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Single-Echelon Supply Chain Two Stage Distribution Inventory Optimization Model for the Confectionery Industry

K. Balaji Reddy [1], S. Narayanan [1] and P. Pandian [2]

[1] School of Mechanical and Building Sciences,
VIT University, Vellore-14, Tamilnadu, India

[2] School of Advanced Sciences,
VIT University, Vellore-14, Tamilnadu, India
pandian61@rediffmail.com

Abstract

In this paper, a supply chain two stage distribution inventory optimization model with interactive-lateral transshipment among retailers in a particular period is considered. Interactive-lateral transshipment among retailers means that once the stocks are allocated to retailers from ware houses, we allow lateral stocks transfer among retailers if the further optimization is possible which will enable us to further minimize overall cost in the supply chain. The proposed model is formulated as a linear programming model which is validated with help of a confectionary industry data. This model is to provide an optimal inventory level for the warehouses and retailers and also, minimizing the total cost of the entire supply chain for a finite planning horizon.

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1. Introduction

Now-a-days, the Supply Chain Management (SCM) is attracting much research attention, because of its ability to reap more benefits by efficiently managing it. Supply Chain Management (SCM) is a set of synchronized decisions

and activities utilized to efficiently integrate suppliers, manufacturers, warehouses, transporters, retailers, and customers so that the right product or service is distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying customer service level requirements. The objective of SCM is to achieve sustainable competitive advantage. To stay competitive, organizations should improve customer service, reduction of costs across the supply chain and efficient use of resources available in the supply chain

One-warehouse and multi-retailer systems have attracted much research attention in the literature because of their application in all business models. The discrete time models of one-warehouse, multi-retailer systems have been studied under different assumptions [4, 5, 7]. Archibald et al. [1] developed a model which was motivated by the inventory problem faced by a UK car part retailer that groups its depots into pairs. Avijit Banerjee et al. [2] studied the effects, in terms of some selected criteria, of two lateral transshipment approaches in a two-echelon supply chain network, with a single supply source at the higher echelon and multiple retail locations at the lower, through a series of simulation experiments under different operating conditions. Noorul Haq and Kannan [9] considered a two-echelon distribution inventory supply chain model for a bread industry using genetic algorithm.

Young Hae Lee et al. [11] proposed a new lateral transshipment policy, called Service Level Adjustment (SLA), which differs from previous policies by integrating emergency lateral transshipment with preventive lateral transshipment to efficiently respond to customer demands. Additionally, the proposed policy considers the service level to decide the quantity for lateral transshipment. Fredrick Olsson [6] developed a model in which the transshipment rule is to always transship when there is a shortage at one location and stock on hand at the other. They assumed that the locations apply (R, Q) policies for normal replenishments and showed that the optimal policies are not necessarily symmetric even though the locations are identical.

Emmanouil et al. [3] proposed a model to determine the timing and size of the replenishment services together with the vehicle routes, so that the total transportation and inventory holding cost of the system is minimized. Sheng-Yuan Shen and Masakazu Honda [10] focused on developing an integrated replenishment and routing plan that takes into account lateral transfers of both vehicles and inventory for a three-echelon supply chain system including a single plant, multiple distribution centers and multiple retailers by mixed integer programming techniques. Lorenzo Tiacci and Stefano Saetta [8] considered a preventive transshipment case, where the inventory of different locations at the same echelon is balanced through lateral transshipments, before a shortage happens. A heuristic for deciding on transshipment policy trying to minimize overall expected costs was presented.

In this paper, a supply chain distribution two stage inventory optimization model is proposed with interactive-lateral transshipment among retailers. The overall objective of the model is to minimize the total cost of the supply chain for a finite planning horizon. Interactive-lateral transshipment among retailers means that once the stocks are allocated to retailers from ware houses, lateral stocks transfer among retailers may be allowed if the further optimization is possible. This will enable us to further minimize overall cost in the supply chain. The key cost components which considered here are, inventory carrying cost at warehouse and retailers, transportation costs from ware house to retailers and among retailers, and the back order cost at retailers in a particular period. The proposed work is formulated as a linear programming model in two stages and is validated by considering the case study of a confectionary industry.

2. Model Description

The proposed model is a single-echelon two stage distribution inventory model with N_w warehouses and N_r retailers with limited capacities. It is assumed that a single product is being distributed from the warehouse to retailers in the system. The demand for the product will be forecasted before beginning of every period and will be used as the reference for ware houses to transfer stocks from them to retailers in a particular period p. All retailers and customers in the downstream are identical; i.e., they have identical cost structures and demand distributions. Any demand that is not met is considered as the back order. All warehouses assume the instantaneous supply from an outside source. All Warehouses and retailers will be managed by a single organisation only. Random demands occur at retailers, which were then transferred to warehouses for replenishing inventories. Excess demands at retailers are completely backlogged. Further more, the lead times of retailers from warehouses are different and depending on their geographical positions. This model is suitable to any kind of industry that follows the two-stage supply chain model with finite number of ware houses and retailers.

The problem is divided into two stage optimization model, and in the first stage we consider only ware houses and retailers and interaction between them. Based on the forecasted demand values, warehouses will try to satisfy the need of all retailers by transferring stocks to them. The capacities of all warehouses and retailers will also be taken into consideration in optimising the model. Once the optimal solution is found then these values will be used in the second stage to further optimize the model. In the second stage, we consider the interaction among retailers. We use optimal values from first stage to find excess inventories and back ordered units at each retailer. We calculate stage inventories and stage back orders for each retailer after realizing the actual demand, and a second stage constrained optimization model is developed to further optimize the total cost by considering

the lateral transhipment among retailers. For modelling simplicity, we did not consider the lead times between retailers at the end of every period, and we assume it as negligible. That is, we assume that this lateral transhipment of stocks among retailers will be taken place before starting of the next period.

The purpose of this model is to provide an optimal inventory level for the warehouses and retailers and also minimising the total cost of the supply chain at each period. In designing the proposed model, the three subsystems need to be analysed.

(i) The inventory level at each warehouse for all periods.
(ii) The inventory level at each retailers for all periods and
(iii) Route between entities where the product is transferred from warehouse to retailers and among retailers with back orders for all periods.

3. Two Stage Constrained Optimization Model

The proposed model deals with optimizing the inventory levels of the supply chain entities with consideration of single-echelon warehouse to retailers and among retailers. The diagrammatic representation of the proposed model for the period '*p*' is shown in Figure 1.

Let, TC_p , TC_{p_1} and TC_{p_2} be the minimum total cost in a particular period p, the total cost in the first stage of period p and the total cost in the second stage of period p respectively.

Then, $TC_p = MinimizeTC_{p_1} + MinimizeTC_{p_2}$

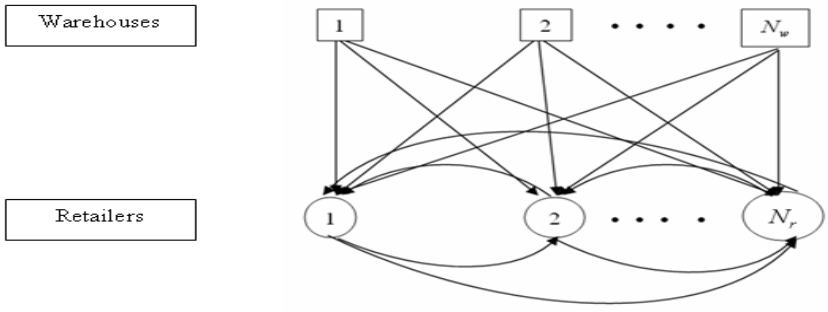

Figure 1 Two Stage Constrained Optimization Model

First Stage problem :

Let $ICWH_p$ be total inventory carrying cost of all warehouses in period ' p ' and $TPWR_p$ be total transportation cost from all warehouses to all retailers in period ' p '

Let IWH_{pj} be inventory at j^{th} ware house at the end of the period p, $UICCW_j$ be unit inventory carrying cost at j^{th} warehouse, $UTWR_{pjk}$ be units transferred from j^{th} warehouse to k^{th} retailer in period ' p ' and $UTCWR_{jk}$ be unit transportation cost from j^{th} ware house to k^{th} retailer.

Now, the first stage objective function is given below:

$$TC_{p_1} = \sum_{j=1}^{N_w} IW_{pj} \times UICCW_j + \sum_{j=1}^{N_w} \sum_{k=1}^{N_r} UTWR_{pjk} \times UTCWR_{jk}$$

In the first stage, we have the following constraints:

1. The sum of the units transferred from a ware house to all retailers should be less than or equal to the ware house inventory for a particular period ' p '

$$\sum_{k=1}^{N_r} UTWR_{pjk} \le IWH_{pj}$$

where $UTWR_{pjk}$ is units transferred from j^{th} ware house to k^{th} retailer in period ' p ' and IWH_{pj} is inventory of j^{th} ware house in period ' p '

2. Inventory at the warehouse should be less than or equal to the ware house capacity in period p.

$$IWH_{pj} \le WHCAP_{pj}$$

where IWH_{pj} is inventory of j^{th} ware house in period ' p ' and $WHCAP_{pj}$ is Capacity of the j^{th} ware house in period ' p '.

3. The sum of the units transferred from all ware houses to a particular retailer should be greater than or equal to the forecasted demand of that particular retailer in period p.

$$\sum_{j=1}^{N_w} UTWR_{pjk} \ge FDR_{pk}$$

where FDR_{pk} is the forecasted demand of the k^{th} retailer in period p and $UTWR_{pjk}$ is units transferred from j^{th} ware house to k^{th} retailer in period ' p '

4. The total retailers forecasted demand for a period p should be less than or equal to all ware houses inventory in that particular period.

$$\sum_{j=1}^{N_w} IWH_{pj} \ge \sum_{k=1}^{N_r} FDR_{pk}$$

where IWH_{pj} is inventory at j^{th} ware house at the end of the period p and FDR_{pk} is demand of k^{th} retailer in period ' p ' .

5. The total number units transferred from all ware houses to a particular retailer should be less than or equal to that retailer capacity.

$$\sum_{j=1}^{N_w} UTWR_{pjk} \le RCAP_{pk}$$

where $RCAP_{pk}$ is the capacity of the retailer for making stock and $UTWR_{pjk}$ is units transferred from j^{th} ware house to k^{th} retailer in period ' p '.

Therefore, the mathematical model for the first stage problem is to minimize the objective function TC_{p_1} subjected to above said constraints.

Second Stage problem :

Let, $ICCR_p$ be total inventory carrying cost of all retailers in period ' p ' $TPCRR_p$ be total transportation cost from all retailers to all retailers in period ' p ' and $BOCR_p$ be total back order cost of all retailers in period ' p '

Now, $ICCR_P = \sum_{k=1}^{N_c} ICR_{pk} \times UICCR_k \quad , \quad k = 1, 2, ...N_r$

where $ICR_{pk} = sICR_{pk} - \sum_{m=1}^{N_r} UTRR_{pkm}$ is inventory at k^{th} retailer at the end of the period p , $sICR_{pk}$ is the stage inventory at k^{th} retailer at the end of the stage 1, $UTRR_{pkm}$ is units transferred from k^{th} retailer to m^{th} retailer and $UICCR_k$ is unit Inventory carrying cost at k^{th} retailer

Now, $BOCR_p = \sum_{k=1}^{N_r} BO_{pk} \times UCBO_k \quad k = 1, 2, ...N_r$

where $BO_{pk} = sBO_{pk} - \sum_{m=1}^{N_r} URRR_{pkm}$ is back order at k^{th} retailer at the end of period p , sBO_{pk} is the stage back order at k^{th} retailer at the end of the stage 1, $URRR_{pkm}$ is units received by k^{th} retailer from m^{th} retailer and $UCBO_k$ is unit cost of back order at k^{th} retailer.

$$\text{Now, } TPCRR_p = \sum_{k=1}^{N_r}\sum_{m=1}^{N_r} UTRR_{pkm} \times UTCRR_{km} \qquad k, m = 1, 2, ...N_r$$

where $UTRR_{pkm}$ is units transferred from k^{th} retailer to m^{th} retailer in period ' p ' with $UTRR_{pkm} = 0, \ if \ k = m$ and $UTCRR_{km}$ is Unit Transportation cost from k^{th} retailer to m^{th} retailer with $UTCRR_{km} = UTCRR_{mk}$.

Therefore, the second stage objective function is given below:

$$TC_{p_2} = \left(ICCR_P + TPCRR_p + BOCR_{pk}\right).$$

In the second stage , we have the following constraints.

1. The inventory at a particular retailer at the end of stage one, is equal to the total number units received from all warehouses by that retailer minus the actual demand of that particular retailer in period p.

$$sICR_{pk} = \sum_{j=1}^{N_w} UTWR_{pjk} - ADR_{pk}$$

(If this value is negative then, $sICU_{pk} = 0$)

2. The back order of a retailer at the end of the stage 1 is equal to the actual demand of that particular retailer minus the total number units received from all warehouses by that retailer in the period p.

$$sBO_{pk} = ADR_{pk} - \sum_{j=1}^{N_w} UTWR_{pjk}$$

(If this value is negative then, $sBO_{pk} = 0$)

3. The sum of the units transferred from a retailer to all retailers should be less than or equal to the excess inventory of a retailer at the end of the stage 1

$$\sum_{m=1}^{N_r} UTRR_{pkm} \le sICU_{pk} \qquad \text{where, } UTRR_{pkm} = 0 \ , \ if \ k = m$$

where, $UTRR_{pkm}$ is units transferred from k^{th} retailer to m^{th} retailer in period ' p ' and $sICR_{pk}$ = Inventory of the k^{th} retailer at the end of the stage 1

4. The sum of the units received by a retailer from all retailers should be less than or unequal to the back order of that retailer for a particular period ' p '

$$\sum_{m=1}^{N_r} URRR_{pkm} \le sBO_{pk} \qquad k = 1, 2, ...N_r$$

where, $URRR_{pkm}$ is units received by k^{th} retailer from m^{th} retailer in period 'p' and sBO_{pk} is back order of k^{th} retailer at the end of the stage 1

5. The sum of all transportation costs from all retailers to a particular retailer should be less than or equal to the transportation cost from any ware house to that retailer plus the back ordering cost of that retailer.

$$\sum_{m=1}^{N_r} TPRR_{pkm} \leq TPWR_{pjk} + BOCR_{pk}$$

where, $TPRR_{pkm} = URRR_{pkm} * UTCRR_{pkm}$ is the transportation cost to k^{th} retailer from m^{th} retailer in period p, $TPWR_{pjk} = sBO_{pk} * UTCWR_{pjk}$ is the transportation cost from j^{th} ware house to k^{th} retailer in period p, $BOCR_{pk} = sBO_{pk} * BOC_k$ is Back Order cost of k^{th} retailer in period p

Therefore, the mathematical model for the second stage problem is to minimize the objective function TC_{p_2} subjected to above said constraints.

At the end of the period *p*, we have

$$TC_p = MinTC_{p_1} + MinTC_{p_2}$$

4. Industry Case study for validation of model

The company chosen for the application of supply chain distribution inventory model in this work is a famous confectionery company located in the southern part of India. The company has planned to build a mathematical model to minimize the total cost of the supply chain. The total cost can be minimized by optimizing the inventory levels at warehouse and retailers and optimizing the transportation cost in the supply chain. This can be done by transporting the product through the optimum route and by maintaining an optimum inventory at ware houses and retailers. The following data is collected for validating the above proposed model. Number of periods N_p = 3, Number of Ware houses N_w = 1 and Number of retailers N_r = 3 .

The input data for ware houses and retailers are given in Tables 1 to 5 respectively.

Table 1 Input Data for Warehouse (DPG)

	Warehouse
Inventory carrying cost per units in Rupees $ICWH_p$	5
Warehouse Capacity in units WHS_P	5,00,000

Table 2 Retailer Demand

Retailers Demand in Units						
	R_1		R_2		R_3	
	Forecasted	Actual	Forecasted	Actual	Forecasted	Actual
Period P_1	115000	100058	110000	101100	75000	95685
Period P_2	125000	110516	100000	126875	130000	120431
Period P_3	90000	97752	125000	110010	85000	91920

Table 3 Input Data for retailers

Retailers			
	R_1	R_2	R_3
Inventory Carrying cost in rupees per unit	5	4	6

Table 4 Transportation Cost from Warehouse to Retailers per unit in Rs

	Retailers		
	R_1	R_2	R_3
Transportation Cost in Rupees from Warehouse $TPWC_p$ to	19	15	11

Table 5 Transportation Cost from Retailers to Retailers per unit in Rs

Retailers	R_1	R_2	R_3
R_1	0	12	9
R_2	12	0	8
R_3	9	8	0

and back Order Cost at each retailer per Kg = Rs. 9.

4.1. Results and Discussions

After solving the model for two stages, using Premium Solver Software, the optimal solutions and results for the case study are obtained. The problem is solved for 3 periods, by using the industry data by considering a single warehouse and three retailers. These results are tabulated, compared with the real time existing solutions of the industry and are given in tables 6 to 11 respectively .

Table 6 Optimal Ware House Stock

Period	Number of units at W_1	
	Proposed Methodology	Existing(Industry)
p_1	310000	400000
p_2	355000	400000
p_3	300000	400000

Table 7 Optimal number of units transferred from ware house to retailer

Period	Retailer	W_1, Proposed Methodology	W_1, Existing(Industry)
p_1	R_1	115000	130000
	R_2	110000	118000
	R_3	75000	90000
p_2	R_1	125000	136000
	R_2	100000	110000
	R_3	130000	145000
p_3	R_1	90000	105000
	R_2	125000	130000
	R_3	85000	95000

Table 8 Optimal number of units transferred from retailer to retailer

Period		Proposed Methodology			Existing(Industry)		
	Customer	R_1	R_2	R_3	R_1	R_2	R_3
p_1	R_1	-	0	11785	0	0	0
	R_2	0	-	8900	0	0	0
	R_3	0	0	-	0	0	0
p_2	R_1	-	14484	0	0	0	0
	R_2	0	-	0	0	0	0
	R_3	0	9569	-	0	0	0
p_3	R_1	-	0	0	0	0	0
	R_2	7752	-	6920	0	0	0
	R_3	0	0	-	0	0	0

Table 9 Optimal Inventory in units at Retailers

Period	Proposed Methodology Inventory in Units			Existing(Industry) Inventory in Units		
	R_1	R_2	R_3	R_1	R_2	R_3
p_1	3157	0	0	29942	16900	0
p_2	0	0	0	25484	0	24569
p_3	0	318	0	7248	19990	3080

Table 10 Number of Back Ordered units at Retailers

Period	Proposed Methodology Back Order in Units			Existing(Industry) Back Order in Units		
	R_1	R_2	R_3	R_1	R_2	R_3
p_1	0	0	0	0	0	5685
p_2	0	2822	0	0	16875	0
p_3	0	0	0	0	0	0

Table 11 Optimal Total Cost

Component	Proposed Methodology in Rs			Existing(Industry) in Rs		
	p_1	p_2	p_3	p_1	p_2	p_3
$ICWH_p$	15,50,000	17,75,000	15,00,000	20,00,000	20,00,000	20,00,000
$TPWC_p$	46,60,000	53,05,000	45,20,000	52,30,000	58,29,000	49,90,000
$ICCU_p$	15,785	0	1,272	2,17,310	2,74,834	1,34,680
$TPCC_p$	1,77,265	2,50,360	1,48,384	0	0	0
BOC_p	0	25,398	0	51,165	1,51,875	0
TC	64,03,050	73,55,758	61,69,656	74,98,475	82,55,709	71,24,680

The optimal cost comparison graph is given in Figure 2. The major advantage of the proposed model over existing model is the maintenance of the optimum inventory levels at the ware house and retailers, and minimizing the back order costs at retailers level. Because of this, there is a significant decrease in total cost using the proposed model over existing model, as shown in the Figure 2.

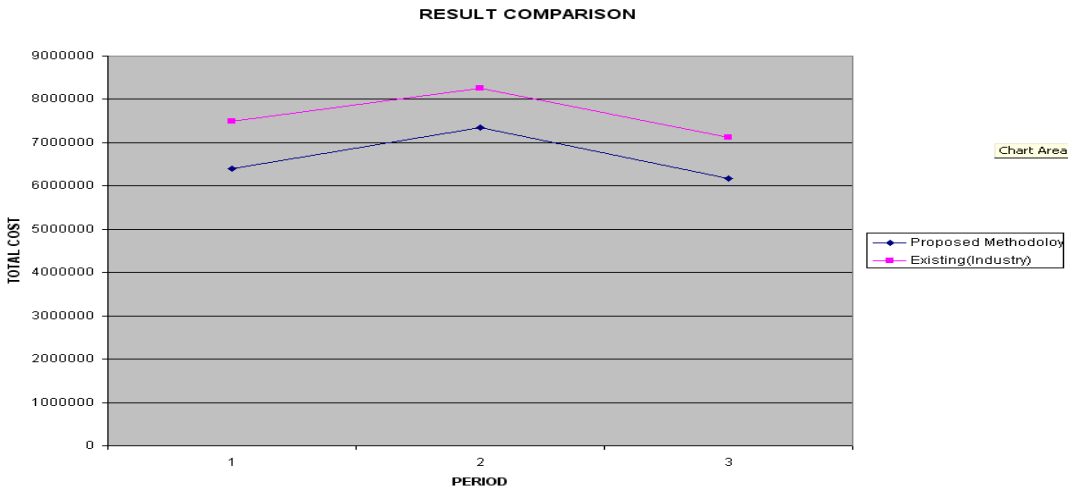

Figure 2 Comparison of Proposed Methodology and Existing (Industry)

5. Conclusion

This paper has presented a supply chain two stage distribution inventory optimization model for a distribution network with multiple ware houses supplying multiple retailers, who in turn serve a large number of customers. This model has taken the distribution and inventory carrying costs into account in the supply chain network at each period. With the validation of case study using the confectionery industry data, it is clear that the results obtained are encouraging and reduced over all system costs. By making retailers to interact and taking a decision on lateral transshipment, the inventory level of different locations at the same echelon is balanced.

However, this work has few limitations. Future research can address some of the following issues.

- The model assumes lead time between retailers is negligible at the end of the period, for modeling simplicity. So, better models can be developed to address this issue.

- The model also assumes that there is a single product in the study area. Models can be developed for multi-product scenarios.

- This model is limited to single echelon and extensions of two and more echelons can be addressed with different approaches.

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