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Robust $\beta-$Stability and $\beta-$Stabilization of Impulsive Switched System with Time-varying Delays

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Abstract

In this paper, we aim to study the problems of robust $\beta-$stability and $\beta-$stabilization for uncertain impulsive switched control systems with time-varying delays. By using the descriptor model transformation, Lyapunov-Krasovskii function method and linear matrix inequality (LMI) technique, sufficient conditions for robust $\beta-$stability and $\beta-$stabilization are obtained. Numerical example is presented to illustrate the effectiveness of the theoretical results. The new stability conditions are less conservative and more general than some existing results.

Keywords: $\beta-$stability, Impulsive switched system, Linear matrix inequality, Lyapunov method, Descriptor model

1 Introduction

In recent years, the problem of robust stability and stabilization of uncertain dynamic systems (with or without control, with or without impulsive effects and switchings) has been considered by many researchers [1]-[19]. Uncertain impulsive switched systems often encountered in physical systems, biological systems and engineering systems. There are various stability conditions for uncertain impulsive switched systems, delay-independent or delay-dependent. For example, the asymptotic stability conditions of uncertain impulsive switched systems are presented by using an LMI approach in [15]. In [7], the authors investigated unified approach for the stability analysis of impulsive hybrid systems. Robust H_∞ stability and stabilization with definite attendance for impulsive switched systems with time-varying uncertainty are presented by using the LMI approach in [16].

In this paper, we shall consider the problem of robust exponential stability and stabilization with a given convergence rate β of uncertain impulsive switched control system with time-varying delays. We use appropriate Lyapunov functions and derive stability conditions in terms of linear matrix inequalities (LMIs). Numerical examples will be presented to illustrate the effectiveness of the theoretical results and we shall compare our results with other existing results.

We introduce some notations and definitions that will be used throughout the paper. R^+ denotes the set of all real non-negative numbers; Z^+ denotes the set of all non-negative integer numbers; R^n denotes the n-dimensional space with the vector norm $\|\cdot\|$; $\|x\|$ denotes the Euclidean vector norm of $x \in R^n$; $R^{n\times r}$ denotes the set $n\times r$ real matrices; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; matrix A is called semi-positive definite $(A \geq 0)$ if $x^T Ax \geq 0$, for all $x \in R^n$; A is positive definite $(A > 0)$ if $x^T Ax > 0$ for all $x \neq 0$; matrix B is called semi-negative definite $(B \leq 0)$ if $x^T Bx \leq 0$, for all $x \in R^n$; B is negative definite $(B < 0)$ if $x^T Bx < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$; $A \geq B$ means $A - B \geq 0$; $C([-h, 0], R^n)$ denotes the space of all continuous vector functions mapping $[-h, 0]$ into R^n; $*$ represents the elements below the main diagonal of a symmetric matrix.

2 Preliminary

Consider the uncertain impulsive switched control system with time-varying delays of the form

$$\begin{cases} \dot{x}(t) = \Big[A_{i_k} + \Delta A_{i_k}(t)\Big]x(t) + \Big[B_{i_k} + \Delta B_{i_k}(t)\Big]x(t - h_{i_k}(t)) & \\ \qquad\quad +\Big[C_{i_k} + \Delta C_{i_k}(t)\Big]u(t), & t \neq t_k; \\ \Delta x(t) = I_k(x(t)) = D_k x(t), & t = t_k; \\ x(t_0 + t) = \phi(t), \quad \dot{x}(t_0 + t) = \psi(t) & \forall t \in [-h, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state and $u(t) \in R^n$ is the control. The initial condition functions $\phi(t), \psi(t) \in C([-h, 0], R^n)$ denote continuous vector-valued initial functions of $t \in [-h, 0]$ with the norm $\|\phi\| = \sup_{s\in[-h,0]} \|\phi(s)\|$, $\|\psi\| = \sup_{s\in[-h,0]} \|\varphi(s)\|$. A_{i_k}, B_{i_k}, C_{i_k} and D_k are given real matrices of appropriate dimensions. $\Delta x(t) = x(t^+) - x(t^-)$, $x(t^+) = \lim\limits_{\nu\to 0^+} x(t+\nu)$, $x(t^-) = \lim\limits_{\nu\to 0^+} x(t-\nu)$. We assume the solution of the impulsive switched system (1) is left continuous, i.e., $\lim\limits_{\nu\to 0^+} x(t_k - \nu) = x(t_k^-) = x(t_k)$. $i_k \in \{1, 2, ..., m\}$, $k, m \in Z^+$, i_k is an impulsive switching time and $t_0 < t_1 < t_2 < \cdots < t_k < \cdots, t_k \to +\infty$ as $k \to +\infty$. Under the switching law of system (1), at the time t_k, the system switches to the i_k subsystem from the i_{k-1} subsystem. The delay $h_{i_k}(t)$ is a time varying bounded continuous function satisfying

$$0 \leq h_{i_k}(t) \leq h, \quad \dot{h_{i_k}}(t) \leq \delta < +\infty,$$

where h, δ are given positive constants for all i_k and $t \in R^+$. The uncertainty $\Delta A_{i_k}(t)$, $\Delta B_{i_k}(t)$ and $\Delta C_{i_k}(t)$ are time varying matrices and satisfy the condition $\Delta A_{i_k}(t) = E_{i_k}\Delta_{i_k}(t)H_{i_k}$, $\Delta B_{i_k}(t) = E_{i_k}\Delta_{i_k}(t)R_{i_k}$, $\Delta C_{i_k}(t) = E_{i_k}\Delta_{i_k}(t)S_{i_k}$, $E_{i_k}, R_{i_k}, S_{i_k} \in R^{n\times n}$. The class of parametric uncertainties $\Delta_{i_k}(t)$ which satisfies

$$\Delta_{i_k}(t) = F_{i_k}(t)[I - JF_{i_k}(t)]^{-1}, \tag{2}$$

is said to be admissible where J is a known matrix satisfying

$$I - JJ^T > 0, \tag{3}$$

and $F_{i_k}(t)$ is uncertain matrix satisfying

$$F_{i_k}(t)^T F_{i_k}(t) \leq I, \quad t \in R^+. \tag{4}$$

Remark 1. The conditions (3) and (4) guarantee that $I - JF_{i_k}(t)$ is invertible. It is easy to show that when $J = 0$, the parametric uncertainty of linear fractional form reduces to a norm bounded one.

Definition 2.1 *When $u(t) = 0$, the system (1) is said to be robustly $\beta-$stable, if there exists a function $\xi(.) : R^+ \times R^+ \to R^+$ such that for each $\phi(t), \psi(t) \in C([-h, 0], R^n)$, the solution $x(t, \phi, \psi)$ of the system satisfies*

$$\|x(t, \phi, \psi)\| \leq \xi(\|\phi\|, \|\psi\|)e^{-\beta t}, \quad \forall t \in R^+.$$

If there exists the state feedback controller $u(t) = Kx(t)$, where K is a constant gain matrix to be designed, the closed-loop system (1) is robustly $\beta-$stable, then we say system (1) is robustly $\beta-$stabilizable.

Lemma 2.2 *[6] Suppose that $\Delta(t)$ is given by (2)-(4) without i_k. Given constant matrices $\Sigma_1 = \Sigma_1^T, \Sigma_2, \Sigma_3$ of appropriate dimensions, the inequality*

$$\Sigma_1 + \Sigma_2\Delta(t)\Sigma_3 + \Sigma_3^T\Delta(t)^T\Sigma_2^T < 0,$$

holds for all $F(t)$ such that $F(t)^T F(t) \leq I$, $t \in R^+$, if and only if there exists $\epsilon > 0$ such that

$$\begin{bmatrix} \Sigma_1 & \Sigma_2 & \epsilon\Sigma_3^T \\ \Sigma_2^T & -\epsilon I & \epsilon J^T \\ \epsilon\Sigma_3 & \epsilon J & -\epsilon I \end{bmatrix} < 0.$$

3 Main Results

3.1 Robust stability

The nominal system (1) is defined to be

$$\begin{cases} \dot{x}(t) = A_{i_k}x(t) + B_{i_k}x(t-h_{i_k}(t)), & t \neq t_k; \\ \Delta x(t) = I_k(x(t)) = D_k x(t), & t = t_k; \\ x(t_0+t) = \phi(t), \quad \dot{x}(t_0+t) = \psi(t) & \forall t \in [-h, 0]. \end{cases} \tag{5}$$

Next, we take the change of the state variable

$$y(t) = e^{\beta t}x(t), \quad t \in R^+. \tag{6}$$

The system (5) is transformed to the delayed system

$$\dot{y}(t) = (A_{i_k} + \beta I)y(t) + e^{\beta h_{i_k}(t)}B_{i_k}y(t-h_{i_k}(t)). \tag{7}$$

Rewrite the system (7) in the following descriptor system

$$\dot{y}(t) = z(t) \tag{8}$$
$$z(t) = (A_{i_k} + \beta I)y(t) + e^{\beta h_{i_k}(t)}B_{i_k}y(t-h_{i_k}(t)). \tag{9}$$

Theorem 3.1 *For given positive constants δ, β and h, the nominal system (1) is $\beta-$ stable, if there exist symmetric positive definite matrices P_{i_k}, Q_{i_k} and W_{i_k} for all $i_k \in \{1, 2, ..., m\}$, $m, k \in Z^+$ and any appropriate dimensional matrices N, M, K_i, $i = 1, 2, 3$ such that the following LMIs hold.*

$$(i) \quad \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ * & \Delta_{22} & \Delta_{23} \\ * & * & \Delta_{33} \end{bmatrix} < 0,$$
$$(ii) \quad \begin{bmatrix} P_{i_{k-1}} & (I+D_k)^T P_{i_k} \\ P_{i_k}(I+D_k) & P_{i_k} \end{bmatrix} > 0,$$
$$(iii) \quad Q_{i_k} - Q_{i_{k-1}} < 0,$$
$$(iv) \quad W_{i_k} - W_{i_{k-1}} < 0,$$

where
$\Delta_{11} = (A_{i_k} + \beta I)^T(N + K_1) + (N + K_1)^T(A_{i_k} + \beta I) + Q_{i_k} - e^{\beta h}W_{i_k}$,
$\Delta_{22} = -M^T - M - K_2^T - K_2 + h^2 e^{2\beta h}W_{i_k}$,
$\Delta_{33} = -e^{-2\beta h}Q_{i_k} + \delta Q_{i_k} - W_{i_k} + K_3^T B_{i_k} + B_{i_k}^T K_3$,
$\Delta_{12} = (A_{i_k} + \beta I)^T(M + K_2) + P_{i_k} - N^T - K_1^T$,
$\Delta_{13} = N^T B_{i_k} + e^{\beta h}W_{i_k} + K_1^T B_{i_k} + (A_{i_k} + \beta I)^T K_3$,
$\Delta_{23} = M^T B_{i_k} + K_2^T B_{i_k} - K_3$.

Proof. For $t \in (t_k, t_{k+1}]$, we define the following Lyapunov function for the system (8), (9) of the form

$$V(t) = \sum_{i=1}^{3} V_i(t), \tag{10}$$

where

$$\begin{aligned} V_1(t) &= y^T(t)P_{i_k}y(t) = \begin{bmatrix} y(t) \\ z(t) \end{bmatrix}^T \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{i_k} & 0 \\ N & M \end{bmatrix} \begin{bmatrix} y(t) \\ z(t) \end{bmatrix}, \\ V_2(t) &= \int_{t-h_{i_k}(t)}^{t} y^T(s)Q_{i_k}y(s)ds, \\ V_3(t) &= he^{2\beta h}\int_{-h}^{0}\int_{t+s}^{t} \dot{y}^T(\alpha)W_{i_k}\dot{y}(\alpha)d\alpha ds. \end{aligned}$$

The derivative of $V(.)$ along the trajectories for system (8), (9) is given by

$$\begin{aligned} D^+V_1(t) &= 2\begin{bmatrix} y(t) \\ z(t) \end{bmatrix}^T \begin{bmatrix} P_{i_k} & N^T \\ 0 & M^T \end{bmatrix} \begin{bmatrix} \dot{y}(t) \\ 0 \end{bmatrix} \\ &= 2y^T(t)P_{i_k}z(t) \\ &\quad +2y^T(t)N^T\Big[-z(t) + A_{i_k}(\beta)y(t) + B_{i_k}(\beta)y(t-h_{i_k}(t))\Big] \\ &\quad +2z^T(t)M^T\Big[-z(t) + A_{i_k}(\beta)y(t) + B_{i_k}(\beta)y(t-h_{i_k}(t))\Big], \quad (11) \\ D^+V_2(t) &= y^T(t)Q_{i_k}y(t) - (1-\dot{h}_{i_k}(t))y^T(t-h_{i_k}(t))Q_{i_k}y(t-h_{i_k}(t)) \\ &\le y^T(t)Q_{i_k}y(t) - e^{-2\beta h}e^{2\beta h_{i_k}(t)}y^T(t-h_{i_k}(t))Q_{i_k}y(t-h_{i_k}(t)) \\ &\quad +\delta e^{2\beta h_{i_k}(t)}y^T(t-h_{i_k}(t))Q_{i_k}y(t-h_{i_k}(t)), \quad (12) \\ D^+V_3(t) &= h^2e^{2\beta h}\dot{y}^T(t)W_{i_k}\dot{y}(t) - he^{2\beta h}\int_{t-h}^{t}\dot{y}^T(s)W_{i_k}\dot{y}(s)ds \\ &\le h^2e^{2\beta h}z^T(t)W_{i_k}z(t) - e^{2\beta h}\int_{t-h_{i_k}(t)}^{t}\dot{y}^T(s)dsW_{i_k}\int_{t-h_{i_k}(t)}^{t}\dot{y}(s)ds \\ &\le h^2e^{2\beta h}z^T(t)W_{i_k}z(t) \\ &\quad -e^{2\beta h}[y(t)-y(t-h_{i_k}(t))]^TW_{i_k}[y(t)-y(t-h_{i_k}(t))] \\ &\le h^2e^{2\beta h}z^T(t)W_{i_k}z(t) - e^{\beta h}y^T(t)W_{i_k}y(t) \\ &\quad +2e^{\beta h}e^{\beta h_{i_k}(t)}y^T(t)W_{i_k}y(t-h_{i_k}(t)) \\ &\quad -e^{2\beta h_{i_k}(t)}y^T(t-h_{i_k}(t))W_{i_k}y(t-h_{i_k}(t)). \quad (13) \end{aligned}$$

where $A_{i_k}(\beta) = (A_{i_k} + \beta I)$, $\quad B_{i_k}(\beta) = e^{\beta h_{i_k}(t)}B_{i_k}$. From system (9), the following equation is true for any real matrices K_i, $i = 1, 2, 3$ with appropriate dimensions

$$\begin{aligned} &\Big[y^T(t)K_1^T + z^T(t)K_2^T + e^{\beta h_{i_k}(t)}y^T(t-h_{i_k}(t))K_3^T\Big] \\ &\times\Big[-z(t) + (A_{i_k}+\beta I)y(t) + e^{\beta h_{i_k}(t)}B_{i_k}y(t-h_{i_k}(t))\Big] = 0 \end{aligned} \tag{14}$$

From (10)-(14), we obtain

$$D^+V(t) \leq \begin{bmatrix} y(t) \\ z(t) \\ e^{\beta h_{i_k}(t)} y(t-h_{i_k}(t)) \end{bmatrix}^T \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ * & \Delta_{22} & \Delta_{23} \\ * & * & \Delta_{33} \end{bmatrix} \begin{bmatrix} y(t) \\ z(t) \\ e^{\beta h_{i_k}(t)} y(t-h_{i_k}(t)) \end{bmatrix},$$

where $\Delta_{11}, \Delta_{12}, \Delta_{13}, \Delta_{22}, \Delta_{23}$ and Δ_{33} are defined in (i). If the condition (i) holds then

$$D^+V(t) < 0. \tag{15}$$

Integrating both sides of (15) from 0 to t, we obtain

$$V(t) \leq V(0),$$

and hence

$$\begin{aligned} y^T(t)P_{i_k}y(t) \leq V(t) &\leq y^T(0)P_{i_k}y(0) + \int_{-h_{i_k}(0)}^{0} y^T(s)Q_{i_k}y(s)ds \\ &\quad + he^{2\beta h}\int_{-h}^{0}\int_{s}^{0} \dot{y}^T(\alpha)W_{i_k}\dot{y}(\alpha)d\alpha ds, \end{aligned}$$

$$\int_{-h}^{0} y^T(s)Q_{i_k}y(s)ds \leq \lambda_{\max}(Q_{i_k})\|\phi\|^2 \int_{-h}^{0} e^{2\beta s}ds = \frac{\lambda_{\max}(Q_{i_k})}{2\beta}(1-e^{-2\beta h})\|\phi\|^2,$$

$$he^{2\beta h}\int_{-h}^{0}\int_{s}^{0} \dot{y}^T(\alpha)W_{i_k}\dot{y}(\alpha)d\alpha ds \leq 2h^3e^{2\beta h}\lambda_{\max}(W_{i_k})\max\{\beta\|\phi\|, \|\psi\|\}^2.$$

It follows that

$$\begin{aligned} \lambda_{\min}(P_{i_k})\|y(t)\|^2 &\leq \lambda_{\max}(P_{i_k})\|y(0)\|^2 + \frac{\lambda_{\max}(Q_{i_k})}{2\beta}(1-e^{-2\beta h})\|\phi\|^2 \\ &\quad + 2h^3e^{2\beta h}\lambda_{\max}(W_{i_k})\max\{\beta\|\phi\|, \|\psi\|\}^2. \end{aligned} \tag{16}$$

Therefore, the solution $y(t, \phi, \psi)$ is bounded. Returning to the solution $x(t, \phi, \psi)$ of the nominal system (1), it is easy to see that

$$\|y(0)\| = \|x(0)\| = \phi(0) \leq \|\phi\|,$$

we get

$$\|x(t, \phi, \psi)\| \leq \xi(\|\phi\|, \|\psi\|)e^{-\beta t},$$

where

$$\begin{aligned} \xi(\|\phi\|, \|\psi\|) &:= \left\{\frac{\lambda_{\max}(P_{i_k})\|\phi\|^2 + \frac{\lambda_{\max}(Q_{i_k})}{2\beta}(1-e^{-2\beta h})\|\phi\|^2}{\lambda_{\min}(P_{i_k})}\right\}^{\frac{1}{2}} \times \\ &\quad \left\{\frac{+2h^3e^{2\beta h}\lambda_{\max}(W_{i_k})\max\{\beta\|\phi\|, \|\psi\|\}^2}{\lambda_{\min}(P_{i_k})}\right\}^{\frac{1}{2}}. \end{aligned}$$

This means that the nominal system (1) is $\beta-$ stable. We now consider the case at the time $t_k, k = 1, 2, 3, ...,$ when the system switches form the t_{k-1} subsystem to the t_k subsystem. To ensure the $\beta-$ stability, we need to show that $V(t_k^+) - V(t_k) < 0$. Consider

$$\begin{aligned} V(t_k^+) - V(t_k) &= y^T(t_k^+)P_{i_k}y(t_k^+) - y^T(t_k)P_{i_{k-1}}y(t_k) \\ &\quad + \int_{t-h_{i_k}(t)}^{t} e^{2\beta s}x^T(s)[Q_{i_k} - Q_{i_{k-1}}]x(s)ds \\ &\quad + he^{2\beta h}\int_{-h}^{0}\int_{t+s}^{s} \dot{y}^T(\alpha)\Big[W_{i_k} - W_{i_k-1}\Big]\dot{y}(\alpha)d\alpha ds \\ &= x(t_k)^T e^{2\beta t_k}\Big[(I + D_k)^T P_{i_k}(t_k)(I + D_k) - P_{i_{k-1}}\Big]x(t_k) \\ &\quad + \int_{t-h_{i_k}(t)}^{t} e^{2\beta s}x^T(s)[Q_{i_k} - Q_{i_{k-1}}]x(s)ds \\ &\quad + he^{2\beta h}\int_{-h}^{0}\int_{t+s}^{t} \dot{y}^T(\alpha)\Big[W_{i_k} - W_{i_k-1}\Big]\dot{y}(\alpha)d\alpha ds. \end{aligned}$$

By assumptions (ii), (iii) and (iv), we get

$$\Big[(I + D_k)^T P_{i_k}(I + D_k) - P_{i_{k-1}}\Big] < 0, \quad Q_{i_k} - Q_{i_{k-1}} < 0, \quad W_{i_k} - W_{i_k-1} < 0.$$

Therefore, we obtain $V(t_k^+) - V(t_k) < 0$. The proof of the theorem is complete.

Theorem 3.2 *For given positive constants δ, β and h, the system (1) where $u(t) = 0$ is robustly $\beta-$ stable, if there exist symmetric positive definite matrices P_{i_k}, Q_{i_k} and W_{i_k} for all $i_k \in \{1, 2, ..., m\}$, $m, k \in Z^+$, any appropriate dimensional matrices N, M, K_i, $i = 1, 2, 3$ and positive constant ϵ such that the following LMIs hold.*

$$\begin{array}{ll} (i) & \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & N^T E_{i_k} + K_1^T E_{i_k} & \epsilon H_{i_k}^T \\ * & \Delta_{22} & \Delta_{23} & M^T E_{i_k} + K_2^T E_{i_k} & 0 \\ * & * & \Delta_{33} & K_3^T E_{i_k} & \epsilon R_{i_k}^T \\ * & * & * & -\epsilon I & \epsilon J^T \\ * & * & * & * & -\epsilon I \end{bmatrix} < 0, \\ (ii) & \begin{bmatrix} P_{i_{k-1}} & (I + D_k)^T P_{i_k} \\ P_{i_k}(I + D_k) & P_{i_k} \end{bmatrix} > 0, \\ (iii) & Q_{i_k} - Q_{i_{k-1}} < 0, \\ (iv) & W_{i_k} - W_{i_{k-1}} < 0, \end{array}$$

where $\Delta_{11}, \Delta_{12}, \Delta_{13}, \Delta_{22}, \Delta_{23}$ *and* Δ_{33} *are defined in Theorem 3.1.*

Proof. The stability conditions for system (1) where $u(t) = 0$ can prove the following Theorem 3.1. Replace A_{i_k}, B_{i_k} in (i) for Theorem 2.1 with $A_{i_k} +$

$E_{i_k}\Delta_{i_k}(t)H_{i_k}$, $B_{i_k} + E_{i_k}\Delta_{i_k}(t)R_{i_k}$, respectively, we find that condition (i) of Theorem 2.1 for system (1) where $u(t) = 0$ is equivalent to the following condition:

$$\Omega + \Gamma\Delta_{i_k}\Pi + \Pi^T\Delta_{i_k}^T\Gamma^T < 0, \tag{17}$$

where $\Omega = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ * & \Delta_{22} & \Delta_{23} \\ * & * & \Delta_{33} \end{bmatrix}$, $\Gamma^T = [E_{i_k}^T N + E_{i_k}^T K_1, E_{i_k}^T M + E_{i_k}^T K_2, E_{i_k}^T K_3]$ and $\Pi = [H_{i_k}, 0, R_{i_k}]$. By Lemma 2.2, there exists a positive constant $\epsilon > 0$ such that (17) is equivalent to

$$\begin{bmatrix} \Omega & \Gamma & \epsilon\Pi^T \\ \Gamma^T & -\epsilon I & \epsilon J^T \\ \epsilon\Pi & \epsilon J & -\epsilon I \end{bmatrix} < 0. \tag{18}$$

We find that (18) is equivalent to the LMI (i). The proof of the theorem is complete.

3.2 Robust stabilization

In this section, as consequence of Theorem 3.1 and Theorem 3.2, we obtain robust $\beta-$stabilization conditions of (1) in terms of LMIs. We recall that (1) is robustly $\beta-$stabilizable if there exists a feedback control $u(t) = Kx(t), K \in R^{m\times n}$. The closed-loop system (1)

$$\dot{x}(t) = \Big[A_{i_k} + C_{i_k}K + \Delta A_{i_k}(t) + \Delta C_{i_k}(t)K\Big]x(t) + \Big[B_{i_k} + \Delta B_{i_k}(t)\Big]x(t - h_{i_k}(t))$$

is robustly $\beta-$stable.

Theorem 3.3 *For given positive constants δ, β and h, the system (1) is robustly $\beta-$stabilizable, if there exist symmetric positive definite matrices P_{i_k}, Q_{i_k} and W_{i_k} for all $i_k \in \{1, 2, ..., m\}$, $m, k \in Z^+$, any appropriate dimensional matrices N, M, K, K_i, $i = 1, 2, 3$ and positive constant ϵ such that the following LMIs hold.*

$$(i) \quad \begin{bmatrix} \overline{\Delta}_{11} & \overline{\Delta}_{12} & \overline{\Delta}_{13} & N^T E_{i_k} + K_1^T E_{i_k} & \epsilon H_{i_k}^T + \epsilon K^T S_{i_k}^T \\ * & \Delta_{22} & \Delta_{23} & M^T E_{i_k} + K_2^T E_{i_k} & 0 \\ * & * & \Delta_{33} & K_3^T E_{i_k} & \epsilon R_{i_k}^T \\ * & * & * & -\epsilon I & \epsilon J^T \\ * & * & * & * & -\epsilon I \end{bmatrix} < 0,$$

$$(ii) \quad \begin{bmatrix} P_{i_{k-1}} & (I + D_k)^T P_{i_k} \\ P_{i_k}(I + D_k) & P_{i_k} \end{bmatrix} > 0,$$

$$(iii) \quad Q_{i_k} - Q_{i_{k-1}} < 0,$$

$$(iv) \quad W_{i_k} - W_{i_{k-1}} < 0,$$

where
$\overline{\Delta}_{11} = (A_{i_k}+\beta I+C_{i_k}K)^T(N+K_1)+(N+K_1)^T(A_{i_k}+\beta I+C_{i_k}K)+Q_{i_k}-e^{\beta h}W_{i_k}$,
$\overline{\Delta}_{12} = (A_{i_k}+\beta I+C_{i_k}K)^T(M+K_2)+P_{i_k}-N^T-K_1^T$,
$\overline{\Delta}_{13} = N^TB_{i_k}+e^{\beta h}W_{i_k}+K_1^TB_{i_k}+(A_{i_k}+\beta I+C_{i_k}K)^TK_3$,
Δ_{22}, Δ_{23} *and* Δ_{33} *are defined in Theorem 3.1. The stabilizing feedback control is given by* $u(t) = Kx(t)$.

4 Numerical examples

Example 4.1 We consider the following uncertain impulsive switched system with Time-varying delays (1) where $u(t) = 0$ under a given switching law. That is, the switching status alternates as $i_1 \to i_2 \to i_1 \to i_2 \to \cdots$. We consider robust performance of the system (1) where $u(t) = 0$ by using Theorem 3.2. The system (1) where $u(t) = 0$ is specified as follows:

$$A_1 = \begin{bmatrix} -7 & 1 \\ -1 & -6 \end{bmatrix}, B_1 = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}, H_1 = \begin{bmatrix} 0.4 & 0.1 \\ 0 & -0.4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -6 & 1 \\ 2 & -8 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, H_2 = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.7 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.3 & 0 \\ 0.1 & -0.3 \end{bmatrix}, E_2 = \begin{bmatrix} -0.3 & 0 \\ 0.2 & 0.4 \end{bmatrix}, R_1 = \begin{bmatrix} 0.5 & 0.3 \\ 0.1 & 0.4 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}, J = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

We choose $h_1(t)) = 1.5\sin^2(\frac{0.4}{1.5})t$, $h_2(t)) = 1.3\sin^2(\frac{0.4}{1.3})t$, i.e., $h = 1.5$, $F_1(t) = F_2(t) = I$, $\epsilon = 1$, $\delta = 0.4$ and $\beta = 0.1$. By using LMI Toolbox in MATLAB, the solutions of LMI are as follows:

$$P_1 = \begin{bmatrix} 291.6408 & -27.0810 \\ -27.0810 & 152.0843 \end{bmatrix}, P_2 = \begin{bmatrix} 250.9963 & -28.1677 \\ -28.1677 & 179.9650 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 672.3144 & 42.6933 \\ 42.6933 & 709.0446 \end{bmatrix}, Q_2 = \begin{bmatrix} 470.1000 & 4.3411 \\ 4.3411 & 524.8286 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 11.3728 & -0.0063 \\ -0.0063 & 4.9535 \end{bmatrix}, W_2 = \begin{bmatrix} 5.0922 & -0.0992 \\ -0.0992 & 2.3476 \end{bmatrix},$$

$$M = 10^4 \times \begin{bmatrix} 1.0884 & -1.3367 \\ -1.3367 & -5.4558 \end{bmatrix}, N = 10^5 \times \begin{bmatrix} 1.1656 & 0.1053 \\ 0.1053 & -0.9712 \end{bmatrix},$$

$$K_1 = 10^5 \times \begin{bmatrix} -1.1650 & -0.1052 \\ -0.1052 & 0.9720 \end{bmatrix}, K_2 = 10^4 \times \begin{bmatrix} -1.0848 & 1.3365 \\ 1.3365 & 5.4578 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -3.8495 & -5.6164 \\ -5.6164 & 17.0656 \end{bmatrix}, \epsilon = 125.3232.$$

Therefore, the system (1) where $u(t) = 0$ is robustly $0.1-$stable.

Numerical experiments are carried out to investigate dynamical system by using program dde45lin in Matlab. In Fig. 1, the parameters of the system are specified as in Example 4.1 and the initial condition $\phi(t) = [3 \quad -3]^T, \forall t \in [-1.5, 0]$,

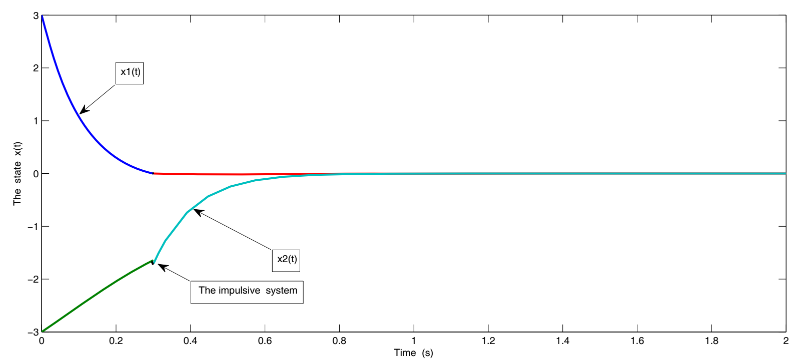

Figure 1: The simulation of solutions of the impulsive switched time-varying delay system in example 4.1

Example 4.2 (Exponential stability). Consider the linear system with time delay of the form

$$\dot{x}(t) = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} -0.5 & 0.1 \\ 0.3 & 0 \end{bmatrix} x(t-h). \tag{19}$$

We use the MATLAB LMI Toolbox for this example, we can compare with the results of other researchers, a summary is given in the following Table 1 by applying the conditions in Theorem 3.1:

Methods	$h_{\max}$ (β=0)	$h_{\max}$ (β=0.2)	$h_{\max}$ (β=0.6)	$h_{\max}$ (β=0.8)
Liu[8]	0.964	0.541	0.124	0.048
Kwon[5]	∞	5.525	1.765	1.345
Our results	∞	5.543	1.786	1.353

Example 4.3 (Asymptotical stability ($\beta = 0$)). Consider the uncertain linear system with time-varying delays of the form

$$\dot{x}(t) = [A + EF(t)H]x(t) + [B + EF(t)R]x(t - h(t)),$$
$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$E = I, \quad H = diag\{1.6, 0.05\}, \quad R = diag\{0.1, 0.3\}.$$

Table 2 shows the results of the upper bounds of delay for different δ. By applying the conditions in Theorem 3.2, we obtain greater the allowable bound on the delay than the previous results given in Table 2.

Methods	$h_{\max}$ (δ=0)	$h_{\max}$ (δ=0.5)	$h_{\max}$ (δ=0.9)	$h_{\max}$ (δ=1.5)
Fridman [2]	1.1490	0.9247	0.6710	-
Wu et al. [13]	1.1490	0.9247	0.6954	-
Our results	1.1500	0.9247	0.6954	0.6274

5 Conclusions

The problems of robust $\beta-$stability and $\beta-$stabilization for uncertain impulsive switched control systems with time-varying delays was studied. By using the descriptor model transformation, Lyapunov-Krasovskii function method and linear matrix inequality (LMI) technique, sufficient conditions for robust $\beta-$stability and $\beta-$stabilization are obtained. As an illustration, two numerical examples were solved and compared using the results obtained in this paper.

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