z))*(D(x,z)*0)) \\ &= (x*D(0,z))*((x*D(x,z))*D(0,z)) \\ &\leq x*(x*D(x,z)) = D(x,z) = D(x,z) \wedge x \end{aligned}$$

On the other hand $D(x,z) \wedge x \leq D(x,z)$, and so (i) holds.

(ii) Let D be a $(r,l) - symmetric\ bi - derivation.$ If $D(x,z) = x \wedge D(x,z)$ for all $x,z \in X$, then

$$D(0,z) = 0 \wedge D(0,z) = D(0,z)*(D(0,z)*0) = 0$$

Conversely if $D(0,z) = 0$ for all $z \in X$, then

$$\begin{aligned} D(x,z) &= D(x*0,z) = x*D(0,z) \wedge D(x,z)*0 \\ &= x*0 \wedge D(x,z)*0 = x \wedge D(x,z) \end{aligned}$$

□

Proposition 3.5. *Let X be a BCI-algebra and $D(.,.) : X \times X \to X$ be a $(l, r) - symmetric\ bi - derivation$. Then*

(i) *$D(0,z) \in L_p(X)$ for all $z \in X$.*

(ii) *$D(a,z) = D(0,z)*(0*a) = D(0,z)+a$ for all $a \in L_p(X)$, $z \in X$.*

(iii) *$D(a,z) \in L_p(X)$ for all $a \in L_p(X)$, $z \in X$.*

(iv) *$D(a+b,z) = D(a,z)+D(b,z)-D(0,z)$, for all $a,b \in L_p(X)$, $z \in X$.*

(v) $D(.,.) : L_p(X) \times X \to L_p(X)$, $D(a,z) = a$, *for all* $a \in L_p(X)$, $z \in X$ *if and only if* $D(0,z) = 0$.

Proof. (i) If we show that $D(0,z) = 0 * (0 * D(0,z))$, the proof is completed. From Proposition 3.4.(i) we know that $D(x,z) = D(x,z) \wedge x$ for all $x, z \in X$, so

$$D(0,z) = D(x,z) = D(0,z) \wedge 0 = 0 * (0 * D(0,z))$$

(ii) Let $a \in L_p(X)$. Hence $a = 0 * (0 * a)$. Then

$$\begin{aligned} D(a,z) &= D(0*(0*a),z) = D(0,z)*(0*a) \wedge 0 * D(0*a,z) \\ &= (0*D(0*a,z))*((0*D(0*a,z))*(D(0,z)*(0*a))) \\ &= D(0,z)*(0*a) = D(0,z)+a \end{aligned}$$

(iii) Let $a \in L_p(X)$. From (ii)

$$D(a,z) = D(0,z) * (0 * a)$$

Because of $D(0,z) \in L_p(X)$, $D(0,z)*(0*a) \in L_p(X)$ so $D(a,z) \in L_p(X)$.

(iv) Let $a, b \in L_p(X)$. Note that $a + b = a * (0 * b) \in L_p(X)$ so from (ii)

$$\begin{aligned} D(a+b,z) &= D(0,z)+(a+b) \\ &= D(0,z)+a+D(0,z)+b-D(0,z) \\ &= D(a,z)+D(b,z)-D(0,z) \end{aligned}$$

(v) If $D(a,z) = a$, for all $a \in L_p(X)$, $z \in X$, clearly for $0 \in L_p(X)$ $D(0,z) = 0$. Conversely if $D(0,z) = 0$, then for all $a \in L_p(X)$, $z \in X$

$$D(a,z) = D(0,z) + a = 0 + a = a$$

□

By Proposition 3.5 we can give the following corollary.

Corollary 3.6. *Let* X *be a BCI-algebra,* $D(.,.) : X \times X \to X$ *be a* $(l,\, r)-$ *symmetric* $bi-derivation$ *and* $d : X \longrightarrow X$ *be trace of* D. *Then*

(i) $d(0) \in L_p(X)$

(ii) $d(a) \in L_p(X)$, *for all* $a \in L_p(X)$

Proposition 3.7. *Let* X *be a BCI-algebra and* $D(.,.) : X \times X \to X$ *be a* $(r,l)-$ *symmetric* $bi-derivation$. *Then*

(i) $D(a,z) \in G(X)$ *for all* $a \in G(X)$.

(ii) $D(a,z) \in L_p(X)$ *for all* $a \in L_p(X)$, $z \in X$.

(iii) $D(a,z) = a * D(0,z) = a + D(0,z)$ *for all* $a \in L_p(X)$, $z \in X$.

(iv) $D(a+b,z) = D(a,z) + D(b,z) - D(0,z)$, *for all* $a, b \in L_p(X)$, $z \in X$.

(v) $D(.,.) : L_p(X) \times X \to L_p(X)$, $D(a, z) = a$, *for all* $a \in L_p(X)$, $z \in X$ *if and only if* $D(0, z) = 0$.

Proof. (i) Let $a \in G(X)$. Hence $a = 0 * a$. Then

$$\begin{aligned} D(a, z) &= D(0 * a, z) = 0 * D(a, z) \wedge D(0, z) * a \\ &= (D(0, z) * a) * ((D(0, z) * a) * (0 * D(a, z)) \\ &= 0 * D(a, z) \end{aligned}$$

(ii) For any $a \in L_p(X)$, $a = 0 * (0 * a)$, so

$$\begin{aligned} D(a, z) &= D(0 * (0 * a), z) = 0 * D(0 * a, z) \wedge D(0, z) * (0 * a) \\ &= (D(0, z) * (0 * a)) * ((D(0, z) * (0 * a)) * (0 * D(0 * a, z))) \\ &= 0 * D(0 * a, z) \in L_p(X) \end{aligned}$$

(iii) For any $a \in L_p(X)$ and $z \in X$

$$\begin{aligned} D(a, z) &= D(a * 0, z) = a * D(0, z) \wedge D(a, z) * 0 \\ &= a * D(0, z) \wedge D(a, z) \\ &= D(a, z) * (D(a, z) * (a * D(0, z)) \\ &= a * D(0, z) = a * (0 * D(0, z)) \\ &= a + D(0, z) \end{aligned}$$

(iv) For all $a, b \in L_p(X)$, $a + b \in L_p(X)$, so

$$\begin{aligned} D(a + b, z) &= (a + b) + D(0, z) \\ &= a + D(0, z) + b + D(0, z) - D(0, z) \\ &= D(a, z) + D(b, z) - D(0, z) \end{aligned}$$

(v) If $D(a, z) = a$, for all $a \in L_p(X)$, $z \in X$, clearly for $0 \in L_p(X)$ $D(0, z) = 0$. Conversely if $D(0, z) = 0$, then for all $a \in L_p(X)$, $z \in X$

$$D(a, z) = a + D(0, z) = a + 0 = a$$

□

By Proposition 3.7. we can give the following corollary.

Corollary 3.8. *Let X be a BCI-algebra, $D(.,.) : X \times X \to X$ be a $(r, l)-$symmetric $bi-derivation$ and $d : X \longrightarrow X$ be the trace of D. Then*

(i) $d(a) \in G(X)$, *for all* $a \in G(X)$

(ii) $d(a) \in L_p(X)$, *for all* $a \in L_p(X)$

Definition 3.9. Let X be a BCI-algebra and $D(.,.) : X \times X \to X$ be a symmetric mapping. If $D(0, z) = 0$, for all $z \in X$, D is called componentwise regular. In particular if $D(0, 0) = d(0) = 0$, D is called $d-regular$.

Example 3.4. *We know that symmetric* $bi-derivation$ D *in Example 3.1. is* $d-regular$.

Example 3.5. *We know that symmetric* $bi-derivation$ D *in Example 3.2. is componentwise regular.*

In the following we show that a $(l,r)-(resp.(r,l-)$ *symmetric* $bi-derivation$ with certain conditions is *componentwise regular* $(l,r)-(resp.(r,l-))$ *symmetric* $bi-derivation$.

Proposition 3.10. *Let* X *be a BCI-algebra and* $D(.,.) : X \times X \rightarrow X$ *be a* $(l,r)-$ *symmetric* $bi-derivation$. *If there exist* $a \in X$ *such that* $D(x,z)*a = 0$, *for all* $x,z \in X$, *then* D *is componentwise regular* $(l,r)-$ *symmetric* $bi-derivation$.

Proof. Let X be a BCI-algebra and $D(.,.) : X\times X \rightarrow X$ be a $(l,r)-$ *symmetric* $bi-derivation$. Assume that there exist $a \in X$ such that $D(x,z)*a = 0$, for all $x,z \in X$. Since D is $(l,r)-$ *symmetric* $bi-derivation$ we get

$$\begin{aligned} 0 &= D(x*a,z)*a = (D(x,z)*a \wedge x*D(a,z))*a \\ &= (0 \wedge x*D(a,z))*a = 0*a \end{aligned}$$

thus $0 \leq a$. This show that

$$\begin{aligned} D(0,z) &= D(0*a,z) = (D(0,z)*a) \wedge (0*D(a,z)) \\ &= 0 \wedge (0*D(a,z)) = 0 \end{aligned}$$

thus D is *componentwise regular*. □

Corollary 3.11. *Let* X *be a BCI-algebra and* $D(.,.) : X\times X \rightarrow X$ *be a* $(l,r)-$ *symmetric* $bi-derivation$.*If there exist* $a \in X$ *such that* $D(x,z)*a = 0$, *for all* $x,z \in X$, *then* D *is* $d-regular$ $(l,r)-$ *symmetric* $bi-derivation$.

Similarly the following can be proved.

Proposition 3.12. *Let* X *be a BCI-algebra and* $D(.,.) : X \times X \rightarrow X$ *be a* $(r,l)-$ *symmetric* $bi-derivation$. *If there exist* $a \in X$ *such that* $a*D(x,z) = 0$, *for all* $x,z \in X$, *then* D *is componentwise regular* $(r,l)-$ *symmetric* $bi-derivation$.

Proof. Let X be a BCI-algebra and $D(.,.) : X\times X \rightarrow X$ be a $(r,l)-$ *symmetric* $bi-derivation$. Assume that there exist $a \in X$ such that $a*D(x,z) = 0$, for all $x,z \in X$. Since D is $(r,l)-$ *symmetric* $bi-derivation$ we get

$$\begin{aligned} 0 &= a*D(x*a,z) = a*(a*D(x,z) \wedge D(a,z)*x) \\ &= a*(0 \wedge D(a,z)*x)*a = a*0 \end{aligned}$$

This show that

$$\begin{aligned} D(0,z) &= D(a*0,z) = (a*D(0,z) \wedge D(a,z)*0) \\ &= 0 \wedge D(a,z) = 0 \end{aligned}$$

thus D is *componentwise regular*. □

Corollary 3.13. *Let X be a BCI-algebra and $D(.,.) : X\times X \to X$ be a $(r,l)-$ symmetric $bi-derivation$.If there exist $a \in X$ such that $a * D(x,z) = 0$, for all $x,z \in X$, then D is $d-regular$ $(r,l)-$ symmetric $bi-derivation$.*

Theorem 3.14. *Let X be a BCI-algebra and $D(.,.) : X \times X \to X$ be a $(l,r)-$ symmetric $bi-derivation$. Then X be a BCK-Algebra if and only if D is componentwise regular.*

Proof. Let X be a BCK-Algebra and $D(.,.) : X \times X \to X$ be a $(l,r)-$ *symmetric* $bi-derivation$. Then for all $x,z \in X$, we have

$$\begin{aligned} D(0,z) &= D(0*x,z) = D(0,z)*x \wedge 0*D(x,z) \\ &= D(0,z)*x \wedge 0 \\ &= 0*(0*(D(0,z)*x)) \\ &= 0*0 \end{aligned}$$

Conversely Let X be a BCI-algebra and $D(.,.) : X\times X \to X$ be a *componentwise regular* $(l,r)-$ *symmetric* $bi-derivation$. Let for a $x \in X$ $0*x \neq 0$. Since D is *componentwise regular* $D(0*x,0) = 0$. And

$$\begin{aligned} D(0,0)*x \wedge 0*D(x,0) &= 0*x \wedge 0*0 \\ &= 0*x \wedge 0 \\ &= 0*(0*0(0*x)) = (0*x) \neq 0 \end{aligned}$$

But it is not possible since D is $(l,r)-$ *symmetric* $bi-derivation$. Thus for all $x \in X$ $0*x = 0$, i.e. X is BCK-algebra. □

Theorem 3.15. *Let X be a BCI-algebra and $D(.,.) : X\times X \to X$ be a $(r,l)-$ symmetric $bi-derivation$. Then X be a BCK-Algebra if and only if D is componentwise regular.*

Proof. Let X be a BCK-Algebra and $D(.,.) : X \times X \to X$ be a $(r,l)-$ *symmetric* $bi-derivation$. Then for all $x,z \in X$, we have

$$\begin{aligned} D(0,z) &= D(0*x,z) = 0*D(x,z) \wedge D(0,z)*x \\ &= 0 \wedge D(0,z)*x \\ &= (D(0,z)*x)*((D(0,z)*x)*0) \\ &= (D(0,z)*x)*(D(0,z)*x) = 0. \end{aligned}$$

Conversely Let X be a BCI-algebra and $D(.,.) : X\times X \to X$ be a *componentwise regular* $(r,l)-$ *symmetric* $bi-derivation$. Suppose $a \in L_p(X)$ and $a \neq 0$. Since D is *componentwise regular*

$$D(a*0,0) = D(a,0) = 0.$$

But

$$\begin{aligned} a * D(0,0) \wedge D(a,0) * 0 &= a * 0 \wedge 0 * 0 \\ &= a \wedge 0 = 0 * (0 * a) = a \neq 0. \end{aligned}$$

But it is not possible since D is $(r,l)-$ *symmetric* $bi-derivation$. Thus the unique p-atom is 0. Assume that for a $x \in X$ $0 * x \neq 0$ then $a_{0*x} = 0 * (0 * (0 * x)) = 0$, so $0 * x \in L_p(X)$ but this is a contradiction. Thus for all $x \in X$ $0 * x = 0$, i.e. X is BCK-Algebra. □

Corollary 3.16. *Let X be a BCI-algebra and $D(.,.) : X \times X \rightarrow X$ be a symmetric $bi-derivation$. Then X be a BCK-Algebra if and only if D is componentwise regular.*

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