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An Optimal More-for-Less Solution to Fuzzy Transportation Problems with Mixed Constraints

P. Pandian and G. Natarajan

Department of Mathematics, School of Advanced Sciences
VIT University, Vellore, Tamil Nadu, India

Corresponding author. pandian61@rediffmail.com (P. Pandian)

Abstract

A new algorithm based on fuzzy zero point method [14] for finding an optimal more-for-less solution for a fuzzy transportation problem with mixed constraints where the transportation cost, supply and demand are triangular fuzzy numbers. The proposed method is very simple, easy to understand and apply. The new proposed algorithm is illustrated with the help of numerical example. The more-for-less situation exists in reality and it could present managers with an opportunity for shipping more units for less or the same cost.

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Keywords: Fuzzy transportation problem; Triangular fuzzy number; Mixed constraints; Optimal solution; Fuzzy zero point method ; More-for-less solution

1 Introduction

A fuzzy transportation problem (FTP) is a transportation problem (TP) in which the transportation costs, supply and demand quantities are fuzzy quantities. The objective of the FTP is to determine the shipping schedule that minimize the total fuzzy transportation cost while satisfying fuzzy supply and fuzzy demand limits. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [7] and Zadeh [18] introduced the notion of fuzziness. In the literature, many researchers [10, 11, 17, 14] have developed various algorithms to solve FTP with equality constraints.

In real life, most of the TPs have mixed constraints accommodating many applications that go beyond transportation related problems to include job scheduling, production inventory, production distribution, allocation problems, and investment analysis. The TPs with mixed constraints are not addressed in the literature because of the rigor required to solve these problems optimally. A literature search revealed no systematic method for finding an optimal solution of TPs with mixed constraints.

The more-for-less (MFL) paradox in a TP occurs when it is possible to ship more total goods for less (or equal) total cost while shipping the same amount or more from each origin and to each destination keeping all shipping costs non-negative. The occurrence of MFL in distribution problems is observed in nature. The existing literature [6, 10, 12, 16, 17, 2-5] has demonstrated the identifying cases where MFL paradoxical situation exists and also, has provided various methods for finding MFL solution for TPs. The primary goal of the MFL method is to minimize the total cost and not merely maximize the shipment load transported.

Arsham [6] developed an approach to post optimality analysis of the TPs through the use of perturbation analysis. Adlakha and Kowalski [2,3,4] introduced a theory of absolute points for solving a TP and used these points for search opportunities to ship more for less in TP. Adlaka et al.[5] developed an algorithm for finding an optimal MFL solution for TPs which builds upon any existing basic feasible solution. Recently, Pandian and Natarajan [14] have proposed the fuzzy zero point method for finding an optimal solution for fuzzy transportation problems.

In this paper, we propose a new algorithm for finding optimal MFL solution for a FTP with mixed constraints where all parameters are triangular fuzzy numbers. The new method is based on fuzzy zero point method [14] and also, it is very simple, easy to understand and apply. The solution procedure is illustrated with the help of numerical example. The MFL analysis could be useful for managers in making important decisions such as increasing warehouse/plant capacity, or advertising efforts to increase demand at certain markets.

2 Fuzzy number and Fuzzy transportation problem with mixed constraints

We need the following definitions of triangular fuzzy number and membership function and also, definitions of basic arithmetic operation on fuzzy triangular numbers which can be found in [11,13] .

Definition 1. A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its member ship function $\mu_{\tilde{a}}(x)$ is given below.

$$\mu_{\tilde{a}}(x)=\begin{cases}(x-a_1)/(a_2-a_1) & \text{for } a_1\le x\le a_2\\ (a_3-x)/(a_3-a_2) & \text{for } a_2\le x\le a_3\\ 0 & \text{otherwise}\end{cases}$$

Definition 2. Let $\tilde{a}=(a_1,a_2,a_3)$ and $\tilde{b}=(b_1,b_2,b_3)$ be two triangular fuzzy numbers. Then

(i) $\tilde{a}\oplus\tilde{b} = (a_1+b_1,a_2+b_2,a_3+b_3)$;

(ii) $\tilde{a}\,\Theta\,\tilde{b} = (a_1-b_3,a_2-b_2,a_3-b_1)$;

(iii) $k\tilde{a}=(ka_1,ka_2,ka_3)$, for $k\ge 0$;

(iv) $k\tilde{a}=(ka_3,ka_2,ka_1)$, for $k<0$;

(v) $\tilde{a}\otimes\tilde{b} = (t_1,t_2,t_3)$, where $t_1=\text{minimum}\{a_1b_1,a_1b_3,a_3b_1,a_3b_3\}$; $t_2=a_2b_2$ and $t_3=\text{maximum}\{a_1b_1,a_1b_3,a_3b_1,a_3b_3\}$;

(vi) $\dfrac{1}{\tilde{b}}=\left(\dfrac{1}{b_3},\dfrac{1}{b_2},\dfrac{1}{b_1}\right)$, where b_1,b_2 and b_3 are all non zero positive real numbers and

(vii) $\dfrac{\tilde{a}}{\tilde{b}}=\tilde{a}\otimes\dfrac{1}{\tilde{b}}$, where b_1,b_2 and b_3 are all non zero positive real numbers.

We need the following definitions of magnitude of a triangular fuzzy number and ordering on E, the set of the fuzzy numbers based on the magnitude of a fuzzy number which can be found in [1].

Definition 3. The magnitude of the triangular fuzzy number $\tilde{u}=(a,b,c)$ is given by

$$Mag(\tilde{u})=\frac{a+10b+c}{12}.$$

Definition 4. Let $\tilde{u}$ and $\tilde{v}$ be two triangular fuzzy numbers. The ranking of $\tilde{u}$ and $\tilde{v}$ by the $Mag(.)$ on E is defined as follows:

(i) $Mag(\tilde{u}) > Mag(\tilde{v})$ if and only if $\tilde{u}\succ\tilde{v}$;

(ii) $Mag(\tilde{u}) < Mag(\tilde{v})$ if and only if $\tilde{u}\prec\tilde{v}$ and

(iii) $Mag(\tilde{u}) = Mag(\tilde{v})$ if and only if $\tilde{u}\approx\tilde{v}$.

Consider the following FTP with mixed constraints,

(P) Minimize $\tilde{z}=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\,\tilde{x}_{ij}$

subject to

$$\sum_{i=1}^{m} \tilde{x}_{ij} \succeq \tilde{a}_j, \ j \in Q \tag{1}$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \preceq \tilde{a}_j, \ j \in T \tag{2}$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{a}_j, \ j \in S \tag{3}$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} \succeq \tilde{b}_i, \ i \in U \tag{4}$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} \preceq \tilde{b}_i, \ i \in V \tag{5}$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{b}_i, \ i \in W \tag{6}$$

$$\tilde{x}_{ij} \succeq \tilde{0}, \ i = 1,2,...,m \ \text{and} \ j = 1,2,...,n \ \text{and integers} \tag{7}$$

where
m = the number of supply points ; n = the number of demand points ;
Q, T and S are pairwise disjoint subsets of $\{1,2,3,...,n\}$ such that $Q \cup T \cup S = \{1,2,3,...,n\}$;
U, V and W are pairwise disjoint subsets of $\{1,2,3,...,m\}$ such that $U \cup V \cup W = \{1,2,3,...,m\}$;
$\tilde{x}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3)$ is the uncertain number of units shipped from supply point i to demand point j ;
$\tilde{c}_{ij} = (c_{ij}^1, c_{ij}^2, c_{ij}^3)$ is the uncertain cost of shipping one unit from supply point i to the demand point j ;
$\tilde{a}_i = (a_i^1, a_i^2, a_i^3)$ is the uncertain supply at supply point i and
$\tilde{b}_j = (b_j^1, b_j^2, b_j^3)$ is the uncertain demand at demand point j.

Remark 1. If $Q = \phi$, $R = \phi$, $U = \phi$ and $V = \phi$, the problem (P) becomes the FTP with equality constraints.

The FTP with mixed constraints can be represented by a table form called transportation table. For $m = 3$ and $n = 4$, the structure of the transportation table is

	1	2	3	4	Supply
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\tilde{c}_{13}$	$\tilde{c}_{14}$	$\preceq \tilde{b}_1$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\tilde{c}_{23}$	$\tilde{c}_{24}$	$\approx \tilde{b}_2$
3	$\tilde{c}_{31}$	$\tilde{c}_{32}$	$\tilde{c}_{33}$	$\tilde{c}_{34}$	$\succeq \tilde{b}_3$
Demand	$\approx \tilde{a}_1$	$\succeq \tilde{a}_2$	$\preceq \tilde{a}_3$	$\succeq \tilde{a}_4$	

Now, we follow the rules given below for finding the maximum possible allotment to a cell in the transportation table whose corresponding demand and supply limits are $\tilde{a}$ and $\tilde{b}$ respectively:

(i) The maximum possible allotment to the cell whose limits pair is $\left\{\preceq \tilde{a}, \approx \tilde{b}\right\}$,

$$= \begin{cases} \tilde{b} & : \tilde{a} \succeq \tilde{b} \\ \tilde{a} & : \tilde{a} \prec \tilde{b} \end{cases}.$$

(ii) The maximum possible allotment to the cell whose limits pair is $\left\{\succeq \tilde{a}, \approx \tilde{b}\right\}$,

$$= \tilde{b}.$$

(iii) The maximum possible allotment to the cell whose limits pair is $\left\{\preceq \tilde{a}, \preceq \tilde{b}\right\} = \tilde{0}$.

(iv) The maximum possible allotment to the cell whose limits pair is $\left\{\approx \tilde{a}, \approx \tilde{b}\right\}$,

$=$ minimum of $\left\{\tilde{a}, \tilde{b}\right\}$.

(v) The maximum possible allotment to the cell whose limits pair is $\left\{\succeq \tilde{a}, \succeq \tilde{b}\right\}$,

$=$ maximum of $\left\{\tilde{a}, \tilde{b}\right\}$ and

(vi) The maximum possible allotment to the cell whose limits pair is $\left\{\succeq \tilde{a}, \preceq \tilde{b}\right\}$,

$$= \begin{cases} \tilde{b} & : \tilde{a} \succeq \tilde{b} \\ \tilde{a} & : \tilde{a} \prec \tilde{b} \end{cases}.$$

3 A new proposed algorithm

Arsham [6] proved that the existence of a MFL situation in a TP requires only one condition namely, the existence of a location with negative plant-to-market shipping shadow price. The shadow prices are easily calculated from the basic feasible solution of the TP with mixed constraints. The MFL solution is obtained from the basic feasible solution distribution by increasing and decreasing the shipping quantities while maintaining the minimum requirements for both supply and demand. In the FTP, the plant-to-market shipping shadow price (also called fuzzy Modi index) at a cell (i, j) is $\tilde{u}_i + \tilde{v}_j$ where $\tilde{u}_i$ and $\tilde{v}_j$ are fuzzy shadow prices corresponding to the cell (i, j). The negative fuzzy Modi index at a cell (i, j) indicates that we can increase the ith plant capacity / the demand of the jth market at the maximum possible level.

Theorem 1. The optimal MFL solution of a FTP with mixed constraints is an optimal solution of a new FTP with mixed constraints which is obtained from the given FTP with mixed constraints by changing the sign of columns and rows having negative Modi indices from $=$ to $\geq$ and $\leq$ to $=$.

Proof: Since the existence of a MFL situation in a FTP with mixed constraints requires only one condition namely, the existence of a location with negative fuzzy Modi index. Therefore, the negative fuzzy Modi index at a cell (i, j), $\tilde{u}_i + \tilde{v}_j$ indicates that we can achieve the supply of the i^{th} source / the demand of the j^{th} destination at the maximum possible level. Construct a new FTP with mixed constraints obtained from the given problem by changing the sign of columns and rows having negative fuzzy Modi indices from $=$ to $\geq$ and $\leq$ to $=$ in the given problem. The newly constructed FTP with mixed constraints is a FTP with mixed constraints such that all the columns and rows having negative fuzzy Modi indices can be achieved at the maximum level. Therefore, any solution of the newly constructed FTP with mixed constraints is an MFL solution to the given problem. Thus, the optimal solution of the newly constructed FTP with mixed constraints is an optimal MFL solution to the given FTP with mixed constraints.
Hence the theorem.

We, now introduce a new algorithm based on the fuzzy zero point method for finding an optimal MFL solution for FTPs with mixed constraints.

The proposed algorithm proceeds as follows.

Step 1: Find a basic feasible solution of the FTP with mixed constraints using the fuzzy zero point method.

Step 2: Prepare the Modi index matrix for the basic feasible solution of the FTP with mixed constraints obtained in the Step 1.

Step 3: Identify the negative Modi indices and related columns and rows. If none exists, this is an optimal solution to the FTP with mixed constraints (no MFL paradox is present). STOP.

Step 4: Form a new TP with mixed constraints by changing the sign of columns and rows having negative Modi indices from $=$ to $\geq$ and $\leq$ to $=$ in the given problem.

Step 5: Repeat the Step 1 to Step 4 until to obtain an optimal solution for the new FTP with mixed constraints.

Step 6. The optimal solution obtained in the Step 5 for the new FTP with mixed constraints is an optimal MFL solution of the given TP with mixed constraints. (by the Theorem 1.).

Remark 2. At the time of using the fuzzy zero point method [14] for finding a basic feasible solution of the given problem, we change the step 4 of the fuzzy zero point method by the following new step:

Step 4. Check if each column fuzzy demand can be *accomplished* from the union of row supplies whose reduced costs in that column are fuzzy zero. Also, check if each row supply can be *accomplished* from the union of column demands whose reduced costs in that row are fuzzy zero. If so, go to Step 7. (Such reduced transportation table is called the allotment table).
If not, go to Step 5.

Remark 3. For calculating Modi indices, we need $n+m-1$ loading cells. So, we keep the cells that would be loaded using the fuzzy zero point method even with a load of zero.

4 Numerical Example

The proposed method is illustrated by the following example.

Example 1. Consider the following FTP with mixed constraints.

	1	2	3	Supply
1	(1,2,3)	(2,5,8)	(2,4,6)	$\approx$(2,5,8)
2	(2,6,10)	(1,3,5)	(0,1,2)	$\succeq$ (3,6,9)
3	(4,8,12)	(3,9,15)	(1,2,3)	$\preceq$ (3,9,15)
Demand	$\approx$ (4,8,12)	$\succeq$ (8,10,12)	$\preceq$ (3,5,7)	

Now, we obtain the following allotment table for the given problem by using the Step 1 to the Step 5 of the fuzzy zero point algorithm.

	1	2	3	Supply
1	$\tilde{0}$	(-4,6,18)	(-1,7,15)	$\approx$(2,5,8)
2	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\succeq$ (3,6,9)
3	(-9,1,11)	(-5,5,15)	$\tilde{0}$	$\preceq$ (3,9,15)
Demand	$\approx$ (4,8,12)	$\succeq$ (8,10,12)	$\preceq$ (3,5,7)	

Now, we obtain the following allotment for the given problem by using allotment rules of the fuzzy zero point method.

	1	2	3	Supply
1	(2,5,8)			$\approx$(2,5,8)
2	(-4,3,10)	(8,10,12)	$\tilde{0}$	$\succeq$ (3,6,9)
3			$\tilde{0}$	$\preceq$ (3,9,15)
Demand	$\approx$ (4,8,12)	$\succeq$ (8,10,12)	$\preceq$ (3,5,7)	

Therefore, a basic feasible solution for the given TP with mixed constraints is $\tilde{x}_{12} = (2,5,8)$, $\tilde{x}_{21} = (-4,3,10)$, $\tilde{x}_{22} = (8,10,12)$, $\tilde{x}_{23} = \tilde{0}$, $\tilde{x}_{33} = \tilde{0}$ and the fuzzy transportation cost is $\tilde{z} = (-30,58,184)$.

The fuzzy Modi index matrix for the above basic feasible solution is given below

	$\tilde{v}_1$	$\tilde{v}_2$	$\tilde{v}_3$	$\tilde{u}_i$
$\tilde{u}_1$	(1,2,3)	(-8,-1,6)	(-9,-3,3)	(-9,-4,1)
$\tilde{u}_2$	(2,6,10)	(1,3,5)	(0,1,2)	$\tilde{0}$
$\tilde{u}_3$	(1,7,13)	(0,4,8)	(1,2,3)	(-1,1,3)
$\tilde{v}_j$	(2,6,10)	(1,3,5)	(0,1,2)	

Since the first row and the second and third columns have negative fuzzy Modi indices, we consider the following new FTP with mixed constraints.

	1	2	3	Supply
1	(1,2,3)	(2,5,8)	(2,4,6)	$\succeq$ (2,5,8)
2	(2,6,10)	(1,3,5)	(0,1,2)	$\succeq$ (3,6,9)
3	(4,8,12)	(3,9,15)	(1,2,3)	$\preceq$ (3,9,15)
Demand	$\approx$ (4,8,12)	$\succeq$ (8,10,12)	$\approx$ (3,5,7)	

Now, we obtain the following allotment table for the above new TP with mixed constraints by using the Step 1 to the Step 5 of the fuzzy zero point method.

	1	2	3	Supply
1	$\tilde{0}$	(-6,1,8)	(-1,2,5)	$\succeq$ (0,5,10)
2	(0,5,10)	$\tilde{0}$	$\tilde{0}$	$\succeq$ (1,6,11)
3	(1,6,11)	(-5,3,15)	$\tilde{0}$	$\preceq$ (3,9,15)
Demand	$\approx$ (4,8,12)	$\succeq$ (4,10,16)	$\approx$ (1,5,9)	

Now, we obtain the following allotment for the new TP with mixed constraints by using allotment rules of the fuzzy zero point method.

	1	2	3	Supply
1	(4,8,12)	$\tilde{0}$		$\succeq$ (0,5,10)
2		(4,10,16)	$\tilde{0}$	$\succeq$ (1,6,11)
3			(1,5,9)	$\preceq$ (3,9,15)
Demand	$\approx$ (4,8,12)	$\succeq$ (4,10,16)	$\approx$ (1,5,9)	

Therefore, a basic feasible solution for the new TP with mixed constraints is $\tilde{x}_{11} = (4,8,12)$, $\tilde{x}_{22} = (4,10,16)$, $\tilde{x}_{23} = \tilde{0}$, $\tilde{x}_{33} = (1,5,9)$ and the fuzzy transportation cost is $\tilde{z} = (9,56,143)$.

The fuzzy Modi index matrix for the above basic feasible solution of the new TP with mixed constraints is given below.

	$\tilde{v}_1$	$\tilde{v}_2$	$\tilde{v}_3$	$\tilde{u}_i$
$\tilde{u}_1$	(1,2,3)	(2,5,8)	(-3,3,9)	(-3,2,7)
$\tilde{u}_2$	$\tilde{0}$	(1,3,5)	(0,1,2)	$\tilde{0}$
$\tilde{u}_3$	(-1,1,3)	(0,4,8)	(1,2,3)	(-1,1,3)
$\tilde{v}_j$	$\tilde{0}$	(1,3,5)	(0,1,2)	

Since all fuzzy Modi indices are non-negative, the current solution is an optimal solution of the new FTP with mixed constraints. Therefore, the optimal MFL solution for the given TP with mixed constraints is $\tilde{x}_{11} = (4,8,12)$, $\tilde{x}_{22} = (4,10,16)$, $\tilde{x}_{23} = \tilde{0}$, $\tilde{x}_{33} = (1,5,9)$ and the minimum fuzzy transportation cost is $\tilde{z} = (9,56,143)$. We are shipping (9,23,37) units at a lower cost of (9,56,143).

Remark 4. The proposed algorithm can also be used for finding an optimal MFL solution of FTPs with equality constraints.

5 Conclusion

We have attempted to develop a new algorithm based on the fuzzy zero point method to find an optimal MFL solution to FTPs with mixed constraints. The proposed method for an optimal MFL solution is very simple, easy to understand and apply. The MFL analysis could be useful for managers in making strategic decisions such as increasing a ware-house stocking level or plant production capacity and advertising efforts to increase demand at certain markets. So, the new method for an optimal MFL solution using fuzzy zero point method can serve managers by providing one of the best MFL solutions to a variety of distribution problems.

References

[1] S.Abbasbandy and T.Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, Computers and Mathematics with Applications, 57 (2009), 413-419.

[2] V.Adlakha and K. Kowalski, A quick sufficient solution to the more –for-less paradox in transportation problems, Omega, 26 (1998), 541-547.

[3] V.Adlakha and K. Kowalski, A note on the procedure MFL for a more –for-less solution in transportation problems, Omega, 28 (2000), 481-483.

[4] V.Adlakha and K. Kowalski, A heuristic method for more –for- less in distribution related problems, International Journal of Mathematical Education in Science and Technology, 32 (2001), 61-71.

[5] V.Adlakha, K.Kowalski, R.R.Vemuganti and B. Lev, More- for -less algorithm for fixed charge transportation problems, Omega, 35 (2007), 116-127.

[6] H. Arsham, Postoptimality analyses of the transportation problem, Journal of the Operational Research Society, 43 (1992), 121–139.

[7] R.E. Bellman and L.A. Zadeh, Decision-making in a fuzzy environment, Management Science, 17 (1970), B141–B164.

[8] S. Chanas and D. Kuchta, A concept of the optimal solution of the transportation problem with fuzzy cost coefficients, Fuzzy Sets and Systems, 82 (1996), 299–305.

[9] S. Chanas, W. Kolodziejczyk and A. Machaj, A fuzzy approach to the transportation problem, Fuzzy Sets and Systems, 13 (1984), 211–221.

[10] A. Charnes and D. Klingman, The more-for-less paradox in the paradox in the distribution models, Cahiers du Centre d'Etudes Recherche Operationnelle, 13 (1977), 11-22.

[11] S. H. Chen, Operations on fuzzy numbers with function principle, Tamkang Journal of Management Sciences, 6(1985), 13-25.

[12] A.Gupta, S. Khanna and M. Puri, Paradoxical situations in transportation problems, Cahiers du Centre d'Etudes de Recherche Operationnell, 34 (1992), 37–49.

[13] A. Kaufmann, Introduction to the Theory of Fuzzy Sets, Vol. I Academic Press, New York, 1976.

[14] P. Pandian and G. Natarajan, A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems , Applied Mathematical Sciences, 4 (2010), 79 - 90

[15] M. Rayan, More- for- less paradox in distribution model, Extremal methods and systems analysis, Springer, New York, 1978.

[16] D.J. Robb, The more for less paradox in distribution models: an intuitive explanation, IIE Transactions, 22 (1990), 377-378.

[17] Shiang-Tai Liu and Chiang Kao, Solving fuzzy transportation problems based on extension principle, European Journal of Operational Research, 153 (2004), 661–674.

[18] L.A. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems, 1 (1978), 3–28.

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