Applied Mathematical Sciences, Vol. 5, 2011, no. 52, 2549 - 2558

Estimation of the Weibull Distribution Based on Type-II Censored Samples

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Abstract

In this paper, the estimation of parameters based on Type-II censored sample from a Weibull distribution is studied. The maximum likelihood estimators are developed for estimating the unknown parameters**.** The observed Fisher information matrix, as well as the asymptotic variance-covariance matrix of the MLEs are derived. Approximate confidence intervals for the parameters, based on *s*-normal approximation to the asymptotic distribution of MLEs are derived. Also, the Bayes estimators relative to asymmetric loss function (Linex loss) are obtained using Lindley's approximation technique. A Monte Carlo simulation study is carried out to compare the performances of the different methods.

Keywords: Maximum likelihood estimators; *s*-normal approximation; Bayes estimators; Asymmetric loss function (Linex loss); Type-II censoring

1 Introduction

Type-I and Type-II censoring schemes are the two most popular censoring schemes used in the reliability and life testing experiments. In the Type II censoring scheme, the experiment items *n* placed on test are set first and the

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number of uncensored data r is predetermined. Instead of continuing experiments until all n items have failed, the experiment is terminated when the rth item fails [7]. The remaining $n-r$ items are regarded as censored data. In this article we consider the conventional Type II censored lifetime data, when the lifetime of the experimental unit follows a Weibull distribution.

The Weibull distribution is one of the most popular distributions in analyzing the lifetime data. Much of the popularity of the Weibull distribution is due to the wide variety of shapes it can assume by varying its parameters. Extensive work has been done on this distribution, see for example, Ahmadi, Doostparast and Parsian [1], Banerjee and Kundu [2], Berger and Sun [4], Gupta and Kundu [5], Kandinsky and Krivtsov [6], McCool [9], Nordman and Meeker [10] and Singh, Gupta and Upadhyay [11;12].

The Weibull distribution has the distribution function;

$$F(x;\alpha,\beta)=1-e^{-\beta x^{\alpha}} \qquad x>0 \qquad \alpha,\beta>0 \tag{1}$$

Therefore, the Weibull distribution has the density function;

$$f(x;\alpha,\beta)=\alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}} \qquad x>0 \tag{2}$$

The survival function

$$S(x;\alpha,\beta)=1-\left(1-e^{-\beta x^{\alpha}}\right) \qquad x>0$$

and the hazard function

$$h(x;\alpha,\beta)=\frac{\alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}}}{e^{-\beta x^{\alpha}}}$$

Here α and β are the shape and the scale parameters, respectively. It is known that the density function of the Weibull distribution (PDF) can be decreasing or unimodal, and the hazard function (HF) can be either decreasing or increasing depending on the shape parameter.

The aim of this paper is two fold. First we consider the maximum likelihood estimators of the unknown parameters when the data are type-II censored. It is observed that the MLEs cannot be obtained in closed form. We propose to use the Newton-Raphson method to compute the MLEs, and the proposed method works quite well. We have also obtained the Fisher information matrix and it has been used to construct the confidence intervals of the unknown parameters based on a s-normal approximation. The second aim of this paper is to consider the Bayesian inference for the unknown parameters. It is remarkable that most of the Bayesian inference procedures have been developed with the usual squared error loss function, which is symmetrical and associates equal importance to the losses due to overestimation and underestimation of equal magnitude. However, such a restriction may be impractical in most situations of practical importance. For example, in the estimation of reliability and failure rate functions, an overestimation is usually much more serious than an underestimation. In this case, the use of symmetrical loss function might be inappropriate as also emphasized by Basu and Ebrahimi [3]. So, this paper, we obtain the Bayes estimates under the Linex loss function using Lindley's approximation technique.

The rest of the paper is organized as follows. In Section 2, we derive the MLE of the unknown parameters and the interval estimation of parameters based on a

s-normal approximation of the distribution of the MLE. Bayes estimates of the parameters of Weibull distribution under the Linex loss function using Lindley's approximation technique are computed in section 3. Different proposed methods are compared using Monte Carlo simulation and the results are presented in section 4 and finally conclusions appear in section 5.

2 Maximum Likelihood Estimation

Suppose that $X_1 < ... < X_r$ is a type II censored sample of size *r* obtained from a life test on *n* items whose lifetimes have the $Weibull(\alpha, \beta)$ model. The likelihood function may be written as

$$l(\alpha,\beta) = \frac{n!}{(n-r)!}(\alpha\beta)^r \prod_{i=1}^{r} e^{-\beta x_i^{\alpha}} \prod_{i=1}^{r} x_i^{\alpha-1} \times \left[1-(1-e^{-\beta x_r^{\alpha}})\right]^{n-r} \tag{3}$$

Then, the log likelihood function can be expressed

$$\begin{aligned} L(\alpha,\beta) = {} & Lnn! - Ln(n-r)! + rLn\alpha + rLn\beta - (n-r)\beta x_r^{\alpha} \\ & - \beta\sum_{i=1}^{r} x_i^{\alpha} + (\alpha-1)\sum_{i=1}^{r} \ln x_i \end{aligned} \tag{4}$$

The MLE's of α and β say $\hat{\alpha}$ and $\hat{\beta}$ respectively, can be obtained as the solutions of

$$\frac{\partial L}{\partial \beta} = \frac{r}{\beta} - (n-r)x_r^{\alpha} - \sum_{i=1}^{r} x_i^{\alpha} = 0 \tag{5}$$

$$\frac{\partial L}{\partial \alpha} = \frac{r}{\alpha} - (n-r)\beta x_r^{\alpha} \ln x_r - \beta\sum_{i=1}^{r} x_i^{\alpha} Lnx_i + \sum_{i=1}^{r} Lnx_i = 0 \tag{6}$$

From (5), we obtain the maximum likelihood estimator of β as a function of α, say $\hat{\beta}$ where

$$\hat{\beta} = \frac{r}{(n-r)x_r^{\alpha} + \sum_{i=1}^{r} x_i^{\alpha}} \tag{7}$$

and $\hat{\alpha}$, can be obtained as the solution of

$$\frac{r(n-r)}{(n-r)x_r^{\alpha} + \sum_{i=1}^{r} x_i^{\alpha}} x_r^{\alpha} \ln x_r + \left(\frac{r}{(n-r)x_r^{\alpha} + \sum_{i=1}^{r} x_i^{\alpha}}\right)\sum_{i=1}^{r} x_i^{\alpha} Lnx_i - \frac{r}{\alpha} = \sum_{i=1}^{r} Lnx_i \tag{8}$$

In order to solve the above equation, the Newton- Raphson iterative procedure is employed. If we define

$$S_{1,f} = \sum_{i=1}^{r} Lnx_i$$

$$S_{2,f}^{(l)} = \sum_{i=1}^{r} x_i^{\hat{\alpha}}, \qquad S_{3,f}^{(l)} = \sum_{i=1}^{r} x_i^{\hat{\alpha}} Lnx_i, \qquad S_{4,f}^{(l)} = \sum_{i=1}^{r} x_i^{\hat{\alpha}} (Lnx_i)^2,$$

$$S_{2,s}^{(l)} = (n-r)x_r^{\hat{\alpha}}, \quad S_{3,s}^{(l)} = (n-r)x_r^{\hat{\alpha}} \ln x_r, \quad S_{4,s}^{(l)} = (n-r)x_r^{\hat{\alpha}} (\ln x_r)^2$$

Therefore, $\hat{\alpha}$, can be obtained as a solution of the equation of the form:

$$\hat{\alpha}_{i+1} = \hat{\alpha}_i + \frac{\left\{ \frac{r}{\hat{\alpha}_i} + S_{1,f} - S_{3,f}^{(l)} - \frac{r}{S_{2,f}^{(l)} + S_{2,s}^{(l)}} (S_{3,f}^{(l)} + S_{3,s}^{(l)}) \right\}}{\left\{ \frac{r}{\hat{\alpha}_i^2} + S_{4,f}^{(l)} - \frac{r}{(S_{2,f}^{(l)} + S_{2,s}^{(l)})^2} (S_{3,f}^{(l)} + S_{3,s}^{(l)})^2 + \frac{r}{S_{2,f}^{(l)} + S_{2,s}^{(l)}} (S_{4,f}^{(l)} + S_{4,s}^{(l)}) \right\}} \tag{9}$$

The iterative procedure works very well. Once we obtain $\hat{\alpha}$, $\hat{\beta}$ can be obtained of:

$$\hat{\beta} = \frac{r}{S_{2,f}^{(l)} + S_{2,s}^{(l)}} \tag{10}$$

2.1 Observed Fisher Information

In this subsection, we compute the observed Fisher information for the MLE. These will enable us to construct confidence intervals for the parameters based on the limiting *s*-normal distribution. We now derive the observed Fisher information for the likelihood using (5) and (6). We have

$$I(\hat{\alpha}, \hat{\beta}) = \left[\begin{array}{cc} -\frac{\partial^2 \ln L}{\partial \alpha^2} & -\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} \\ -\frac{\partial^2 \ln L}{\partial \beta \partial \alpha} & -\frac{\partial^2 \ln L}{\partial \beta^2} \end{array} \right]_{(\alpha,\beta)=(\hat{\alpha},\hat{\beta})} \tag{11}$$

Where

$$\frac{\partial^2 L}{\partial \beta^2} = -\frac{r}{\beta^2} \tag{12}$$

$$\frac{\partial^2 L}{\partial \alpha^2} = -\frac{r}{\alpha^2} - (n-r)\beta x_r^{\alpha} (\ln x_r)^2 - \beta \sum_{i=1}^{r} x_i^{\alpha} (Lnx_i)^2 \tag{13}$$

$$\frac{\partial^2 L}{\partial \alpha \partial \beta} = \frac{\partial^2 L}{\partial \beta \partial \alpha} = -(n-r)x_r^{\alpha} \ln x_r - \sum_{i=1}^{r} x_i^{\alpha} Lnx_i \tag{14}$$

The observed Fisher information matrix can be inverted to obtain a local estimate of the asymptotic variance-covariance matrix of the MLE as

$$I(\hat{\alpha}, \hat{\beta})^{-1} = \left[\begin{array}{cc} \hat{\sigma}^2(\hat{\alpha}) & \hat{\sigma}(\hat{\alpha}, \hat{\beta}) \\ \hat{\sigma}(\hat{\beta}, \hat{\alpha}) & \hat{\sigma}^2(\hat{\beta}) \end{array} \right]$$

Following the general asymptotic theory of MLE, the sampling distribution of $(\hat{\alpha} - \alpha) \Big/ \sqrt{\hat{\sigma}^2(\hat{\alpha})}$ can be approximating by a standard *s*-normal distribution. A two-sided $100(1-\gamma)\%$ *s*-normal approximation confidence interval for α can

then be constructed as

$$\hat{\alpha} \pm Z_{1-\gamma/2}\sqrt{\hat{\sigma}^2(\hat{\alpha})} \tag{15}$$

Similarly, the two-sided $100(1-\gamma)\%$ *s*-normal approximation confidence interval for β can be constructed as

$$\hat{\beta} \pm Z_{1-\gamma/2}\sqrt{\hat{\sigma}^2(\hat{\beta})} \tag{16}$$

3 Bayes Estimator under Linex loss

Varian [13] introduced the most popular asymmetric loss function, called the Linex loss function (LLF), which is given by

$$d\{e^{k\Delta} - k\Delta - 1\}; \quad k \neq 0, d > 0 \tag{17}$$

Where, $(\hat{\theta} - \theta)$ is a scalar estimation error using $\hat{\theta}$ to estimate θ. Under the above loss function, the Bayes estimator of θ is given by

$$\hat{\theta}_{Linex} = -\frac{1}{k}\ln E_{\theta}(e^{-k\theta}) \tag{18}$$

The sign of shape parameter *k* reflects the direction of asymmetry, and its magnitude reflects the degree of asymmetry.

For a Bayesian estimation, we need prior distribution for the parameters α and β. It is assumed that α and β each have independent $Gamma(a_1, b_1)$ and $Gamma(a_2, b_2)$ priors respectively, for $a_1, b_1, a_2, b_2 > 0, i.e.$

$$\pi_1(\alpha) \propto \alpha^{a_1-1}e^{-b_1\alpha} \quad \text{and} \quad \pi_2(\beta) \propto \beta^{a_2-1}e^{-b_2\beta}$$

Based on the priors, the joint posterior density of α and β given the data is

$$\pi(\alpha, \beta|data) = \frac{l(data|\alpha, \beta) \times \pi_2(\beta) \times \pi_1(\alpha)}{\int_0^{\infty}\int_0^{\infty} l(data|\alpha, \beta) \times \pi_2(\beta) \times \pi_1(\alpha) d\alpha d\beta} \tag{19}$$

It may be noted here that the posterior distribution of (α, β) takes a ratio form that involves an integration in the denominator and cannot be reduced to a closed form. Therefore, we adopt Lindley's approximation [8] procedure to approximate the Bayes estimate of $g(\alpha, \beta)$ under a Linex loss function. Based on Lindley's approximation, any ratio of the integral of the form

$$\begin{aligned} &\hat{g}_B(\alpha, \beta) \\ &= E_{\alpha,\beta|data}(g(\alpha, \beta)) \\ &= \frac{\int_0^{\infty}\int_0^{\infty} g(\alpha, \beta) l(data|\alpha, \beta) \times \pi_2(\beta) \times \pi_1(\alpha) d\alpha d\beta}{\int_0^{\infty}\int_0^{\infty} l(data|\alpha, \beta) \times \pi_2(\beta) \times \pi_1(\alpha) d\alpha d\beta} \end{aligned} \tag{20}$$

can be evaluated as

$$\hat{g} = \hat{g}\left(\hat{\alpha},\hat{\beta}\right)+\frac{1}{2}\left(\sum_{j=1}^{2}\sum_{i=1}^{2}v_{ij}\tau_{ij}+\upsilon_{30}A_{12}+\upsilon_{03}A_{21}+\upsilon_{21}B_{12}+\upsilon_{12}B_{21}\right)$$
$$+\omega_1 C_{12}+\omega_2 C_{21}$$

Where

$$\upsilon_{ij}=\frac{\partial^{i+j}L(\alpha_1,\alpha_2)}{\partial\alpha^i\partial\beta^j},\qquad i,j=0,1,2,3,\quad i+j=3$$

$$\omega_1=\frac{\partial\ln\pi(\alpha,\beta)}{\partial\alpha},\omega_2=\frac{\partial\ln\pi(\alpha,\beta)}{\partial\beta},\quad v_{12}=\frac{\partial^2 g(\alpha,\beta)}{\partial\alpha\partial\beta},\quad v_{11}=\frac{\partial^2 g(\alpha,\beta)}{\partial\alpha^2},\quad v_{22}=\frac{\partial^2 g(\alpha,\beta)}{\partial\beta^2}$$

$$v_{21}=\frac{\partial^2 g(\alpha,\beta)}{\partial\beta\partial\alpha},\quad v_1=\frac{\partial g(\alpha,\beta)}{\partial\alpha},\quad v_2=\frac{\partial g(\alpha,\beta)}{\partial\beta}$$

$$A_{ij}=\left(v_i\tau_{ii}+v_j\tau_{ij}\right)\tau_{ii},\ B_{ij}=3v_i\tau_{ii}\tau_{ij}+v_j\left(\tau_{ii}\tau_{jj}+2\tau_{ij}^2\right);\quad i,j=1,2$$

$$C_{ij}=v_i\tau_{ii}+v_j\tau_{ji};\ i,j=1,2$$

Therefore, the approximate Bayes estimates of α and β for the Linex loss functions are respectively

$$\hat{\alpha}_{Linex}=-\frac{1}{k}\ln\Bigg[e^{-k\hat{\alpha}}+\frac{1}{2}\Bigg\{(k^2e^{-k\hat{\alpha}})\tau_{11}+\left(\frac{2r}{\hat{\alpha}^3}-\hat{\beta}(\sum_{i=1}^{r}x_i^{\hat{\alpha}}(\ln x_i)^3+(n-r)x_r^{\hat{\alpha}}(\ln x_r)^3)\right)(-ke^{-k\hat{\alpha}})\tau_{11}^2$$
$$+\frac{2r}{\hat{\beta}^3}(-ke^{-k\hat{\alpha}})\tau_{21}\tau_{22}-3\tau_{11}\tau_{12}(-ke^{-k\hat{\alpha}})(\sum_{i=1}^{r}x_i^{\hat{\alpha}}(\ln x_i)^2+(n-r)x_r^{\hat{\alpha}}(\ln x_r)^2)\Bigg\}$$
$$+\left(\frac{a-1}{\hat{\alpha}}-b\right)(-ke^{-k\hat{\alpha}})\tau_{11}+\left(\frac{c-1}{\hat{\beta}}-d\right)(-ke^{-k\hat{\alpha}})\tau_{12}\Bigg] \tag{21}$$

and

$$\hat{\beta}_{Linex}=-\frac{1}{k}\ln\Big[e^{-k\hat{\beta}}+\frac{1}{2}\Bigg\{(k^2e^{-k\hat{\beta}})\tau_{22}+\left(\frac{2r}{\hat{\alpha}^3}-\hat{\beta}(\sum_{i=1}^{r}x_i^{\hat{\alpha}}(\ln x_i)^3+(n-r)x_r^{\hat{\alpha}}(\ln x_r)^3)\right)(-ke^{-k\hat{\beta}})\tau_{11}\tau_{12}$$
$$+\frac{2r}{\hat{\beta}^3}(-ke^{-k\hat{\beta}})\tau_{22}^2-(\tau_{11}\tau_{22}+2\tau_{12}^2)(-ke^{-k\hat{\beta}})(\sum_{i=1}^{r}x_i^{\hat{\alpha}}(\ln x_i)^2+(n-r)x_r^{\hat{\alpha}}(\ln x_r)^2)\Bigg\}$$
$$+\left(\frac{a-1}{\hat{\alpha}}-b\right)(-ke^{-k\hat{\beta}})\tau_{21}+\left(\frac{c-1}{\hat{\beta}}-d\right)(-ke^{-k\hat{\beta}})\tau_{22}\Bigg] \tag{22}$$

Where

$$\tau_{11}=\frac{V}{UV-W^2},\qquad \tau_{22}=\frac{\mathrm{U}}{UV-W^2}$$

$$\tau_{12}=\tau_{21}=-\frac{W}{UV-W^2}$$

and

$$U=\frac{r}{\hat{\alpha}^2}+\hat{\beta}\left[\sum_{i=1}^{r}x_i^{\hat{\alpha}}(\ln x_i)^2+(n-r)x_r^{\hat{\alpha}}(\ln x_r)^2\right]$$

$$W=\sum_{i=1}^{n}x_i^{\hat{\alpha}}\ln x_i+(n-r)x_r^{\hat{\alpha}}\ln x_r\ ;\ \mathrm{V}=\frac{r}{\hat{\beta}^2}$$

4 Simulation Results

In order to compare the proposed Bayes estimators with the MLEs, we perform a Monte Carlo Simulation study using different sample sizes (*n*), different effective

sample sizes (r) and for different parameter values(β). In computing the estimates we generated 1000 samples from the Weibull distribution and we replicated the process 1000 times. We estimate the MLE from (9) and (10) and Bayes estimators obtained by Lindley's approximation from (21) and (22). We also compute the local estimate of the variance- covariance matrix by inverting the observed Fisher information matrix (11), and use that to construct two-sided approximate 95% confidence intervals for the parameters from (15) and (16).

For computing the Bayes estimators under the Linex loss function, it is assumed that α and β have $Gamma(a_1,b_1)$, and $Gamma(a_2,b_2)$ priors respectively. Moreover we use the non-informative priors of both α and β, i.e. $a_1 = b_1 = a_2 = b_2 = 0$. All the results are reported in Tables I–IV.

From the results of the simulation study, we observe the following.

1- For fixed effective sample size (r), as sample size (n) increases the biases and the average relative MSE of the estimates decrease. This is indicative of the fact that the estimators are consistent and approaches the true parameter values as the sample size increases.

2- For fixed effective sample size (r), as sample size (n) increases the length of the confidence intervals also decrease.

3- For fixed (β) and sample size (n) as r increases, the average relative MSE of the estimates and the length of the respective confidence intervals increase.

Furthermore, the performance of the Bayes estimators obtained by Lindley's approximation are quite similar to the MLE in terms of biases and MSE.

Table I. Average estimates and the corresponding MSEs for n=20

$\beta\downarrow$	r		MLE	Bayes (Lindley)
0.5	4	$\hat{\alpha}$	0.9976(0.1456)	0.9916(0.1238)
		$\hat{\beta}$	1.1696(0.2759)	1.1665(0.2722)
	2	$\hat{\alpha}$	0.9885(0.1224)	0.9823(0.1112)
		$\hat{\beta}$	1.1668(0.2645)	1.1643(0.2598)
1.0	4	$\hat{\alpha}$	1.0651(0.2431)	1.0611(0.2413)
		$\hat{\beta}$	1.0901(0.2573)	1.0875(0.3265)
	2	$\hat{\alpha}$	1.0144(0.1905)	1.0129(0.1886)
		$\hat{\beta}$	1.0443(0.2361)	1.0393(0.2309)
1.5	4	$\hat{\alpha}$	1.1482(0.4231)	1.1375(0.4199)
		$\hat{\beta}$	1.0576(0.2142)	1.0503(0.2148)
	2	$\hat{\alpha}$	1.0655(0.2876)	1.0618(0.2869)
		$\hat{\beta}$	1.0021(0.1299)	0.9987(0.1310)

Table II. Average estimates and the corresponding MSEs for n=40

$\beta\downarrow$	r		MLE	Bayes (Lindley)
0.5	8	$\hat{\alpha}$	0.9153(0.0451)	0.9086(0.0426)
		$\hat{\beta}$	0.9487(0.0754)	0.9465(0.0735)
	4	$\hat{\alpha}$	0.9039(0.0379)	0.8973(0.0333)
		$\hat{\beta}$	0.9324(0.0725)	0.9303(0.0705)
1.0	8	$\hat{\alpha}$	0.9182(0.0504)	0.9172(0.0478)
		$\hat{\beta}$	0.9254(0.0373)	0.9238(0.0322)
	4	$\hat{\alpha}$	0.9177(0.0447)	0.9159(0.0365)
		$\hat{\beta}$	0.9211(0.0355)	0.9193(0.0312)
1.5	8	$\hat{\alpha}$	0.9331((0.0705)	0.9315(0.0688)
		$\hat{\beta}$	0.9164(0.0497)	0.9149(0.0464)
	4	$\hat{\alpha}$	0.9275(0.0652)	0.9286(0.0601)
		$\hat{\beta}$	0.9115(0.0375)	0.9087(0.0352)

Table III. Approximate Confidence Intervals Based on MLE, for n=20

$\beta\downarrow$	r	$[\hat{\alpha}_l, \hat{\alpha}_u]$	$[\hat{\beta}_l, \hat{\beta}_u]$
0.5	4	[0.1049, 0.5159]	[0.0012, 2.7568]
	2	[0.1154, 0.5032]	[0.0023, 3.0562]
1.0	4	[0.1598, 1.1533]	[0.2609, 2.4465]
	2	[0.1906, 1.0980]	[0.2764, 2.3703]
1.5	4	[0.1274, 2.7363]	[0.4034, 2.1125]
	2	[0.2235, 2.5405]	[0.4301, 2.1129]

Table IV. Approximate Confidence Intervals Based on MLE, for n=40

$\beta\downarrow$	r	$[\hat{\alpha}_l, \hat{\alpha}_u]$	$[\hat{\beta}_l, \hat{\beta}_u]$
0.5	8	[0.1262, 0.4541]	[0.1349, 2.6578]
	4	[0.1373, 0.4372]	[0.1623, 2.4730]
1.0	8	[0.2237, 1.0223]	[0.3721, 2.2171]
	4	[0.2496, 0.9312]	[0.3599, 2.0710]
1.5	8	[0.3189, 2.0694]	[0.5416, 1.9589]
	4	[0.4079, 2.0694]	[0.4887, 1.8354]

5 Conclusions

In this paper we consider the classical, and Bayes estimator under the Linex loss of the Weibull distribution based on Type-II censored samples. We obtain the maximum likelihood estimators and it is observed that the MLEs of the unknown parameters can be obtained by using one non-linear equation. Bayesian estimates of the unknown parameters are also proposed and it is observed that the Bayes estimates cannot be obtained in explicit form. We used Lindley's approximations to compute the approximate Bayes estimates. We compare the performance of the different methods by Monte Carlo simulations, and it is observed that the performances are quite similar.

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Received: February, 2011