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Influence of Chemical Reaction and Radiation on Unsteady MHD Free Convective Flow and Mass Transfer through Viscous Incompressible Fluid Past a Heated Vertical Plate Immersed in Porous Medium in the Presence of Heat Source

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Abstract

Aim of the paper is to investigate the influence of chemical reaction and radiation on an unsteady megnetohydrodynamic free convective flow and mass transfer through viscous incompressible fluid past a heated vertical porous plate immersed in porous medium in the presence of uniform transverse magnetic field, oscillating free stream and heat source when viscous dissipation effect is also taken into account. The velocity, temperature and concentration distributions are derived, discussed numerically and their profiles for various values of physical parameters are shown through graphs. The coefficient of skin-friction, Nusselt number and Sherwood number at the plate are derived, discussed numerically and their numerical values for various values of physical parameters are presented through Tables.

Keywords: Unsteady, MHD, radiation, chemical reaction, mass transfer, porous medium, free stream and heat source.

1. Introduction

Coupled heat and mass transfer (i.e. double diffusion) driven by buoyancy due to temperature and concentration variations in a saturated porous medium, has several important applications in geothermal and geophysical engineering such as the migration of moisture through the air contained in the fibrous insulation, the extraction of geothermal energy, underground disposal of numerical wastes and the spreading of chemical contaminants through water saturated soil. Not only this, the study of heat and mass transfer to chemical reacting MHD free convection flow with radiation effects on a vertical plate has received a growing interest during the last decades. This is due to its importance in several engineering, industrial, geophysical and astrophysical applications, such as polymer production, manufacturing of ceramic, packed-bed catalytic reactors, food processing, cooling of nuclear reactors, enhanced oil recovery, underground energy transport, magnetized plasma flow, high speed plasma wind, cosmic jets and steller system. For some industrial applications such as glass production, furnace design, cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effect can also be significant. A clear understanding of the nature of interaction between thermal and concentration buoyancies is necessary to control these processes.

Natural convection boundary layer flow along a heated vertical plate in a stratified environment was studied by Henkes et al.(1989). Takhar et al. (1996) investigated the radiation effects on MHD free convection flow of a radiating gas past a semi-infinite vertical plate. The unsteady hydromagnetic free convection flow with radiative transfer in a rotating fluid was discussed by Bestman and Adjepong (1998). Muthucumaraswamy and Ganesan (2002) studied the diffusion and first-order chemical reaction on impulsively started infinite vertical plate with variable temperature. Chamkha (2004) studied the unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation was presented by Chien (2004). Badruddin et al. (2005) analyzed the free convection and radiation characteristics for a vertical plate embedded in a porous medium. Abd-El-Aziz(2006) investigated the thermal radiation effects on a magnetohydrodynamic mixed convection flow of a micropolar fluid past a continuously moving semi-infinite plate for high temperature differences. The effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical moving porous plate bounded by porous medium was presented by Sharma and Sharma (2007). The influence of chemical reaction on transient MHD free convection over a moving vertical plate was discussed by Al-Odat and Al-Azab(2007). Mbeledoguet et al.(2007) studied the unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer. The heat and mass transfer of unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer was analyzed by Mbeledogu and Ogulu (2007). Orhan and Ahmet(2008) presented radiation effects on MHD mixed convection flow about a permeable vertical plate.

Aim of the paper is to investigate influence of chemical reaction and radiation on an unsteady MHD free convective flow and mass transfer through a viscous incompressible, electrically conducting fluid past an infinite vertical heated porous plate with suction, embedded in porous medium in the presence of a uniform transverse magnetic field, oscillating free stream and heat source by taking into account the viscous dissipation.

2. Formulation of the Problem

Consider unsteady two dimensional hydromegnetic laminar free convective flow of a viscous incompressible, electrically conducting and radiating fluid in an optically thin environment past an infinite heated vertical porous plate embedded in a porous medium, in the presence of thermal and concentration buoyancy effects and heat source. The x-axis is taken along the plate in upward direction and y-axis is normal to the plate. It is assumed that there exist a homogeneous chemical reaction of first order with constant rate k_1 between the diffusing species and the fluid. A uniform magnetic field is applied in the direction perpendicular to the plate. Flow is fully developed under the following assumptions:

(i)The fluid is supposed to be slightly conducting and hence the magnetic Reynolds number is less and the induced magnetic field is negligible in comparison with the transverse magnetic field,

(ii) There is no applied voltage, so that the electric field is absent,

(iii)The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible, and

(iv)The temperature in the fluid flowing is governed by the energy concentration equation involving radiative heat temperature with heat source.

Since the plate is infinite in length, therefore all the physical quantities are functions of y and t only. Within the frame work of these assumptions, the governing equations in dimensionless form for unsteady free convective flow of an electrically conducting fluid with radiation effects and in the presence of heat source under usual Boussinesq approximation and radiative flux [Bestman and Adjepong (1998)] are

$$v = -\left(1 + \in A e^{nt}\right), \qquad ...(1)$$

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \in A e^{nt}\right)\frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + Gr\,\theta + Gc\,C - \left(M + \frac{1}{K}\right)(u - U), \qquad \ldots(2)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \left(1 + \in A e^{nt}\right)\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - N\,\theta + Ec\left(\frac{\partial u}{\partial y}\right)^2 + Q\theta, \qquad \ldots(3)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1 + \in A e^{nt}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - k_1\,C\,, \qquad \ldots(4)$$

where θ is the dimensionless temperature, t the dimensionless time, u the dimensionless velocity along x-axis, C the dimensionless concentration of the fluid, n the frequency, Gr is the Grashof number for heat transfer, Gc the Grashof number for mass transfer, M the Hartmann number, K the permeability parameter, Pr the Prandtl number, N the radiation parameter, Ec the Eckert number, Q the heat source/sink parameter, Sc the Schimdt number, k_l is the chemical reaction parameter, $0<\in$, A<1 and $0<\in A<<1$.
The boundary conditions in dimensionless form are given by

$$y=0 \;:\quad u=0\,,\quad \theta=1\,,\quad C=1\,;$$
$$y\to\infty:\quad u\to U(t)\left(1+\in e^{nt}\right),\;\; \theta\to 0\,,\; C\to 0\,. \qquad \ldots(5)$$

3. Method of Solution

Equations (2) to (4) are second order coupled partial differential equations and solved under the boundary conditions (5). In view of the boundary conditions, the velocity, temperature and concentration distributions of the fluid in the neighborhood of the plate can be expressed as given below

$$u(y,t)=u_0(y)+\in e^{nt}\,u_1(y)+O\left(\in^2\right)+\ldots.,$$
$$\theta(y,t)=\theta_0(y)+\in e^{nt}\,\theta_1(y)+O\left(\in^2\right)+\ldots.,$$
$$C(y,t)=C_0(y)+\in e^{nt}\,C_1(y)+O\left(\in^2\right)+\ldots., \qquad \ldots(6)$$

and the free stream velocity becomes

$$U(t)=1+\in e^{nt}\,. \qquad \ldots.(7)$$

Substituting (6) and (7) into the equations (2) to (4), equating the harmonic and non-harmonic terms, and neglecting the higher order terms of O $\left(\in^2\right)$, we obtain

$$u_0''+u_0'-\left(M+\frac{1}{K}\right)u_0=-\left(M+\frac{1}{K}\right)-Gr\,\theta_0-Gc\,C_0\,, \qquad \ldots(8)$$
$$u_1''+u_1'-\left(M+\frac{1}{K}+\frac{n}{4}\right)u_1=-\left(M+\frac{1}{K}+\frac{n}{4}\right)-A\,u_0'-Gr\,\theta_1-Gc\,C_1\,, \qquad \ldots(9)$$
$$\theta_0''+Pr\,\theta_0'+Pr\,(Q-N)\theta_0=-Pr\,Ec\,u_0^2\,, \qquad \ldots(10)$$
$$\theta_1''+Pr\,\theta_1'+Pr\left(Q-N+\frac{n}{4}\right)\theta_1=-Pr\,A\,\theta_0'-2\,Pr\,Ec\,u_0'\,u_1'\,, \qquad \ldots(11)$$
$$C_0''+Sc\,C_0'-k_1\,Sc\;C_0=0\,, \qquad \ldots(12)$$
$$C_1''+Sc\,C_1'-Sc\left(\frac{n}{4}+k_1\right)C_1=-A\,Sc\,C_0'\,, \qquad \ldots(13)$$

where prime denotes the differentiation with respect to y.
Now, the corresponding boundary conditions are reduced to

$$y=0:\quad u_0=0\,,\quad u_1=0\,,\quad \theta_0=1\,,\quad \theta_1=0\,,\quad C_0=1\,,\quad C_1=0\,;$$

$y \to \infty: \quad u_0 \to 1, \quad u_1 \to 1, \quad \theta_0 \to 0, \; \theta_1 \to 0, \; C_0 \to 0, \; C_1 \to 0.$ …(14)

The equations (8) to (13) are still coupled ordinary second order differential equations. Since the Eckert number Ec is very small for incompressible fluid flows, therefore u_0, u_1, θ_0, θ_1, and C_0, C_1 can be expanded in the powers of Ec as given below

$$F(y) = F_0(y) + Ec\,F_1(y) + O(Ec^2) + \ldots, \qquad \ldots(15)$$

where F stands for any u_0, u_1, θ_0, θ_1, and C_0 and C_1.

Substituting (15) into the equations (8) to (14) and equating the coefficients of like powers of Ec, we get a set of ordinary second order differential equations which are solved under the corresponding boundary conditions. Hence the solutions of $u_{00}(y)$, $u_{01}(y)$, $u_{10}(y)$, $u_{11}(y)$, $\theta_{00}(y)$, $\theta_{01}(y)$, $\theta_{10}(y)$, $\theta_{11}(y)$, $C_{00}(y)$, $C_{01}(y)$, $C_{10}(y)$, $C_{11}(y)$ are known, whose expressions are not included here for the sake of brevity.

4. Skin-friction Coefficient

The coefficient of skin-friction at the plate is given by

$$C_f = \left[\frac{\tau_w}{\rho U_0 V_0}\right]_{y^*=0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y} + \in e^{nt}\frac{\partial u_1}{\partial y}\right)_{y=0}. \qquad \ldots.(16)$$

Table- 1. Numerical values of skin-friction coefficient at the plate for various values of physical parameters when n=0.1, t =1 and A=1.

$\in$	Pr	N	Ec	Q	k_l	Gr	Gc	M	Sc	K	C_f
0.00	0.71	1	0.01	1	2	5	3	1	0.78	0.5	6.521799
0.01	0.71	1	0.01	1	2	5	3	1	0.78	0.5	3.255598
0.01	7.0	1	0.01	1	2	5	3	1	0.78	0.5	-0.03872
0.01	0.71	3	0.01	1	2	5	3	1	0.78	0.5	-3.15347
0.01	0.71	1	0.02	1	2	5	3	1	0.78	0.5	-17.7135
0.01	0.71	1	0.01	0.5	2	5	3	1	0.78	0.5	-25.8557
0.01	0.71	1	0.01	1	4	5	3	1	0.78	0.5	-26.4023
0.01	0.71	1	0.01	1	2	10	3	1	0.78	0.5	-16.7082
0.01	0.71	1	0.01	1	2	5	5	1	0.78	0.5	-29.3706
0.01	0.71	1	0.01	1	2	5	3	3	0.78	0.5	-36.3702
0.01	0.71	1	0.01	1	2	5	3	1	2.62	0.5	-25.9912
0.01	0.71	1	0.01	1	2	5	3	1	0.78	1.0	-29.8419

5. Nusselt Number

The rate of heat transfer in terms of Nusselt Number at the plate is given by

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\left(\frac{\partial\theta_0}{\partial y} + \in e^{nt}\frac{\partial\theta_1}{\partial y}\right)_{y=0}. \quad ...(17)$$

Table- 2. Numerical values of Nusselt number at the plate for various values of physical parameters when n=0.1, t=1 and A=1.

∈	Pr	N	Ec	Q	k_l	Gr	Gc	M	Sc	K	Nu
0.00	0.71	1	0.01	1	2	5	3	1	0.78	0.5	0.775142
0.01	0.71	1	0.01	1	2	5	3	1	0.78	0.5	1.027536
0.01	7.0	1	0.01	1	2	5	3	1	0.78	0.5	0.123739
0.01	0.71	3	0.01	1	2	5	3	1	0.78	0.5	2.683570
0.01	0.71	1	0.02	1	2	5	3	1	0.78	0.5	1.336927
0.01	0.71	1	0.01	0.5	2	5	3	1	0.78	0.5	1.541774
0.01	0.71	1	0.01	1	4	5	3	1	0.78	0.5	2.438049
0.01	0.71	1	0.01	1	2	10	3	1	0.78	0.5	1.532309
0.01	0.71	1	0.01	1	2	5	5	1	0.78	0.5	1.122554
0.01	0.71	1	0.01	1	2	5	3	3	0.78	0.5	0.849508
0.01	0.71	1	0.01	1	2	5	3	1	2.62	0.5	10.791809
0.01	0.71	1	0.01	1	2	5	3	1	0.78	1.0	1.619592

6. Sherwood Number

The rate of mass transfer in terms of Sherwood Number at the plate is given by

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\left(\frac{\partial C_0}{\partial y} + \in e^{nt}\frac{\partial C_1}{\partial y}\right)_{y=0}. \quad ...(18)$$

Table- 3. Numerical values of Sherwood number at the plate for various values of physical parameters when n=0.1, t=1 and A=1, Ec=0.01.

∈	Sc	k_l	Sh
0.00	0.78	2	1.698472
0.01	0.78	2	1.704051
0.01	2.62	2	3.969060
0.01	0.78	4	2.204126

7. Results and Discussion

Fig.1 shows that the amplitude of fluid velocity of mean flow is less in comparison to unsteady flow. Further, it is observed that the fluid velocity of unsteady flow increases with the increase of Eckert number, Prandtl number or heat source parameter; while it decreases due to increase in radiation parameter or chemical reaction coefficient. Fig.2 depicts that the fluid velocity increases with the increase of Grashof number for mass transfer, permeability parameter or Grashof number for heat transfer; while it decreases with the increase of Schmidt number or Hartmann number. It is noted from Fig.3 that the amplitude of fluid temperature of steady flow is more in comparison to unsteady flow. Further, it is observed that the fluid temperature for unsteady flow increases with the increase of heat source parameter; while it decreases due to increase in Eckert number, chemical reaction parameter, radiation parameter or Prandtl number. Fig.4 reflects that the fluid temperature increases with the increase of Hartmann number or Schmidt number; while it decreases with the increase of Grashof number for mass transfer, permeability parameter or Grashof number for heat transfer. It is observed from figure 5 that the fluid concentration for unsteady flow decreases with the increase of chemical reaction parameter or Schmidt number. It is inferred from Table-1 that the skin-friction coefficient at the plate is greater for the steady flow in comparison of unsteady flow. Further, for unsteady flow it increases due to increase in the heat source parameter; while it decreases due to increase in the Prandtl number, radiation parameter, Eckert number, chemical reaction coefficient, Grashof number for heat transfer, Grashof number for mass transfer, Hartmann number, Schmidt number or permeability parameter. It is noted from Table-2 that the Nusselt number at the plate increases with the increase of radiation parameter, Eckert number, chemical reaction parameter, Grashof number for heat transfer, Grashof number for mass transfer or permeability parameter; while it decreases with the increase of Prandtl number, heat source parameter, Hartmann number or Schmidt number. Table-3 reflects that the Sherwood number at the plate increases with the increase of chemical reaction coefficient or Schmidt number.

8. Conclusions

Influence of chemical reaction and radiation on unsteady MHD free convection flow and mass transfer of a viscous incompressible electrically conducting fluid past a heated vertical porous plate in a porous medium with viscous dissipation effect in the presence of heat source, oscillating free stream and transverse magnetic field are carried out and the following conclusions are made

(i) An increase in the Grashof number, leads to a rise in the magnitude of fluid velocity due to enhancement in buoyancy force. The peak value of the velocity increases rapidly near the porous plate as buoyancy force for heat transfer increases and then decays the free stream velocity.
(ii) The magnitude of fluid velocity attains a distinctive maximum value with the increase of buoyancy force for mass transfer in the vicinity of the plate and then decreases properly to approach a free stream velocity.
(iii) The magnitude of fluid velocity decreases with the increase of molecular diffusivity or intensity of the magnetic field, while it increases with the increase of heat source.
(iv) An increase in the chemical reaction parameter tends to decrease the velocity as well as the species concentration. The hydrodynamic and the concentration boundary layer become thin as the reaction parameter increases.
(v)The magnitude of fluid concentration decreases with the increase of molecular diffusivity or chemical reaction parameter.
(vi) An increase in the radiation heat transfer leads to decrease in the magnitude of fluid velocity and the magnitude of fluid temperature within the boundary layer as well as decreases the thickness of the velocity and thermal boundary layers.
(vii) An increase in Prandtl number leads to decrease in the thermal boundary layer and in general lower average temperature within the boundary layer region being the smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence for smaller Pr, the rate of heat transfer is reduced.
(viii)The magnitude of fluid temperature decreases with the increase of chemical reaction parameter, viscous dissipation effect and molecular diffusivity; while it increases with an increase of intensity of magnetic field and heat source.

References

[1] A.C. Cogley., W.G Vincenti and S.E. Gilles, Differential approximation for radiative transfer in a non-gray gas near equilibrium, AIAA J., 6(1968), 551-553.
[2] A.J. Chamkha, Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption, Int. J. of Engg. Science, 42(2004), 217-230.
[3] A. Orhan and K. Ahmet, Radiation effect on MHD mixed convection flow about a permeable vertical plate." J. Heat and Mass Transfer, 45(2008), 239-246.
[4] A. R. Bestman and S.K. Adjepong, Unsteady hydromagnetic free convection flow with radiative transfer in a rotating fluid, Astrophysics Space Sci., 143(1998), 73-80.

[5] C.H. Chien, Combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation, Int. J. Eng. Science, 42(2004), 699-713.
[6] H.S. Takhar, R.S.R. Gorla and V M. Soundalgekar, Radiation effects on MHD free convection flow of a radiating gas past a semi-infinite vertical plate, Int. J. Numerical Methods Heat Fluid Flow, 6(1996), 77-83.
[7] I.U. Mbeledogu and A. Ogulu, Heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer, Int. J. Heat Mass Transfer, 50(2007), 1902-1908.
[8] I.U. Mbeledogu, A.R.C. Amokiri and A. Ogulu, Unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer, Int. J. Heat Mass Transfer, 50(2007), 1668-1674.
[9] I.Z. Badruddin, Z.A. Zainal, P.A.A. Narayana, K.N. Seetharameu and W.S. Lam, Free convection and radiation characteristics for a vertical plate embedded in a porous medium, Int. J. Numerical Methods Engg., 65(2005), 2265-2278.
[10] M. Abd-El-Aziz, Thermal radiation effects on a magnetohydrodynamic mixed convection flow of a micropolar fluid past a continuously moving semi-infinite plate for high temperature differences, Acta Mechanica, 187(2006), 113-127.
[11] M.Q. Al-Odat and T.A. Al-Azab, Influence of chemical reaction on transient MHD free convection over a moving vertical plate, Emirates J. for Engg. Research,12(3)(2007), 15-21.
[12] P.R. Sharma and K. Sharma, Effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical moving porous plate bounded by porous medium, Mod. Meas. & Control 'B', AMSE J., France, 76(5)(2007), 34-60.
[13] R.A. Henkes, W.M. Hoogendoorn and C.J. Laminar, Natural convection boundary layer flow along a heated vertical plate in a stratified environment, Int. J. Heat Mass Trransfer, 32(1)(1989), 147-155.
[14] R. Grief, I.S. Habib and J.C. Lin, Laminar convection of a radiating gas in a vertical channel, J. Fluid Mech., 46(1971), 513-520.
[15] R. Muthucumaraswamy and P. Ganesan, Diffusion and first-order chemical reaction on impulsively started infinite vertical plate with variable temperature, Int. J. Therm. Sci., 41(5)(2002), 475-479.

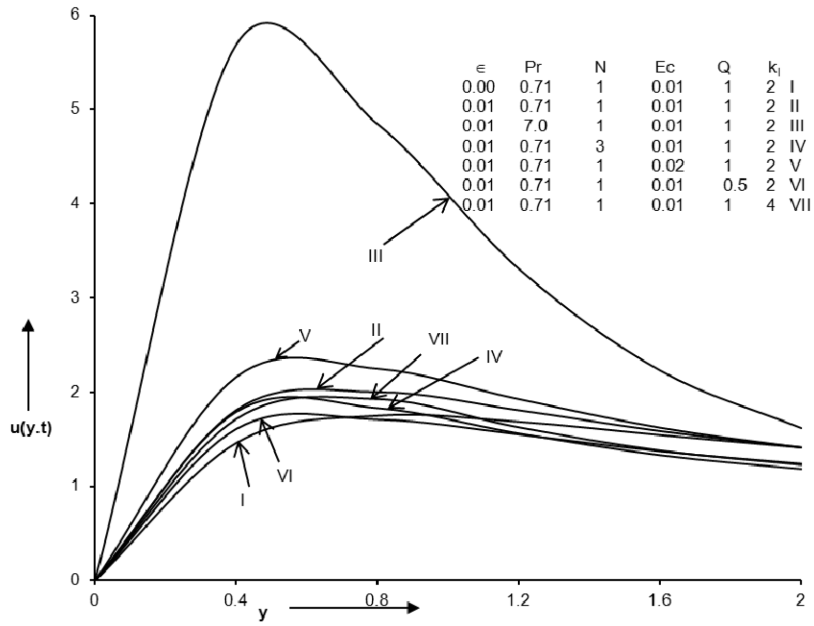

Fig.1 Velocity distribution versus y when A = 1, n = 0.1, t = 1, Gr = 5, Gc = 5, M =1, Sc = 0.78, K = 0.5.

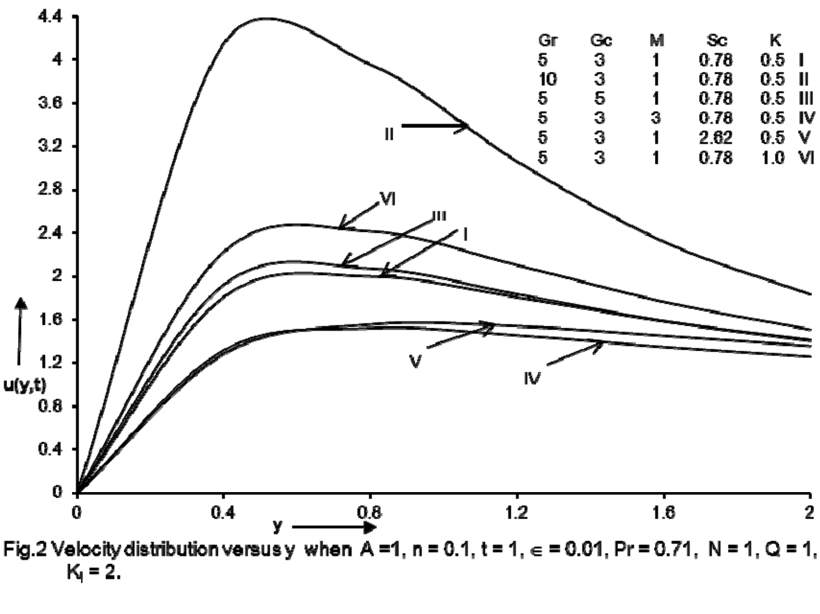

Fig.2 Velocity distribution versus y when A =1, n = 0.1, t = 1, ∈ = 0.01, Pr = 0.71, N = 1, Q = 1, K_l = 2.

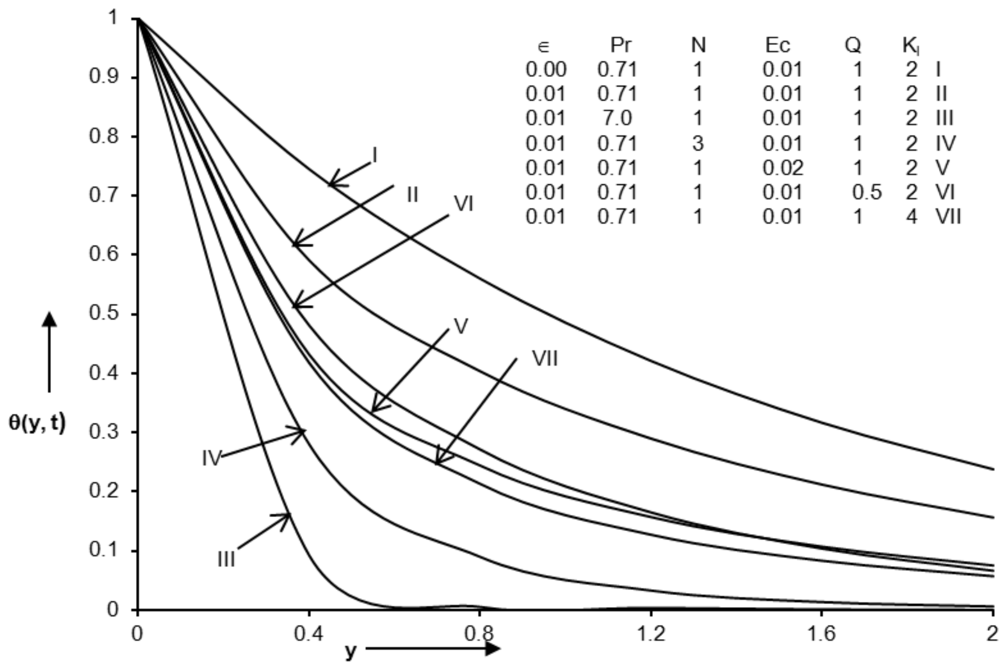

Fig.3 Temperature distribution versus y when A = 1, n = 0.1, t = 1, Gr = 5, Gc = 5, M =1, Sc = 0.78, K = 0.5.

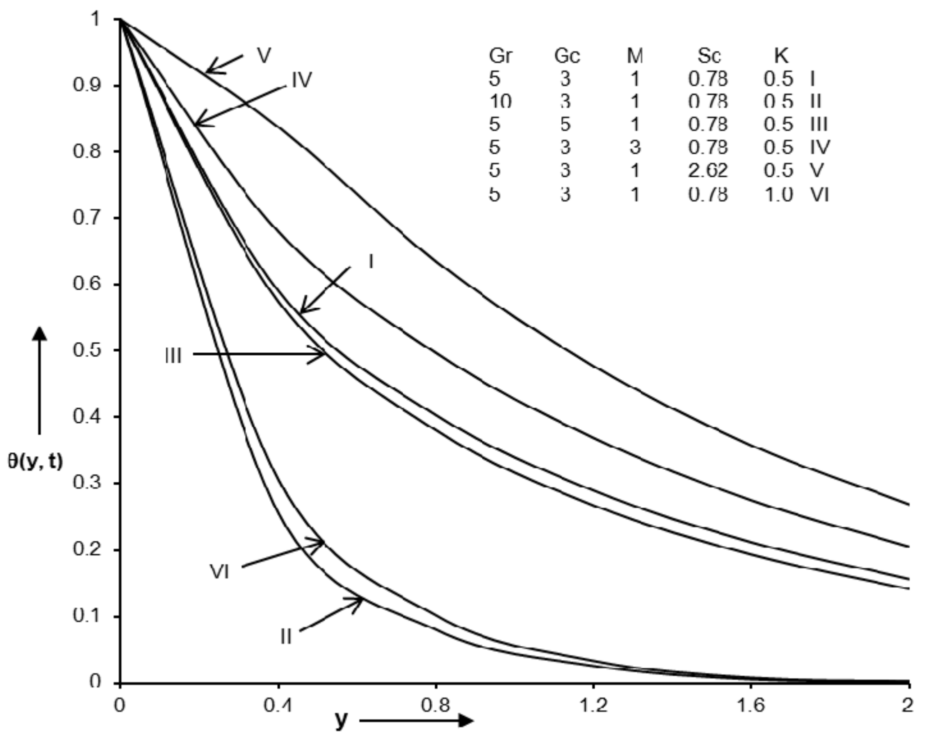

Fig.4 Temperature distribution versus y when A = 1, n = 0.1, t = 1, ∈ = 0.01, Pr = 0.71, N = 1, Q = 1, K_l = 2.

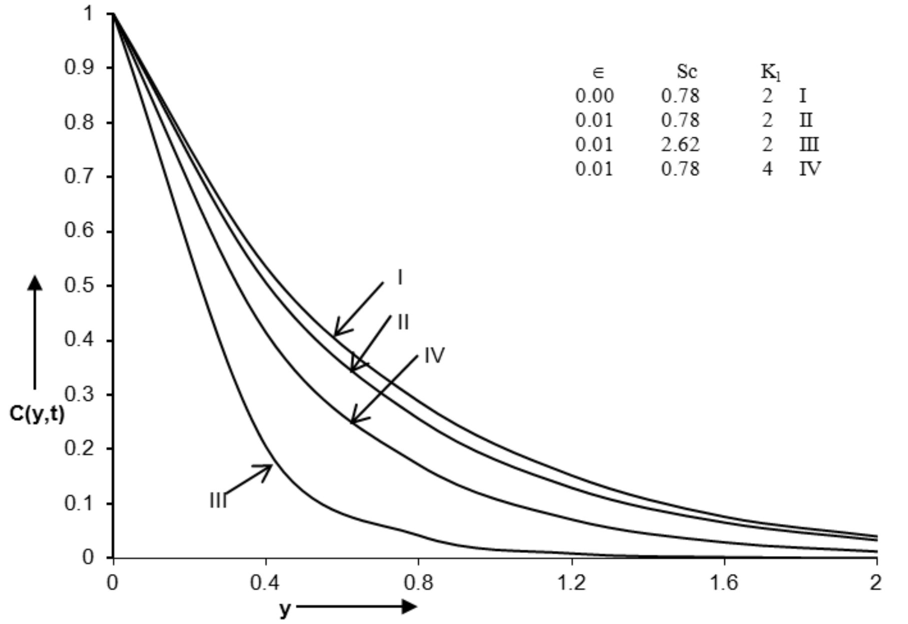

Fig.5 Concentration distribution versus y when A =1, n = 0.1, t = 1, Ec=0.01.

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