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Magnetogasdynamic Laminar Circular Jets

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Abstract

The flow of a laminar viscous electrically conducting fluid in a circular jet, in the presence of a variable radial magnetic field, is studied. The fluid discharged through a circular orifice in the presence of magnetic field which is inversely proportional to the radial direction and mix with an ambient fluid, which is initially at temperature T_1 . The Schlichting-Yih model for non-magnetic flow in a circular jet of viscous incompressible fluid is taken as the basis. The boundary layer equations for momentum and energy have been solved numerically by Newton's shooting techniques with the fourth order Runge-Kutta scheme using computer. The effects of various parameters have been discussed with graphically.

Keywords: Magnetogasdynamics, Laminar, Boundary Layer, and Circular Jet

1. Introduction

The velocity distribution in a laminar non-magnetic incompressible circular jet was obtained by Schlichting [7] and Bickley [20]. They obtained closed-form solutions; the details of which may be found in the book edited by Goldstein [1] for compressible fluid the results were given by Howarth [11], Illingworth [2] and Pack [4]. There a solution for the case when the Prandtle number is unity and the viscosity varies as the temperature. In this direction some efforts have also been made by Pai [16] and Pack [4], which have been summarized in the book by Pai [17]. The Schlichting solution for velocity distribution was supplemented for the temperature distribution in the absence of viscous heating by Yih [3] and for viscous heating by Tak and Bansal [18]. They found that the similar solutions also exist for the temperature distribution. It was Peskin [13] who extended the Schlichting's solutions of plane free jet flow to the case of electrically conducting fluids in the presence of a uniform transverse magnetic field by a perturbation on the stream function of the Schlichting model. Efforts in the interaction of the magnetic field with the jet of an electrically conducting fluid have been made Smith and Cambel [5], Pozzi and Bianchini [1], Bansal [8]. More recently closed form solutions for the same have been obtained by Bansal and Gupta [9]. The effects of compressibility and variable radial Magnetic field on a laminar circular jet of an electrically conducting fluid by taking the Prandtl number of the fluid as unity were studied by Mishra and Bansal [10]. The study of Magnetogasdynamic laminar circular jets also been made by Akerstedt and Lofgren [6], Shtem and Hussain [19], Cinalli and Keppens [12].

In the present paper we have studied the effects of both, compressibility and magnetic field on a laminar circular free jet of an electrically conducting gas issuing in the presence of a variable radial magnetic field. The energy equation is solved numerically for initial heating and viscous and Joule heating for arbitrary values of Prandtl number.

2. Formulation of the problem

Let an electrically conducting, viscous, compressible fluid be discharge through a circular orifice in the presence of a variable radial magnetic field $B(B_0x^{-1},0,0)$ and mix with an ambient fluid having a temperature T_1.
The governing boundary layer equations for a non-relativistic fluid motion may be written as [A.B.Cambel].

$$\frac{\partial}{\partial x}(\rho r u)+\frac{\partial}{\partial r}(\rho r v)=0 \qquad \ldots(1)$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right)-\sigma B^2 u \qquad \ldots(2)$$

$$\rho u\frac{\partial i}{\partial x}+\rho v\frac{\partial i}{\partial r}=\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial r}\left(\mu r\frac{\partial i}{\partial r}\right)+\mu\left(\frac{\partial u}{\partial r}\right)^2+\sigma B^2u^2 \qquad \ldots(3)$$

Where σ denotes the electrical conductivity of the fluid, μ the coefficient of viscosity, ρ the density, i(=C_pT) the enthalpy and Pr is the Prandtle number of the fluid.

The boundary conditions are:

$$r = 0 : \frac{\partial u}{\partial r} = 0, v = 0, \frac{\partial i}{\partial r} = 0 \qquad \ldots(4)$$

$$r \to \infty : \frac{\partial u}{\partial r} \to 0, \frac{\partial u}{\partial x} \to -\frac{\sigma B_0^2 x^{-2}}{\rho_1}, \frac{\partial i}{\partial r} \to 0, \frac{\partial i}{\partial x} \to \frac{\sigma_1 B_0^2 x^{-2} u}{\rho_1} \qquad \ldots(5)$$

$$\text{or } u = m_1 x^{-1}, I = i_1 - \frac{m_1^2 x^{-2}}{2} \qquad \ldots(6)$$

Where $m_1 = \frac{\sigma_1 B_0^2}{\rho_1}$ and $i_1 = C_p T_1$ is a constant. …(7)

The integral condition in which implies constant momentum flux in the absence of the magnetic field.

$$\int_0^\infty \rho u_0^2 dr = \frac{M_0}{2\pi} \qquad \ldots(8) \quad \text{and} \quad 2\pi \int_0^\infty \rho u_0 i_0 dr = \frac{H_0}{T_\infty} \qquad \ldots(9)$$

3. Analysis

Let us transform the independent variables (x,r) to (X,R) such that

$$X=r, \ \rho_1 R^2 = 2\int_0^r \rho r dr \qquad \ldots(10)$$

Where ρ_1 is the density outside and on the boundary of the jet.

By introducing a stream function ψ, such that

$$pru = \frac{\partial \psi}{\partial r} = \frac{\rho r}{\rho_1 R}\frac{d\psi}{dR} \ \ldots(11) \text{ and } \rho r v = -\frac{\partial \psi}{\partial x} = -\frac{\partial \psi}{\partial X} - \frac{\partial \psi}{\partial R}\frac{\partial R}{\partial X} \qquad \ldots(12)$$

The equation of continuity (1) is identically satisfied.

To find the similar solution for the velocity distribution Bansal et.al. (1985) make the following assumptions:

$$\mu = \mu_1, \sigma\rho_1 = \sigma_1 \rho, \rho r^2 = \rho_1 R^2 \text{ and } B^2 = B_0^2 x^{-2} \qquad \ldots(13)$$

and taking

$$\psi = \mu_1 X g_m(\eta), \text{ and } \eta = \frac{R}{X\mu_1^{1/2}} \qquad \ldots(14)$$

then equation of momentum reduces to

$$g_m^{'''} - \frac{1}{\eta} g_m^{''} + \frac{1}{\eta^2} g_m^{'} + \frac{1}{\eta} g_m^{'2} + \frac{g_m g_m^{''}}{\eta} - \frac{g_m g_m^{'}}{\eta^2} - m g_m^{'} = 0 \qquad \ldots(15)$$

The corresponding boundary and integral condition are

$$\eta = 0 : g_m = 0, g_m^{'} = 0$$

$$\eta \to \infty : \lim_{\eta \to \infty} \frac{g_m^{'}}{\eta} \to m \qquad \ldots(16)$$

and $\int_0^\infty \frac{1}{\eta} g_0^{'2} d\eta = \frac{M_0}{2\pi\upsilon_1}$, where $\upsilon_1 = \frac{\mu_1}{\rho}$ …(17)

and a prime denotes differentiation with respect to η.

Integrating (15) once, we get

$$\frac{d}{d\eta}\left(\frac{g_m^{'}}{\eta}\right) + \frac{g_m}{\eta}\frac{g_m^{'}}{\eta} = m\frac{g_m}{\eta} \qquad \ldots(18)$$

Which satisfies the boundary conditions (16) at $\eta \to \infty$ Bansal et.al. found the first and second approximation for (18) as,

$$g_m = \frac{1}{2} m\alpha^2\eta^2 + \frac{\alpha^2\eta^2}{1 + \frac{\alpha^2\eta^2}{4}} \qquad \ldots(19)$$

$$g_m = \frac{1}{2} m\alpha^2\eta^2 + \frac{\alpha^2\eta^2 e^{-m\eta^2/4}}{1 + \frac{\alpha^2\eta^2}{4}} + \frac{m\alpha^2}{2}\int_0^\eta \frac{\eta^3 e^{-m\eta^2/4}}{1 + \alpha^2\eta^2/4} d\eta \qquad \ldots(20)$$

Which satisfies the boundary conditions (16) and when m=0 (Schlichting model).

$g_0 = \frac{\alpha^2\eta^2}{1 + \alpha^2\eta^2/4}$ …(21) where, $\alpha^2 = \frac{3M_0}{16\pi\upsilon_1}$ …(22)

The momentum flux M, across a section of the jet at a distance x from the orifice, is given by

$$M = 2\pi\int_0^\infty \rho r u^2 dr = 2\upsilon_1 \int_0^\infty \frac{g_m^{'2}}{\eta} d\eta \qquad \ldots(23)$$

Which for small value of m/α^2, is obtained as

$$\frac{M}{M_0} = 1 + \frac{2m}{\alpha^2} + \ldots \qquad \ldots(24)$$

Where in a compressible flow $u = \frac{1}{\rho_1 \eta} g_m^{'}(\eta)$ …(25)

4. Analysis of the thermal boundary layer

The energy equation (3) has complete solution of the form

$$i = \theta_1 + \theta_2 \qquad \ldots(26)$$

Where θ_1 is complementary solution, neglecting the Joule heating and viscous dissipation and θ_2 the particular solution of the non homogeneous equation. The two cases of initial heating and Joule heating are taken up separately.

(i) Initial heating (Viscous and Joule heating neglected):

In the present case of initially heated jet the energy equation, with help of the assumptions (13) and (14) and after taking

$$\theta_1 = \frac{h_m}{\rho_1 X} \qquad \ldots(27)$$

becomes,

$$\eta h_m'' + h_m' + \Pr(g_m h_m' + g_m' h_m) = 0 \qquad \ldots(28)$$

or $\eta h_m' + \Pr g_m h_m = \text{constant}$ …(29)

The boundary conditions are:

$$\eta = 0 : h_m' = 0$$
$$\eta \to \infty : h_m = 0 \qquad \ldots(30)$$

and the integral condition is

$$\upsilon_1 \int_0^\infty h_0 g_0' d\eta = 1 \qquad \ldots(31)$$

The solution of (28) is obtained by Yih [3], as

$$h(\eta) = \frac{1 + 2\Pr}{4\upsilon_1}\left(1 + \frac{\alpha^2\eta^2}{4}\right)^{-2\Pr} \qquad \ldots(32)$$

The non-dimensional value of θ_1 is obtained as:

$$\theta_1 = \frac{h(\eta)}{h(0)} \qquad \ldots(33)$$

The graphical representation is also shown in figure 1 for different values of Pr.

(ii) Viscous and Joule heating

In this case, we shall determine the particular solution, $i = \theta_2$, using (13) and (14) and taking

$$\theta_2 = \frac{H_m}{\rho_1^2 X^2} \qquad \ldots(34)$$

the equation of energy becomes,

$$\frac{\eta}{\Pr}H_m'' + \frac{H_m'}{\Pr} + \left(g_m H_m' + 2g_m' H_m\right) = -\frac{1}{\eta}g_m''\left(g_m'' - \frac{2}{\eta}g_m'\right) - \frac{1}{\eta}g_m'^2\left(\frac{1}{\eta^2} + m\right)$$
…(35)

The boundary conditions are:

$$\eta = 0 : H_m' = 0, g_m = 0, g_m' = 0$$
$$\eta \to \infty : H_m \to -\frac{m^2}{2}, \lim_{\eta \to \infty} \frac{g_m'}{\eta} \to m \qquad \ldots(36)$$

The non-dimensional value of θ_2 is given as:

$$\theta_2 = \frac{H(\eta)}{H(0)} \qquad \ldots(37)$$

The contribution θ_2, on account of viscous and Joule heating is plotted against the similarity variable η, for various values of Pr, in figure 2.

5. Conclusion

The effects of a uniform radial magnetic field on the velocity and temperature distribution in a circular jet of viscous incompressible electrically conducting fluid are studied by a perturbation on the Schlichting-Yih model (m=0). From the analysis and the graphical representation it is concluded that the temperature on the axis of the jet increases with the increasing value of the magnetic parameter m and decreases with the value of the Prandtle number Pr of the fluid. Also we see that as the Prandtle number Pr decreases from unity the overall temperature difference near the axis of the jet increases but as we move away from the axis it goes on decreasing.

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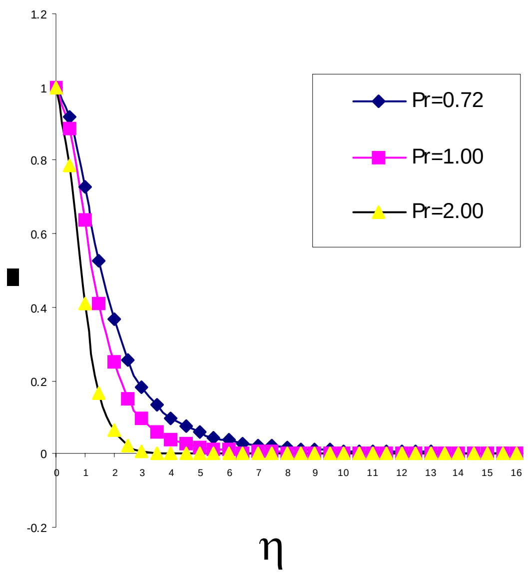

Fig. 1 - Temperature distribution in a circular jet

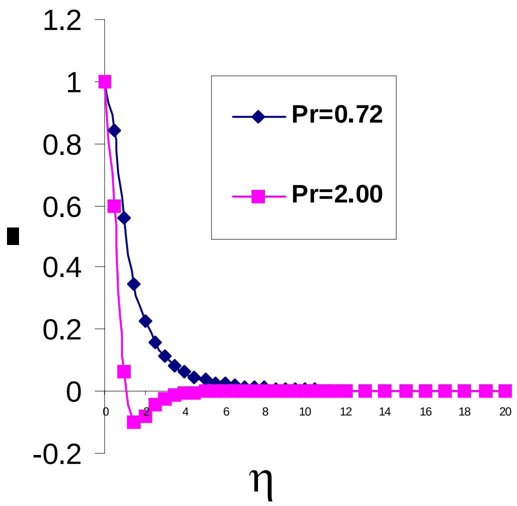

Fig. 2 - Temperature distribution in a circular jet (Viscous and Joule heating)

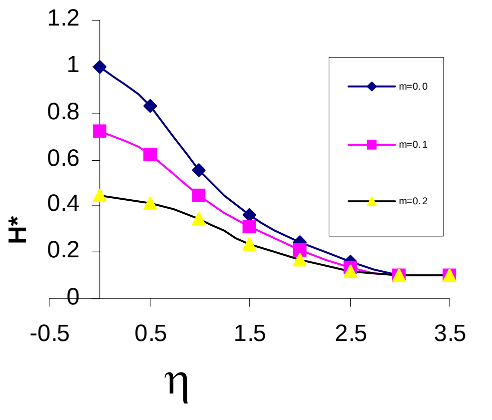

Fig. 3 - Temperature distribution in a circular jet (Viscous and Joule heating) for Pr=0.72

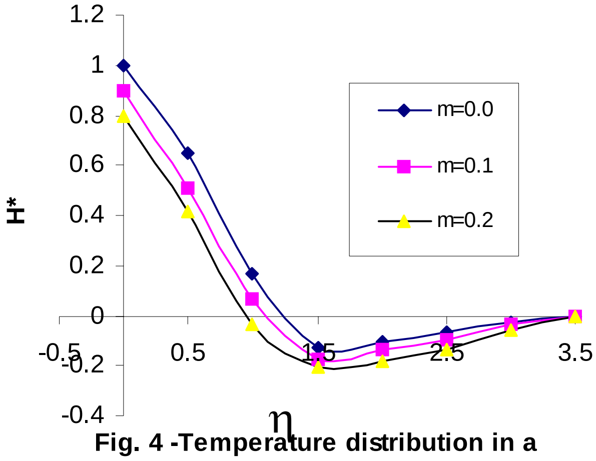

Fig. 4 -Temperature distribution in a circular jet(Viscous and Joule heating) for Pr=2.00

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