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Ranking of Fuzzy Numbers by Using Distance between Convex Combination of Upper and Lower Central Gravity of α-level and the Origin of Coordinate System

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Abstract

So far, many methods have been proposed for ranking of fuzzy numbers. But no one of them has ranked fuzzy numbers completely. Therefore, proposing methods for ranking fuzzy numbers are very essential. In this paper, we obtain the upper and lower central gravity of α-level for every fuzzy number at first, and then the distance between the convex combination and the origin of coordinate system are calculated. It should be noted that the advantage of this proposed method over others is that we will obtain diverse and different ranking by introducing the convex combination of central gravity to make a decision.

Mathematics Subject Classification: 94D05

Keywords: Fuzzy numbers, α-level, Convex Combination

1 Introduction

In many applications, ranking of fuzzy numbers is an important component of decision process. Numerous fuzzy ranking indices have been proposed since 1976. Jain [9, 10] proposed a method using the concept of maximizing set to order the fuzzy numbers in 1976 and 1977. Dubbois and Prade 1978 [8] used maximizing sets to order fuzzy numbers. Chen [6], choobineh [4], Cheng [3], Chu, Tsao [5] and Ma, Kandel and Freidman [11] have presented some methods. A number of these scientists used central gravity for ranking of fuzzy numbers. For instance, Cheng [3] proposed the distance method for ranking of fuzzy number, i.e.,

$$R(\tilde{A}) = \sqrt{\bar{x}^2 + \bar{y}^2},$$

where

$$\bar{x} = \frac{\int_a^b x f_{\tilde{A}}^l dx + \int_b^c x dx + \int_c^d x f_{\tilde{A}}^r dx}{\int_a^b f_{\tilde{A}}^l dx + \int_b^c dx + \int_c^d f_{\tilde{A}}^r dx}, \qquad \bar{y} = \frac{\int_0^1 y g_{\tilde{A}}^l dy + \int_0^1 y g_{\tilde{A}}^r dy}{\int_0^1 g_{\tilde{A}}^l dy + \int_0^1 g_{\tilde{A}}^r dy},$$

where $f_{\tilde{A}}^l$, $f_{\tilde{A}}^r$, $g_{\tilde{A}}^l$, $g_{\tilde{A}}^r$ are illustrated in preliminaries.

2 Preliminary Notes

Fuzzy number can defined as follows [5].

Definition 2.1 *A fuzzy number $\tilde{A}$ is described as any fuzzy subset of the real line R with membership function $f_{\tilde{A}}$ which processes the following properties:*

(a) *$f_{\tilde{A}}$ is a continuous mapping from R to the closed interval $[0, w]$, $0 \leq w \leq 1$.*

(b) *$f_{\tilde{A}}(x) = 0$, for all $x \in (-\infty, a]$.*

(c) *$f_{\tilde{A}}$, is strictly increasing on[a,b].*

(d) *$f_{\tilde{A}}(x) = w$, for all $x \in [b, c]$, where w is constant and $w \in (0, 1]$.*

(e) *$f_{\tilde{A}}$, is strictly decreasing on [c,d].*

(f) *$f_{\tilde{A}}(x) = 0$, for all $x \in [d, \infty)$, where a, b, c ,d are real numbers. we may let $a = -\infty$, or a=b, or c=d, or $d = +\infty$.*

Unless elsewhere specified, it is assumed that $\tilde{A}$ is convex and bounded, i.e, $-\infty < a$, $d < \infty$. If, $w = 1$, in (d) $\tilde{A}$ is a normal fuzzy number, and if $0 < w < 1$, in (d), $\tilde{A}$ is a non-normal fuzzy number. For convenience, the fuzzy number in Definition can be denoted by $\tilde{A} = (a, b, c, d; w)$. The image (opposite) of $\tilde{A} = (a, b, c, d; w)$ can be given by $-\tilde{A} = (-d, -c, -b, -a; w)(see[7, 11])$.

The membership $f_{\tilde{A}}$ of can be expressed as

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^l(x), & a \leq x \leq b \\ w, & b \leq x \leq c \\ f_{\tilde{A}}^r(x), & c \leq x \leq d \\ 0, & otherwise \end{cases}$$

where $f_{\tilde{A}}^l$: [a,b]$\rightarrow [0, w]$ and $f_{\tilde{A}}^r$: [c,d]$\rightarrow [0, w]$. Since $f_{\tilde{A}}^l$ and $f_{\tilde{A}}^r$ are continuous and strictly increasing and decreasing respectively, then inverse function of $f_{\tilde{A}}^l$ and $f_{\tilde{A}}^r$ can be denoted by $g_{\tilde{A}}^l$ and $g_{\tilde{A}}^r$ respectively. Since $g_{\tilde{A}}^l$ and $g_{\tilde{A}}^r$ are continuous on $[0, w]$ so they are integrable on $[0, w]$. That is, both $\int_0^w g_{\tilde{A}}^l dy$ and $\int_0^w g_{\tilde{A}}^r dy$ exist [8].

3 Main Results

Granted that $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n$, n are positive fuzzy numbers. At first, we obtain the upper and lower central gravity of α-level for every fuzzy number. For each A_i, $i = 1, ..., n$, the upper and lower central gravity of α-level should be indicated as $(\bar{x}_{i1}, \bar{y}_{i1})$ and $(\bar{x}_{i2}, \bar{y}_{i2})$ respectively and obtained as follows:

$$\bar{x}_{i1} = \frac{\int_e^b x f_{i1}^l dx + \int_b^c x dx + \int_c^f x f_{i1}^r dx}{\int_e^b f_{i1}^l dx + \int_b^c dx + \int_c^f f_{i1}^r dx}, \qquad \bar{x}_{i2} = \frac{\int_a^e x f_{i2}^l dx + \int_e^f x dx + \int_f^d x f_{i2}^r dx}{\int_a^e f_{i2}^l dx + \int_e^f dx + \int_f^d f_{i2}^r dx},$$

$$\bar{y}_{i1} = \frac{\int_\alpha^{\omega_i} y g_{i1}^l dy + \int_\alpha^{\omega_i} y g_{i1}^r dy}{\int_\alpha^{\omega_i} g_{i1}^l dy + \int_\alpha^{\omega_i} g_{i1}^r dy}, \qquad \bar{y}_{i2} = \frac{\int_0^\alpha y g_{i2}^l dy + \int_0^\alpha y g_{i2}^r dy}{\int_0^\alpha g_{i2}^l dy + \int_0^\alpha g_{i2}^r dy},$$

where

$$\omega_i \in [\alpha, 1], \qquad \alpha \leq \min\{\omega_i, i = 1, ..., n\},$$

then

$$R(\tilde{A}_i) = \sqrt{(\lambda \bar{x}_{i1} + (1-\lambda)\bar{x}_{i2})^2 + (\lambda \bar{y}_{i1} + (1-\lambda)\bar{y}_{i2})^2}\ , \qquad \lambda \in [0, 1].$$

Then, for ranking of $\tilde{A}_i, \tilde{A}_j$ we have,

$$\tilde{A}_i \succ \tilde{A}_j \quad iff \quad R(\tilde{A}_i) > R(\tilde{A}_j)$$

$$\tilde{A}_i \prec \tilde{A}_j \quad iff \quad R(\tilde{A}_i) < R(\tilde{A}_j)$$

$$\tilde{A}_i \sim \tilde{A}_j \quad iff \quad R(\tilde{A}_i) = R(\tilde{A}_j)$$

Finally, each fuzzy number having the greater distance have the better rank.

In above, the denominator of fractions is not to be equal to zero.
It is noticed that if the fuzzy number A_i is negative, our proposition is to multiply $R(A_i)$ by -1 at first, and then we can conduct the ranking approach.

4 Comparison with other ranking methods

We shall now compare our methods with other authors's method. consider that in all the tables, F.N is an indicator of fuzzy number.

Example 4.1 *Let, $\tilde{A} = (1.9, 2, 2.1)$, $\tilde{B} = (2.1, 3, 4)$ consider the two triangular fuzzy number clearly $\tilde{A} \prec \tilde{B}$. By the cv index, $cv(\tilde{A}) = 0.0000834$, $cv(\tilde{B}) = 0.0636$ where $cv(\tilde{A}) < cv(\tilde{B})$ therefore by cv index, $\tilde{A} \succ \tilde{B}$. The results calculated by our proposed method for different α and λ are illustrated in table 1.*

Table 1

	F.N	$\lambda = 0.0$	$\lambda = 0.2$	$\lambda = 0.4$	$\lambda = 0.6$	$\lambda = 0.8$	$\lambda = 1$
$\alpha = 0.0$	$\tilde{A}$	$-$	$-$	$-$	$-$	$-$	2.0616
	$\tilde{B}$	$-$	$-$	$-$	$-$	$-$	3.0740
Results		$-$	$-$	$-$	$-$	$-$	$\tilde{A} \prec \tilde{B}$
$\alpha = 0.2$	$\tilde{A}$	1.9917	2.0014	2.0159	2.0354	2.0595	2.0881
	$\tilde{B}$	2.9381	2.9612	2.9876	3.0171	3.0498	3.0854
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
$\alpha = 0.4$	$\tilde{A}$	1.9857	2.003	2.0252	2.0521	2.0834	2.1190
	$\tilde{B}$	2.8007	2.8546	2.9117	2.9717	3.0346	3.1
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
$\alpha = 0.6$	$\tilde{A}$	1.9779	2.0043	2.0354	2.0708	2.1105	2.1541
	$\tilde{B}$	2.5919	2.6921	2.7951	2.9005	3.0081	3.1177
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
$\alpha = 0.8$	$\tilde{A}$	1.9482	1.9892	2.0345	2.0838	2.1367	2.1932
	$\tilde{B}$	2.0566	2.2709	2.4865	2.7031	2.9205	3.1385
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$

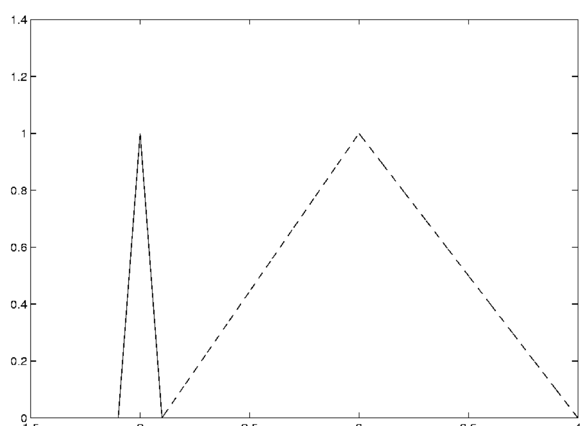

Fig.1. Triangular fuzzy numbers,$\tilde{A} = (1.9, 2, 2.1)$, $\tilde{B} = (2.1, 3, 4)$.

In table 1 we can observe that $\forall \alpha, \lambda$, $\tilde{A} \prec \tilde{B}$. These results is reasonable based on the proposed method.

Example 4.2 *The three triangular fuzzy numbers,* $\tilde{U}_1 = (0.2, 0.3, 0.5)$, $\tilde{U}_2 = (0.17, 0.32, 0.58)$, $\tilde{U}_3 = (0.25, 0.4, 0.7)$, *shown in figure 2, ranked by Chu and Tsao,* $S(\tilde{U}_1) = \bar{x}_{\tilde{U}_1} \times \bar{y}_{\tilde{U}_1} = 0.333 \times 0.4872 = 0.162$, $S(\tilde{U}_2) = \bar{x}_{\tilde{U}_2} \times \bar{y}_{\tilde{U}_2} = 0.357 \times 0.4868 = 0.174$, $S(\tilde{U}_3) = \bar{x}_{\tilde{U}_3} \times \bar{y}_{\tilde{U}_3} = 0.450 \times 0.4857 = 0.219$, *Therefore* $\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$.
The results obtained by our proposed method for different α *and* λ *are illustrated in table 2.*

Table 2

	F.N	$\lambda = 0.0$	$\lambda = 0.4$	$\lambda = 0.8$	$\lambda = 1$
$\alpha = 0.0$	$\tilde{U}_1$	$-$	$-$	$-$	0.5903
	$\tilde{U}_2$	$-$	$-$	$-$	0.6035
	$\tilde{U}_3$	$-$	$-$	$-$	0.6621
Results		$-$	$-$	$-$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$
$\alpha = 0.2$	$\tilde{U}_1$	0.338	0.44	0.5923	0.6772
	$\tilde{U}_2$	0.3552	0.4548	0.6044	0.6883
	$\tilde{U}_3$	0.4457	0.5280	0.6607	0.7380
Results		$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$
$\alpha = 0.4$	$\tilde{U}_1$	0.3501	0.4992	0.6748	0.7670
	$\tilde{U}_2$	0.3578	0.5079	0.6840	0.7764
	$\tilde{U}_3$	0.4307	0.5652	0.7303	0.8187
Results		$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$
$\alpha = 0.6$	$\tilde{U}_1$	0.3739	0.5625	0.7589	0.8584
	$\tilde{U}_2$	0.3721	0.5656	0.7654	0.8663
	$\tilde{U}_3$	0.4152	0.6040	0.8021	0.9027
Results		$\tilde{U}_2 \prec \tilde{U}_1 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$
$\alpha = 0.8$	$\tilde{U}_1$	0.4167	0.5975	0.8279	0.9507
	$\tilde{U}_2$	0.4057	0.6003	0.8345	0.9576
	$\tilde{U}_3$	0.4684	0.5958	0.8452	0.9889
Results		$\tilde{U}_2 \prec \tilde{U}_1 \prec \tilde{U}_3$	$\tilde{U}_3 \prec \tilde{U}_1 \prec \tilde{U}_2$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$	$\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$

Table 2 showed that the ranking for $\forall \alpha, \lambda$ in more cases is similar to Chu and Tsao's method [5], namely, $\tilde{U}_1 \prec \tilde{U}_2 \prec \tilde{U}_3$. In some cases at the end of the table 2, we can see another ranking which is different from Chu and Tsao's method [5], for example, if $\alpha = 0.6, \lambda = 0.0$ and $\alpha = 0.8, \lambda = 0.0$ we have $\tilde{U}_2 \prec \tilde{U}_1 \prec \tilde{U}_3$ and if $\alpha = 0.8$, $\lambda = 0.4$ we have $\tilde{U}_3 \prec \tilde{U}_1 \prec \tilde{U}_2$. This results is also reasonable due to the importance of the lower central gravity of α-level over the upper central gravity.

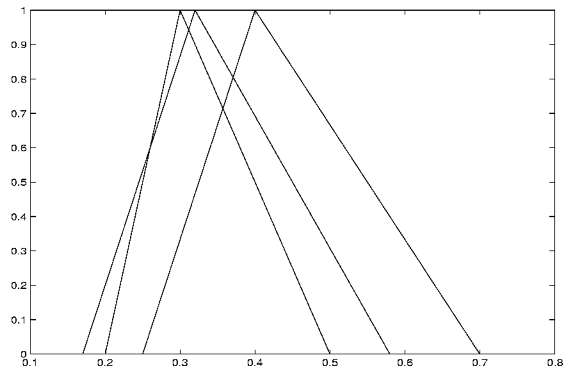

Fig. 2. Triangular fuzzy numbers $\tilde{U}_1 = (0.2, 0.3, 0.5)$, $\tilde{U}_2 = (0.17, 0.32, 0.58)$, $\tilde{U}_3 = (0.25, 0.4, 0.7)$.

Example 4.3 *Consider the following sets.*

$$Set1 : \tilde{A} = (0.1, 0.3, 0.5; 1), \tilde{B} = (0.3, 0.5, 0.7; 1).$$

$$Set2 : \tilde{A} = (0.1, 0.3, 0.5; 0.8), \tilde{B} = (0.1, 0.3, 0.5; 1).$$

$$Set3 : \tilde{A} = (0.3, 0.5, 1; 1), \tilde{B} = (0.1, 0.6, 0.8; 1).$$

$$Set4 : \tilde{A} = (0.0, 0.4, 0.6, 0.8; 1), \tilde{B} = (0.2, 0.5, 0.9; 1), \tilde{C} = (0.1, 0.6, 0.7, 0.8; 1).$$

Which are shown in figure 3.

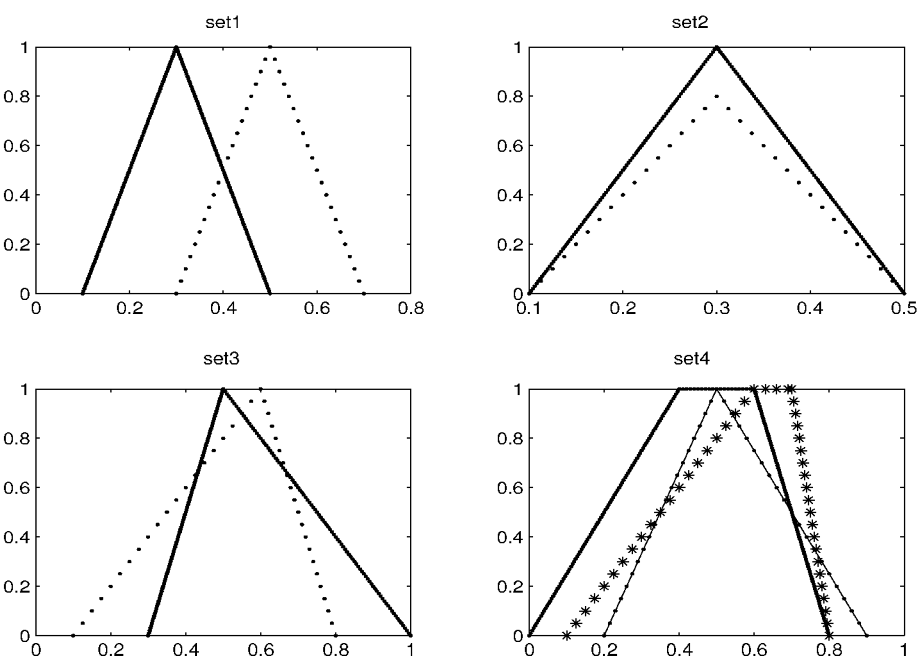

Fig. 3. four sets of normal and non-normal fuzzy numbers.

Table 3: Comparative results of example 4.3

methods	F.N	$set1$	$set2$	$set3$	$set4$
Cheng's method	$\tilde{A}$	0.5831	0.461	0.7673	0.68
(1998)	$\tilde{B}$	0.7071	0.5831	0.7241	0.7257
	$\tilde{C}$	–	–	–	0.7462
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Chu's method	$\tilde{A}$	0.15	0.12	0.287	0.2281
(2002)	$\tilde{B}$	0.25	0.15	0.2619	0.2624
	$\tilde{C}$	–	–	–	0.2784
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Murakami et al's	$\tilde{A}$	0.3	0.2333	0.6	0.44
method(1983)	$\tilde{B}$	0.5	0.3	0.5	0.5333
	$\tilde{C}$	–	–	–	0.525
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Yager's method	$\tilde{A}$	0.3	0.3	0.6	0.44
(1978)	$\tilde{B}$	0.5	0.3	0.5	0.5333
	$\tilde{C}$	–	–	–	0.525
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Chen and Chen's	$\tilde{A}$	0.4456	0.3565	0.4128	0.3719
method(2007)	$\tilde{B}$	0.4884	0.4456	0.4005	0.4155
	$\tilde{C}$	–	–	–	0.3979
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Chen and Chen's	$\tilde{A}$	0.2579	0.2063	0.4428	0.3354
method(2009)	$\tilde{B}$	0.4298	0.2579	0.4043	0.4079
	$\tilde{C}$	–	–	–	0.4196
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$

In set 1 and set 2, other scientists's methods and the proposed method result in $\tilde{A} \prec \tilde{B}$ with the exception of Yager's in set 2 in which $\tilde{A} \sim \tilde{B}$. In set 3, it is reasonable that the $\tilde{B}$ rank is better than $\tilde{A}$ rank. But using other scientists's methods $\tilde{A} \succ \tilde{B}$ and using the proposed method $\tilde{A} \prec \tilde{B}$. In set 4, it is logical that $\tilde{A} \prec \tilde{B} \prec \tilde{C}$. Cheng's method (1998), Chu's method (2002), Chen and Chen's method(2009), our proposed method $\tilde{A} \prec \tilde{B} \prec \tilde{C}$ But Murakami et al's method(1983), Yager's method (1978), Chen and Chen's method (2007) $\tilde{A} \prec \tilde{C} \prec \tilde{B}$.

The results obtained by our proposed method for different α and λ are illustrated in table 4.

Table 4

	F.N	$\lambda = 0.0$	$\lambda = 0.4$	$\lambda = 0.8$	$\lambda = 1$
set(1)	$\tilde{A}$	$-$	$-$	$-$	0.5831
$\alpha = 0.0$	$\tilde{B}$	$-$	$-$	$-$	0.7071
Results		$-$	$-$	$-$	$\tilde{A} \prec \tilde{B}$
set(1)	$\tilde{A}$	0.3420	0.5197	0.7104	0.8078
$\alpha = 0.5$	$\tilde{B}$	0.5003	0.6435	0.8120	0.9014
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
set(2)	$\tilde{A}$	$-$	$-$	$-$	0.500
$\alpha = 0.0$	$\tilde{B}$	$-$	$-$	$-$	0.5831
Results		$-$	$-$	$-$	$\tilde{A} \prec \tilde{B}$
set(2)	$\tilde{A}$	0.3345	0.4820	0.6370	0.7159
$\alpha = 0.5$	$\tilde{B}$	0.3420	0.5197	0.7104	0.8078
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
set(3)	$\tilde{A}$	$-$	$-$	$-$	0.7673
$\alpha = 0.0$	$\tilde{B}$	$-$	$-$	$-$	0.7241
Results		$-$	$-$	$-$	$\tilde{A} \succ \tilde{B}$
set(3)	$\tilde{A}$	0.4873	0.6502	0.8326	0.9276
$\alpha = 0.5$	$\tilde{B}$	0.4944	0.6562	0.8377	0.9322
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
set(4)	$\tilde{A}$	$-$	$-$	$-$	0.68
$\alpha = 0.0$	$\tilde{B}$	$-$	$-$	$-$	0.7257
	$\tilde{C}$	$-$	$-$	$-$	0.7462
Results		$-$	$-$	$-$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
set(4)	$\tilde{A}$	0.4452	0.6096	0.7927	0.8877
$\alpha = 0.5$	$\tilde{B}$	0.4701	0.6328	0.8151	0.9099
	$\tilde{C}$	0.5330	0.6880	0.8643	0.9569
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$

5 Conclusion

In this paper, a simple ranking method is obtained by using distance between convex combination of upper and lower central gravity of α-level and the origin of coordinate system. The advantage of this method is that each α-level and each convex combination result in one ranking index which can rank a group of fuzzy number. It is also noticed that shortcomings found in Cheng's cv index for ranking fuzzy numbers is in this paper (example 4.1). Normal and non-normal fuzzy numbers are also compared with each other in this invetigation.

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