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Heat Transfer in Peristaltic Flow of Viscoelastic Fluid in an Asymmetric Channel

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Abstract

In this paper, we carry out a study of peristaltic flow of Oldroyd fluid in an asymmetric channel in the presence of heat transfer. The governing equations of motion and energy are simplified using along wave length approximation. A closed form solution for the axial velocity and the temperature is obtained using perturbation method. Furthermore, The effect of various parameters of interest on axial velocity, temperature and heat transfer coefficient are discussed numerically and explained graphically.

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1 Introduction

The peristaltic flow of biofluids in different geometries has many application in mathematics, biology and engineering. The initial mathematical models of peristalsis obtained by a train of sinusoidal waves in an infinitely long symmetric channel or tube have been investigated by Shapiro *et al*. [2] and Fung and Yih [17].

After these studies, many investigations were done to understand the peristaltic action for Newtonian and non Newtonian fluids in different situations.

The importance of the study of peristaltic transport in an asymmetric channel has been brought out by Eytan and Elad [13] with an application in intra uterine fluid flow in anon-pregnant uterus. After this study, some investigations were done to understand the mechanism of peristalsis in asymmetric channels. Mishra and Rao [10] have investigated the flow in an asymmetric channel generated by peristaltic waves propagating on the walls with different amplitudes and phases. Rao and Mishra [5] discussed the non-linear and curvature effects on the peristaltic flow of a viscous fluid in asymmetric channel when the ratio of channel width to the wavelength is small. Ebaid [1] studied the effects of magnetic field and wall slip condition on the peristaltic transport of a Newtonian fluid in an asymmetric channel. Also Kothandapani and Srinivas [9] discussed the non-linear peristaltic transport of a Newtonian fluid in an inclined asymmetric channel through a porous medium. Furthermore, Haroun [8] studied the effect of wall compliance on peristaltic transport of a Newtonian fluid in an asymmetric channel . In their paper, Hayat *et al*.[16] investigated the peristaltic mechanism of a Maxwell fluid in asymmetric channel and Subba Reddy *et al*. [11] investigated the peristaltic motion of a power-low fluid in an asymmetric channel. Also Ali and Hayat [12] studied the peristaltic motion of a Carreau fluid in an asymmetric channel. Furthermore, Sobh[3] studied the slip flow in peristaltic transport of a Carreau fluid in an asymmetric channel and Wang *et al*.[18] have studied the magnetohydrodynamic peristaltic flow of a Sisko fluid in a symmetric or asymmetric channel.

Recently, few studies have been done to understand the interaction between heat transfer and peristaltic flow for Newtonian fluids. Srinivas and Kothandapani [15] investigated the peristaltic transport of a Newtonian fluid with heat transfer in an asymmetric channel. Sobh [4] studied the slip flow of peristaltic transport of a magneto-Newtonian fluid through a porous medium with heat transfer. Also Mekhheimer and Abd elmaboud [7] studied the influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus. Furthermore, Radhakrishnamacharya and Srinivasulu [6] investigated the influence of wall properties on peristaltic transport with heat transfer.

Since it is well known that physiological fluids behave like non-Newtonian fluids, we'll study the heat transfer in peristaltic flow of Oldroyd fluid, as a viscoelastic fluid, in an asymmetric channel with peristaltic waves of different amplitudes and phase traveling on its walls. The problem is formulated and analyzed using perturbation series on the wave number as a parameter. Because of the complexity of the governing equation, long wavelength approximation is used to obtain analytic expressions for the axial velocity, the temperature and the heat transfer coefficient. Moreover, the influences of Reynolds number, phase difference, Weissenberg number, wave number, Prandtl number, Eckert number and channel width on axial velocity, temperature and heat transfer coefficient have been discussed.

2 Formulation and Analysis

Let us consider a two-dimensional flow of Oldroyd fluid in an asymmetric channel. We assume sinusoidal wave train moving with speed c along the channel walls. The upper wall is maintained at temperature T_0 and the Lower wall at T_1. Let $d_1 + d_2$ be the channel width. Taking $\overline{X}$ and $\overline{Y}$ as rectangular coordinates, the geometry of the wall surfaces are defined as

$$\overline{h}_1\left(\overline{X},\overline{t}\right) = \overline{d}_1 + \overline{a}_1 \cos\left[\frac{2\pi}{\lambda}(\overline{X} - c\,\overline{t}\,)\right], \qquad \text{upper wall,} \tag{1}$$

$$\overline{h}_2\left(\overline{X},\overline{t}\right) = -\overline{d}_2 - \overline{a}_2 \cos\left[\frac{2\pi}{\lambda}\left(\overline{X} - c\overline{t}\,\right) + \phi\right], \qquad \text{lower wall,} \tag{2}$$

Where $\overline{a}_1$, $\overline{a}_2$ are the amplitudes of the waves, λ is the wavelength, t is the time and $\phi\,(0 \le \phi \le \pi\,)$ is the phase difference. Moreover, $\overline{a}_1$, $\overline{a}_2$, $\overline{d}_1$, $\overline{d}_2$ and ϕ satisfy the following inequality, Mishra and Rao [10],

$$\overline{a}_1^2 + \overline{a}_2^2 + 2\overline{a}_1\,\overline{a}_2 \cos\phi \le \left(\overline{d}_1 + \overline{d}_2\,\right)^2.$$

The constitutive equation for Oldroyd fluid is

$$\tau_{ij} + \Gamma\left[\frac{\partial \tau_{ij}}{\partial t}\left(g^{kk} g_{ii} g_{jj}\right)^{\frac{1}{2}} v_k \frac{\partial}{\partial x^k}(\sqrt{g^{ii} g^{jj}}\,\tau_{ij}) - \sqrt{g^{kk} g_{jj}}\,\tau_{kj}\frac{\partial}{\partial x_k}\left(\sqrt{g^{ii}}\,v_i\right) - \sqrt{g^{kk} g_{ii}}\,\tau_{ik}\frac{\partial}{\partial x_k}\left(\sqrt{g^{jj}}\,v_j\right)\right] = -\mu\dot{\gamma}_{i\,j} \tag{3}$$

where g_{ii} and g^{jj} are respectively the diagonal components of covariant and contravariant metric tensor, i, j =1,2.

Taking moving coordinates $(\overline{x}, \overline{y})$, (wave frame), which travel in the $\overline{X}$ -direction with the same speed, as the wave, the unsteady flow in the laboratory frame $\left(\overline{X},\overline{Y}\right)$ can be treated as steady, Shapiro *et al*. [2]. The coordinates frame are related through

$$\overline{x} = \overline{X} - c\overline{t}, \qquad \overline{y} = \overline{Y}\,, \tag{4}$$

$$\overline{u} = \overline{U} - c, \qquad \overline{v} = \overline{V}\,, \tag{5}$$

where $\left(\overline{U},\overline{V}\right)$ and $\left(\overline{u},\overline{v}\right)$ are the velocity components in the corresponding coordinate system.

Equations of motion in the moving coordinates are, Rathy [14]

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{6}$$

$$\rho\left[\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right]=-\frac{\partial \bar{p}}{\partial \bar{x}}-\frac{\partial \bar{\tau}_{\bar{x}\bar{x}}}{\partial \bar{x}}-\frac{\partial \bar{\tau}_{\bar{y}\bar{x}}}{\partial \bar{y}} \tag{7}$$

$$\rho\left[\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right]=-\frac{\partial \bar{p}}{\partial \bar{y}}-\frac{\partial \bar{\tau}_{\bar{x}\bar{y}}}{\partial \bar{x}}-\frac{\partial \bar{\tau}_{\bar{y}\bar{y}}}{\partial \bar{y}} \tag{8}$$

The constitutive equations of Oldroyd fluid are, Rathy[14]

$$\bar{\tau}_{\bar{x}\bar{x}}+\Gamma\left[\bar{u}\frac{\partial \bar{\tau}_{\bar{x}\bar{x}}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\tau}_{\bar{x}\bar{x}}}{\partial \bar{y}}-2\bar{\tau}_{\bar{x}\bar{x}}\frac{\partial \bar{u}}{\partial \bar{x}}-2\bar{\tau}_{\bar{x}\bar{y}}\frac{\partial \bar{u}}{\partial \bar{y}}\right]=-\mu\,\bar{\dot{\gamma}}_{\bar{x}\bar{x}} \tag{9}$$

$$\bar{\tau}_{\bar{x}\bar{y}}+\Gamma\left[\bar{u}\frac{\partial \bar{\tau}_{\bar{x}\bar{y}}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\tau}_{\bar{x}\bar{y}}}{\partial \bar{y}}-\bar{\tau}_{\bar{y}\bar{y}}\frac{\partial \bar{u}}{\partial \bar{y}}-\bar{\tau}_{\bar{x}\bar{x}}\frac{\partial \bar{v}}{\partial \bar{x}}\right]=-\mu\bar{\dot{\gamma}}_{\bar{x}\bar{y}} \tag{10}$$

$$\bar{\tau}_{\bar{y}\bar{y}}+\Gamma\left[\bar{u}\frac{\partial \bar{\tau}_{\bar{y}\bar{y}}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\tau}_{\bar{y}\bar{y}}}{\partial \bar{y}}-2\bar{\tau}_{\bar{x}\bar{y}}\frac{\partial \bar{v}}{\partial \bar{x}}-2\bar{\tau}_{\bar{y}\bar{y}}\frac{\partial \bar{v}}{\partial \bar{y}}\right]=-\mu\bar{\dot{\gamma}}_{\bar{y}\bar{y}}, \tag{11}$$

The differential energy equation is

$$\rho C_v\left[\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right]=k\nabla^2\bar{T}+\bar{\tau}_{\bar{x}\bar{x}}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{\tau}_{\bar{y}\bar{y}}\frac{\partial \bar{v}}{\partial \bar{y}}+\bar{\tau}_{\bar{x}\bar{y}}\left[\frac{\partial \bar{u}}{\partial \bar{y}}+\frac{\partial \bar{v}}{\partial \bar{x}}\right], \tag{12}$$

where $\bar{p}$ is the pressure, $\bar{\tau}_{\bar{x}\bar{x}}$, $\bar{\tau}_{\bar{x}\bar{y}}$, $\bar{\tau}_{\bar{y}\bar{y}}$ are components of the extra stress tensor, Γ is relaxation time, μ is the coefficient of viscosity of the fluid, σ is the electrical conductivity, C_v is the specific heat at constant volume, ν is kinematic viscosity, k is thermal conductivity of the fluid, $\bar{T}$ is temperature and $\bar{\dot{\gamma}}_{\bar{x}\bar{x}}, \bar{\dot{\gamma}}_{\bar{x}\bar{y}}, \bar{\dot{\gamma}}_{\bar{y}\bar{y}}$ are components of strain-rate tensor and given by

$$\bar{\dot{\gamma}}_{\bar{x}\bar{x}}=2\frac{\partial \bar{u}}{\partial \bar{x}},\quad \bar{\dot{\gamma}}_{\bar{y}\bar{y}}=2\frac{\partial \bar{v}}{\partial \bar{y}},\quad \text{and } \bar{\dot{\gamma}}_{\bar{x}\bar{y}}=\left[\frac{\partial \bar{u}}{\partial \bar{y}}+\frac{\partial \bar{v}}{\partial \bar{x}}\right], \tag{13}$$

Introducing the non-dimensional variables and parameters

$$x=\frac{\bar{x}}{\lambda}, y=\frac{\bar{y}}{\bar{d}_1}, u=\frac{\bar{u}}{c}, v=\frac{\lambda\bar{v}}{\bar{d}_1 c}, t=\frac{c}{\lambda}\bar{t}, h_1=\frac{\bar{h}_1}{\bar{d}_1}, h_2=\frac{\bar{h}_2}{\bar{d}_1}, \tau_{ij}=\frac{\bar{d}_1}{c\mu}\bar{\tau}_{ij},$$

$$\delta=\frac{\bar{d}_1}{\lambda}, R_e=\frac{\rho c\bar{d}_1}{\mu}, W_i=\frac{c\Gamma}{d_1}, p=\frac{\bar{d}_1^{\,2}}{c\lambda\mu}\bar{p}, \dot{\gamma}_{ij}=\frac{\bar{d}_1}{c}\bar{\dot{\gamma}}_{ij}, \theta=\frac{\bar{T}-T_0}{T_1-T_0},$$

$$P_r=\frac{C_v\mu}{k}, E=\frac{c^2}{C_v(T_1-T_0)}, \tag{14}$$

where δ is the wave number, R_e is the Reynolds number, W_i is the Weissenberg number, θ is the dimensionless temperature, P_r is the Prandtl number and E is the Eckert number, equations (6-12) are reduced to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{15}$$

$$R_e\delta\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}-\delta\frac{\partial\tau_{xx}}{\partial x}-\frac{\partial\tau_{yx}}{\partial y}, \tag{16}$$

$$R_e\delta^3\left[\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\right]=-\frac{\partial p}{\partial y}-\delta^2\frac{\partial\tau_{xy}}{\partial x}-\delta\frac{\partial\tau_{yy}}{\partial y}, \tag{17}$$

$$\tau_{xx}+Wi\left[\delta\left(u\frac{\partial\tau_{xx}}{\partial x}+v\frac{\partial\tau_{xx}}{\partial y}-2\tau_{xx}\frac{\partial u}{\partial x}\right)-2\tau_{xy}\frac{\partial u}{\partial y}\right]=-2\delta\frac{\partial u}{\partial x}, \tag{18}$$

$$\tau_{xy}+Wi\left[\delta\left(u\frac{\partial\tau_{xy}}{\partial x}+v\frac{\partial\tau_{xy}}{\partial y}-\delta\tau_{xx}\frac{\partial v}{\partial x}\right)-\tau_{yy}\frac{\partial u}{\partial y}\right]=-\left[\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right], \tag{19}$$

$$\tau_{yy}+Wi\left[\delta\left(u\frac{\partial\tau_{yy}}{\partial x}+v\frac{\partial\tau_{yy}}{\partial y}-2\tau_{yy}\frac{\partial v}{\partial y}-2\delta\tau_{xy}\frac{\partial v}{\partial x}\right)\right]=-2\delta\frac{\partial v}{\partial x}\ . \tag{20}$$

$$R_e\delta\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\frac{1}{P_r}\left[\frac{\partial^2\theta}{\partial y^2}+\delta^2\frac{\partial^2\theta}{\partial x^2}\right]+\delta E\left[\tau_{xx}\frac{\partial u}{\partial x}+\tau_{yy}\frac{\partial v}{\partial y}\right]+$$
$$E\tau_{xy}\left[\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right] \tag{21}$$

Eliminating the pressure from equation (16), (17) we get

$$R_e\delta\left[v\frac{\partial^2 u}{\partial y^2}-u\frac{\partial^2 v}{\partial y^2}+\delta^2\left(u\frac{\partial^2 v}{\partial x^2}-v\frac{\partial^2 u}{\partial x^2}\right)\right]=$$
$$\delta^2\frac{\partial^2\tau_{xy}}{\partial x^2}-\frac{\partial^2\tau_{yx}}{\partial y^2}+\delta\left(\frac{\partial^2\tau_{yy}}{\partial x\partial y}-\frac{\partial^2\tau_{xx}}{\partial y\partial x}\right) \tag{22}$$

3 Rate of Volume Flow and Boundary Conditions

The dimensional volume flow rate in laboratory frame is

$$Q=\int_{\bar{h}_2(\bar{X},\bar{t})}^{\bar{h}_1(\bar{X},\bar{t})}\bar{U}\left(\bar{X},\bar{Y},\bar{t}\right)d\bar{Y}\ , \tag{23}$$

in which $\bar{h}_1$ and $\bar{h}_2$ are function of $\bar{X}$, $\bar{t}$

The rate of volume flow in the wave frame is given by

$$q = \int_{\bar{h}_2(\bar{x})}^{\bar{h}_1(\bar{x})} \bar{u}(\bar{x}, \bar{y})\, d\,\bar{y}\,, \tag{24}$$

where $\bar{h}_1$ and $\bar{h}_2$ are function of $\bar{x}$ only.

On substituting Eqs. (4) and (5) into Eq. (23) and then integrating, we have

$$Q = q + c\bar{h}_1(\bar{x}) - c\bar{h}_2(\bar{x}) \tag{25}$$

The time- mean flow over a period T is defined as

$$\bar{Q} = \frac{1}{T}\int_0^T Q\, d\,\bar{t} \tag{26}$$

Substituting (25) into (26), and integrating, we get

$$\bar{Q} = q + c\bar{d}_1 + c\bar{d}_2\,. \tag{27}$$

Defining the dimensionless mean flows θ and F as follow

$$\theta = \frac{\bar{Q}}{c\bar{d}_1} \text{ and } F = \frac{q}{c\bar{d}_1}\,, \tag{28}$$

Equation (27) may be written as

$$\theta = F + 1 + d\,, \tag{29}$$

where

$$F = \int_{h_2(x)}^{h_1(x)} u\, dy\,. \tag{30}$$

We note that $h_1(x)$ and $h_2(x)$ represent the dimensionless forms of the wall surfaces and defined by

$$h_1(x) = 1 + a\cos 2\pi x \quad \text{and} \quad h_2(x) = -d - b\cos(2\pi x + \phi), \tag{31}$$

where $a = \frac{\bar{a}_1}{\bar{d}_1}$, $b = \frac{\bar{a}_2}{\bar{d}_2}$, and $d = \frac{\bar{d}_2}{\bar{d}_1}$,

and a, b, d and ϕ satisfy the relation

$$a^2 + b^2 + 2ab\cos\phi \le (1 + d)^2\,. \tag{32}$$

The boundary conditions for the dimensionless in the wave frame is

$$u = -1, \quad \text{at} \quad y = h_1(x), \tag{33-a}$$

$$u = -1. \quad \text{at} \quad y = h_2(x). \tag{33-b}$$

$$v = \frac{-dh_1}{dx}, \quad \text{at} \quad y = h_1(x), \tag{34-a}$$

$$v = \frac{dh_2}{dx}. \quad \text{at} \quad y = h_2(x). \tag{34-b}$$

$$\theta = 0, \quad \text{at} \quad y = h_1(x), \tag{35-a}$$

$$\theta = 1. \quad \text{at} \quad y = h_2(x). \tag{35-b}$$

4 Method of Solution

We expand the following in a power series of small parameter δ as follows

$$u = u_0 + \delta u_1 + O(\delta^2),$$
$$v = v_0 + \delta v_1 + O(\delta^2),$$
$$\theta = \theta_0 + \delta \theta_1 + O(\delta^2),$$
$$\tau_{xx} = \tau_{xx}^{(0)} + \delta \tau_{xx}^{(1)} + O(\delta^2),$$
$$\tau_{xy} = \tau_{xy}^{(0)} + \delta \tau_{xy}^{(1)} + O(\delta^2),$$
$$\tau_{yy} = \tau_{yy}^{(0)} + \delta \tau_{yy}^{(1)} + O(\delta^2),$$
$$F = F_0 + \delta F_1 + O(\delta^2),$$
$$\frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + \delta \frac{\partial p_1}{\partial x} + O(\delta^2). \tag{36}$$

The use of expansions (36) with Eqs. (15), (16), (17), (18), (19), (20),(21),(22) and boundary conditions (33),(34),(35) we get

system of order zero

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0, \tag{37}$$

$$\frac{\partial p_0}{\partial x} = -\frac{\partial \tau_{yx}^{(0)}}{\partial y}, \tag{38}$$

$$\tau_{xx}^{(0)} - 2Wi\,\tau_{xy}^{(0)} \frac{\partial u_0}{\partial y} = 0, \tag{39}$$

$$\tau_{xy}^{(0)} - Wi\,\tau_{yy}^{(0)} \frac{\partial u_0}{\partial y} = -\frac{\partial u_0}{\partial y}, \tag{40}$$

$$\tau_{yy}^{(0)} = 0, \tag{41}$$

$$0 = \frac{1}{P_r}\left[\frac{\partial^2 \theta_0}{\partial y^2}\right] + E\,\tau_{xy}^{(0)}\left[\frac{\partial u_0}{\partial y}\right], \tag{42}$$

$$\frac{\partial^2 \tau_{xy}^{(0)}}{\partial y^2} = 0\,. \tag{43}$$

With the dimensionless boundary conditions

$u_0 = -1,$ at $y = h_1(x),$ (44-a)

$u_0 = -1.$ at $y = h_2(x).$ (44-b)

$v_0 = -\frac{dh_1}{dx},$ at $y = h_1(x),$ (45-a)

$v_0 = \frac{dh_2}{dx}.$ at $y = h_2(x).$ (45-b)

$\theta_0 = 0\,,$ at $y = h_1(x),$ (46-a)

$\theta_0 = 1.$ at $y = h_2(x).$ (46-b)

The solution of (41),(40)and (38),subject the boundary conditions(44-a)and(44-b),is

$$u_0(x,y) = \frac{1}{2}\left[\frac{\partial p_0}{\partial x}\right]\left[y^2 - (h_1 + h_2)y + h_1 h_2\right] - 1 \tag{47}$$

The instantaneous volume flow rate, F_0 ,is given by

$$F_0 = \int_{h2}^{h_1} u_0\, dy = \frac{1}{12}\left[\frac{\partial p_0}{\partial x}\right]\left[h_2 - h_1\right]^3 - \left[h_1 - h_2\right] \tag{48}$$

The zero-order pressure gradient can be obtained by solving (48) for $\frac{dp_0}{dx}$,as

$$\frac{dp_0}{dx} = \frac{12\left(F_0 + h_1 - h_2\right)}{\left(h_2 - h_1\right)^3} \tag{49}$$

Substituting $\frac{dp_0}{dx}$ into(47),we obtain the form of u_0 as

$$u_0(x,y) = L_1 y^2 + L_2 y + L_3 \tag{50}$$

where $L_1 = \frac{6(F_0 + h_1 - h_2)}{(h_2 - h_1)^3}$, $L_2 = \frac{-6(F_0 + h_1 - h_2)(h_1 + h_2)}{(h_2 - h_1)^3}$,

and $L_3 = \frac{6(F_0 + h_1 - h_2)h_1 h_2}{(h_2 - h_1)^3} - 1.$ (51)

Substituting $u_0(x,y)$ from (50) into(37),and then solving the equation , subject the boundary condition(45-a),we obtain the form of v_0 as

$$v_0(x,y) = -\frac{1}{3}L_1^1 y^3 - \frac{1}{2}L_2^1 y^2 - L_3^1 y + G(x) \tag{52}$$

where $G(x) = -h_1^1 + \frac{1}{3}L_1^1 h_1^3 + \frac{1}{2}L_2^1 h_1^2 + L_3^1 h_1\,.$ (53)

Substituting $u_0(x,y)$ from (50)and(40) into (42),subject the boundary conditions(46-a)and(46-b),we obtain the form of θ_0 as

$$\theta_0(x,y) = B_1 y^4 + B_2 y^3 + B_3 y^2 + B_4 y + B_5 \qquad (54)$$

where $B_1 = \frac{1}{3}P_r E L_1^2, B_2 = \frac{2}{3}P_r E L_1 L_2, B_3 = \frac{1}{2}P_r E L_2^2,$

$$B_4 = \frac{1}{(h_2 - h_1)}\left[1 - P_r E\left[\frac{1}{3}L_1^2(h_2^4 - h_1^4) + \frac{2}{3}L_1 L_2(h_2^3 - h_1^3) + \frac{1}{2}L_2^2(h_2^2 - h_1^2)\right]\right],$$

and $B_5 = \frac{h_1}{h_2 - h_1}\left[-1 + P_r E h_2\left[\frac{1}{3}L_1^2(h_2^3 - h_1^3) + \frac{2}{3}L_1 L_2(h_2^2 - h_1^2) + \frac{1}{2}L_2^2(h_2 - h_1)\right]\right]$.(55)

System of order one

Equating the coefficients of δ on both sides in Eqs. (15), (16), (17), (18), (19), (20),(21),(22),(33),(34) and (35) we get

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \qquad (56)$$

$$R_e\left[\left(u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}\right)\right] = -\frac{\partial p_1}{\partial x} - \frac{\partial \tau_{xx}^{(0)}}{\partial x} - \frac{\partial \tau_{yx}^{(1)}}{\partial y}, \qquad (57)$$

$$\tau_{xx}^{(1)} + Wi\left[u_0\frac{\partial \tau_{xx}^{(0)}}{\partial x} + v_0\frac{\partial \tau_{xx}^{(0)}}{\partial y} - 2\tau_{xx}^{(0)}\frac{\partial u_0}{\partial x} - 2\tau_{xy}^{(0)}\frac{\partial u_1}{\partial y} - 2\tau_{xy}^{(1)}\frac{\partial u_0}{\partial y}\right] = -2\frac{\partial u_0}{\partial x}, \qquad (58)$$

$$\tau_{xy}^{(1)} + Wi\left[u_0\frac{\partial \tau_{xy}^{(0)}}{\partial x} + v_0\frac{\partial \tau_{xy}^{(0)}}{\partial y} - \tau_{yy}^{(0)}\frac{\partial u_1}{\partial y} - \tau_{yy}^{(1)}\frac{\partial u_0}{\partial y}\right] = -\frac{\partial u_1}{\partial y}, \qquad (59)$$

$$\tau_{yy}^{(1)} + Wi\left[u_0\frac{\partial \tau_{yy}^{(0)}}{\partial x} + v_0\frac{\partial \tau_{yy}^{(0)}}{\partial y} - 2\tau_{yy}^{(0)}\frac{\partial v_0}{\partial y}\right] = -2\frac{\partial v_0}{\partial x}, \qquad (60)$$

$$R_e\left[u_0\frac{\partial \theta_0}{\partial x} + v_0\frac{\partial \theta_0}{\partial y}\right] = \frac{1}{P_r}\left[\frac{\partial^2 \theta_1}{\partial y^2}\right] + E\left[\tau_{xx}^{(0)}\frac{\partial u_0}{\partial x} + \tau_{yy}^{(0)}\frac{\partial v_0}{\partial y}\right] +$$
$$E\left[\tau_{xy}^{(0)}\frac{\partial u_1}{\partial y} + \tau_{xy}^{(1)}\frac{\partial u_0}{\partial y}\right] \qquad (61)$$

$$R_e\left[v_0\frac{\partial^2 u_0}{\partial y^2} - u_0\frac{\partial^2 v_0}{\partial y^2}\right] = -\frac{\partial^2 \tau_{yx}^{(1)}}{\partial y^2} + \frac{\partial^2 \tau_{yy}^{(0)}}{\partial x \partial y} - \frac{\partial^2 \tau_{xx}^{(0)}}{\partial y \partial x}, \qquad (62)$$

with the boundary conditions

$$u_1 = 0 \quad \text{at} \quad y = h_1(x) \qquad (63\text{-a})$$

$u_1 = 0$ at $y = h_2(x)$ (63-b)

$v_1 = 0$ at $y = h_1(x)$ (64-a)

$v_1 = 0$ at $y = h_2(x)$ (64-b)

$\theta_1 = 0$ at $y = h_1(x)$ (65-a)

$\theta_1 = 0$ at $y = h_2(x)$ (65-b)

Substituting (41),(40),(39),(50),(52) into Eqs. (60), (59), (58), (57) and then solving the resulting system along the boundary conditions (63), we get velocity u_1 in the form

$$u_1(x,y) = \frac{A_1}{30}y^6 + \frac{A_2}{20}y^5 + \frac{A_3}{12}y^4 + \frac{A_4}{6}y^3 + \frac{1}{2}A_5 y^2 + \frac{1}{2}\left[\frac{\partial p_1}{\partial x}\right]y^2 +$$
$$\frac{y}{(h_2-h_1)}\left(\frac{A_1}{30}(h_1^6-h_2^6) + \frac{A_2}{20}(h_1^5-h_2^5) + \frac{A_3}{12}(h_1^4-h_2^4) + \frac{A_4}{6}(h_1^3-h_2^3) + \frac{A_5}{2}(h_1^2-h_2^2) +\right.$$
$$\left.\frac{1}{2}\left[\frac{\partial p_1}{\partial x}\right](h_1^2-h_2^2)\right) + \frac{h_1h_2}{(h_2-h_1)}\left(\frac{A_1}{30}(h_2^5-h_1^5) + \frac{A_2}{20}(h_2^4-h_1^4) + \frac{A_3}{12}(h_2^3-h_1^3) + \frac{A_4}{6}(h_2^2-h_1^2)\right.$$
$$\left. + \frac{A_5}{2}(h_2-h_1) + \frac{1}{2}\left[\frac{\partial p_1}{\partial x}\right](h_2-h_1)\right) \tag{66}$$

where

$$A_1 = \frac{1}{3}R_e L_1 L_1^1,\ A_2 = \frac{2}{3}\left[R_e(L_1^1 L_2) + W_i(8L_1 L_1^{11})\right],$$
$$A_3 = R_e(L_1^1 L_3 + \frac{1}{2}L_2^1 L_2 - L_3^1 L_1) + W_i(6L_1 L_2^{11} + 2L_2 L_1^{11} - 12 L_1 L_1^1),$$
$$A_4 = R_e\left(L_3 L_2^1 + 2L_1 G(x)\right) + Wi\left(8L_1 L_3^{11} + 2L_2 L_2^{11} - 4L_2 L_1^1 - 8L_1 L_2^1\right),$$
$$A_5 = R_e\left(L_3 L_3^1 + L_2 G(x)\right) + Wi\left(2L_1^1 L_3 - 3L_2 L_2^1 - 2L_1 L_3^1 - 4L_1 G^1(x) + 2L_2 L_3^{11}\right). \tag{67}$$

The instantaneous volume flow rate, F_1, is given by

$$F_1 = \int_{h2}^{h_1} u_1 dy = \frac{1}{12}\left[\frac{\partial p_1}{\partial x}\right](h_2-h_1)^3 + \frac{A_1}{420}(7h_1^6 h_2 - 5h_1^7 + 5h_2^7 - 7h_2^6 h_1) + \frac{A_2}{120}(3h_2 h_1^5 -$$
$$2h_1^6 + 2h_2^6 - 3h_1 h_2^5) + \frac{A_3}{120}(5h_1^4 h_2 - 3h_1^5 + 3h_2^5 - 5h_1 h_2^4) + \frac{A_4}{24}(2h_1^3 h_2 - h_1^4 + h_2^4 - 2h_{12}^{\ 3})$$
$$+ \frac{A_5}{12}(h_2-h_1)^3. \tag{68}$$

The one-order pressure gradient can be obtained by solving (68) for $\frac{dp_1}{dx}$, as

$$\frac{dp_1}{dx}=\frac{1}{70(h_2-h_1)^3}\begin{bmatrix}840F_1+10A_1h_1^7+14A_1h_1h_2^6-10A_1h_2^7-14A_1h_1^6h_2\\+14A_2h_1^6-21A_2h_1^5h_2-14A_2h_2^6+21A_2h_1h_2^5\\+21A_3h_1^5-35A_3h_1^4h_2-21A_3h_2^5+35A_3h_1h_2^4\\+35A_4h_1^4-70A_4h_2h_1^3-35A_4h_2^4+70A_4h_2^3h_1\\+70A_5h_1^3-210A_5h_1^2h_2+210A_5h_1h_2^2-70A_5h_2^3\end{bmatrix}, \quad (69)$$

Substituting $\frac{dp_1}{dx}$ into(66),we obtain the form of u_1 as

$$u_1(x,y)=\frac{A_1}{30}y^6+\frac{A_2}{20}y^5+\frac{A_3}{12}y^4+\frac{A_4}{6}y^3+\frac{A_5}{2}y^2+\frac{1}{2}G_1(x)y^2+G_2(x)y+G_3(x) \quad (70)$$

where

$$G_1(x)=\frac{dp_1}{dx},$$

$$G_2(x)=\frac{\eta_1}{60(h_2-h)_1}-\frac{(h_1+h_2)G_1(x)}{2},$$

where

$$\eta_1=2A_1(h_1^6-h_2^6)+3A_2(h_1^5-h_2^5)+5A_3(h_1^4-h_2^4)+10A_4(h_1^3-h_2^3)+30A_5(h_1^2-h_2^2).$$

and $G_3(x)=\frac{h_1h_2\eta_2}{60(h_2-h_1)}+\frac{h_1h_2}{2}G_1(x).$

where

$$\eta_2=2A_1(h_2^5-h_1^5)+3A_2(h_2^4-h_1^4)+5A_3(h_2^3-h_1^3)+10A_4(h_2^2-h_1^2)+30A_5(h_2-h_1) \quad (71)$$

Substituting $u_1(x,y)$ from (70) into(56),and then solving the equation , subject the boundary condition(64-a),we obtain the form of v_1 as

$$v_1(x,y)=\frac{-A_1^1y^7}{210}-\frac{A_2^1y^6}{120}-\frac{A_3^1y^5}{60}-\frac{A_4^1y^4}{24}-\frac{A_5^1y^3}{6}-\frac{G_1^1(x)y^3}{6}-\frac{G_2^1(x)y^2}{2}$$
$$-G_3^1(x)y+N(x). \quad (72)$$

where

$$N(x)=\frac{A_1^1h_1^7}{210}+\frac{A_2^1h_1^6}{120}+\frac{A_3^1h_1^5}{60}+\frac{A_4^1h_1^4}{24}+\frac{A_5^1h_1^3}{6}+\frac{G_1^1(x)h_1^3}{6}+\frac{G_2^1(x)h_1^2}{2}+G_3^1(x)h_1 \quad (73)$$

Substituting(39),(40),(41),(50),(52),(54),(59)and(60) into (61),subject the boundary conditions(65-a)and(65-b),we obtain the form of θ_1 as

$$\theta_1(x,y)=\frac{M_1}{56}y^8+\frac{M_2}{42}y^7+\frac{M_3}{30}y^6+\frac{M_4}{20}y^5+\frac{M_5}{12}y^4+\frac{M_6}{6}y^3+\frac{M_7}{2}y^2+$$
$$\frac{1}{(h_2-h_1)}\left(\frac{M_1}{56}(h_1^8-h_2^8)+\frac{M_2}{42}(h_1^7-h_2^7)+\frac{M_3}{30}(h_1^6-h_2^6)+\frac{M_4}{20}(h_1^5-h_2^5)+\frac{M_5}{12}(h_1^4-h_2^4)\right.$$

$$+\frac{M_6}{6}(h_1^3-h_2^3)+\frac{M_7}{2}(h_1^2-h_2^2)\Bigg)y+\frac{h_1h_2}{(h_2-h_1)}\Bigg(\frac{M_1}{56}(h_2^7-h_1^7)+\frac{M_2}{42}(h_2^6-h_1^6)+$$

$$\frac{M_3}{30}(h_2^5-h_1^5)+\frac{M_4}{20}(h_2^4-h_1^4)+\frac{M_5}{12}(h_2^3-h_1^3)+\frac{M_6}{6}(h_2^2-h_1^2)+\frac{M_7}{2}(h_2-h_1)\Bigg). \quad (74)$$

where

$$M_1=R_eP_r(L_1B_1^1-\frac{4}{3}B_1L_1^1)+EP_r(\frac{4}{5}L_1A_1),$$

$$M_2=R_eP_r(L_1B_2^1+L_2B_1^1-2L_2^1B_1-L_1^1B_2)+EP_r(L_1A_2+\frac{2}{5}L_2A_1)-W_iEP_r(\frac{8}{3}L_1^2L_1^{11}),$$

$$M_3=R_eP_r(L_1B_3^1+L_2B_2^1+L_3B_1^1-4L_3^1B_1-\frac{3}{2}L_2^1B_2-\frac{2}{3}L_1^1B_3)+EP_r(\frac{4}{3}L_1A_3+\frac{1}{2}L_2A_2)$$

$$-W_iEP_r(4L_1^2L_2^{11}+\frac{8}{3}L_1L_2L_1^{11}-\frac{16}{3}L_1^1L_1^2),$$

$$M_4=R_eP_r(L_1B_4^1+L_2B_3^1+L_3B_2^1-3L_3^1B_2-L_2^1B_3-\frac{1}{3}L_1^1B_4+4B_1G(x))+$$

$$EP_r(2L_1A_4+\frac{2}{3}L_2A_3)-W_iEP_r(8L_1^2L_3^{11}+\frac{2}{3}L_2^2L_1^{11}+4L_1L_2L_2^{11}-8L_2^1L_1^2-\frac{8}{3}L_1L_2L_1^1),$$

$$M_5=R_eP_r(L_1B_5^1+L_2B_4^1+L_3B_3^1-2L_3^1B_3-\frac{1}{2}L_2^1B_4+3B_2G(x))+EP_r(4L_1A_5+L_2A_4+$$

$$4L_1G_1(x))-W_iEP_r(4L_1L_3L_1^1-6L_1L_2L_2^1+8L_1L_2L_3^{11}-12L_1^2L_3^1+L_2^2L_2^{11}-8L_1^2G^1(x)),$$

$$M_6=R_eP_r(L_2B_5^1+L_3B_4^1-L_3^1B_4+2B_3G(x))+EP_r(4L_1G_2(x)+2L_2G_1(x)+2L_2A_5)$$

$$-W_iEP_r(2L_1L_3L_2^1+4L_1^2G(x)-8L_1L_2G^1(x)+2L_2L_3L_1^1-10L_1L_2L_3^1+2L_2^2L_3^{11}-L_2^1L_2^2),$$

$$M_7=R_eP_r(L_3B_5^1+B_4G(x))+EP_r(2L_2G_2(x))-W_iEP_r(L_2L_3L_2^1+2L_1L_2G(x)-2L_2^2G^1(x)$$

$$-2L_3^1L_2^2). \quad (75)$$

Substituting $u_0(x,y)$ from (50)and $u_1(x,y)$ from(70)into (36) for $u(x,y)$ we get the axial velocity u in the form

$$u(x,y)=L_1y^2+L_2y+L_3+\delta[\frac{A_1}{30}y^6+\frac{A_2}{20}y^5+\frac{A_3}{12}y^4+\frac{A_4}{6}y^3+\frac{A_5y^2}{2}+\frac{1}{2}G_1(x)y^2$$

$$+G_2(x)y+G_3(x)]. \quad (76)$$

Substituting $\theta_0(x,y)$ from (54)and $\theta_1(x,y)$ from(74)into (36) for $\theta(x,y)$ we have temperature θ in the form

$$\theta(x,y)=B_1y^4+B_2y^3+B_3y^2+B_4y+B_5+\delta\Bigg(\frac{M_1}{56}y^8+\frac{M_2}{42}y^7+\frac{M_3}{30}y^6+\frac{M_4}{20}y^5$$

$$+\frac{M_5}{12}y^4+\frac{M_6}{6}y^3+\frac{M_7}{2}y^2+\frac{1}{(h_2-h_1)}\left\{\frac{M_1}{56}(h_1^8-h_2^8)+\frac{M_2}{42}(h_1^7-h_2^7)+\right.$$

$$\left.\frac{M_3}{30}(h_1^6-h_2^6)+\frac{M_4}{20}(h_1^5-h_2^5)+\frac{M_5}{12}(h_1^4-h_2^4)+\frac{M_6}{6}(h_1^3-h_2^3)+\frac{M_7}{2}(h_1^2-h_2^2)\right\}y$$

$$+\frac{h_1h_2}{(h_2-h_1)}\left\{\frac{M_1}{56}(h_2^7-h_1^7)+\frac{M_2}{42}(h_2^6-h_1^6)+\frac{M_3}{30}(h_2^5-h_1^5)+\frac{M_4}{20}(h_2^4-h_1^4)+\right.$$

$$\left.\left.\frac{M_5}{12}(h_2^3-h_1^3)+\frac{M_6}{6}(h_2^2-h_1^2)+\frac{M_7}{2}(h_2-h_1)\right\}\right) \quad (77)$$

The heat transfer coefficient (Z) at the(upper) wall is given by

$$Z=h_{1x}\theta_y. \quad (78)$$

Substituting Eq. (77) in Eq. (78), we get

$$Z=h_{1x}\left(4B_1y^3+3B_2y^2+2B_3y+B_4+\delta\left\{\frac{M_1}{7}y^7+\frac{M_2}{6}y^6+\frac{M_3}{5}y^5+\frac{M_4}{4}y^4+\frac{M_5}{3}y^3\right.\right.$$

$$+\frac{M_6}{2}y^2+M_7y+\frac{1}{h_2-h_1}\left[\frac{M_1}{56}(h_1^8-h_2^8)+\frac{M_2}{42}(h_1^7-h_2^7)+\frac{M_3}{30}(h_1^6-h_2^6)+\frac{M_4}{20}(h_1^5-h_2^5)\right.$$

$$\left.\left.\left.+\frac{M_5}{12}(h_1^4-h_2^4)+\frac{M_6}{6}(h_1^3-h_2^3)+\frac{M_7}{2}(h_1^2-h_2^2)\right]\right\}\right). \quad (79)$$

Results and Discussion

It is seen form equations (76), (77) and (79) that we have obtained the axial velocity, the temperature and the heat transfer coefficient in explicit form. The effect of the physical parameters of the problem on the axial velocity and the temperature is seen through figures (1-10).

Fig (1) shows the effect of the Weissenberg number W_i on the axial velocity u at x=0.2, δ=0.02, E=1, P_r=1, R_e=10, a=0.5, b=0.7, d=1, θ=1, $\phi=\frac{\pi}{4}$ and (W_i=0, 0.04, 0.08). We observe that there is no effect for Weissenberg number W_i on the axial velocity as the curves coincide.

Fig (2) represents the graph of the axial velocity u versus y at x=0.2, E=1, P_r=1, W_i=0.01, R_e=10, a=0.5, b=0.7, d=1, θ=1, $\phi=\frac{\pi}{4}$ and (δ=0, 0.04, 0.08). It can be seen that an increase in the wave number δ increase the magnitude of the axial velocity u.

The effects of Reynolds number R_e on the axial velocity u is seen through Fig.(3) at x=0.2, δ=0.02, E=1, P_r=1, W_i=0.04, a=0.5, b=0.7, d=1, θ=1, $\phi=\frac{\pi}{4}$ and (R_e=0, 10, 20). It is noted that an increase in the Reynolds number R_e increase the magnitude of the axial velocity.

Fig.(4) gives the effects of phase difference ϕ on the axial velocity u at x=0.2, δ=0.02, E=1, P_r=1, W_i=0.04, a=0.3, b=0.5, d=0.7, θ=1, R_e =10 and (ϕ=0, $\frac{\pi}{6}$, $\frac{\pi}{4}$). We have observed that the magnitude of the axial velocity increase with increasing phase difference ϕ.

To see the effects of channel width d on the axial velocity we have prepared Fig.(5). Obviously, the magnitude of the axial velocity increase as d increases.

Fig.(6) displays the influence of the Reynolds number R_e on the temperature distribution for x=0, δ=0.02, E=1, P_r=1, W_i=0.04, a=0.7, b=1.2, d=2, θ=1, $\phi=\frac{\pi}{4}$ and (R_e=90, 80, 70). We note that the temperature θ increases as Reynolds number R_e increases.

Fig.(7) shows the effect of the Prandtl number P_r on the temperature for x=0, δ=0.02, E=1, R_e=50, W_i=0.04, a=0.5, b=0.7, d=2, θ=1, $\phi=\frac{\pi}{4}$ and (P_r=1.5, 1.4, 1.3). It is noticed that the temperature θ increases with increasing the Prandtl number P_r.

Fig.(8) depicts the variation of the fluid temperature θ with y, for different value of the Eckert number and at x=0, δ=0.02, P_r=1.5, W_i=0.04, a=0.5, b=0.7, d=2, θ=1, R_e=50, $\phi=\frac{\pi}{4}$ and (E=1, 0.8, 0.6). It is evident that the temperature θ increases with the increase in Eckert number E.

In Fig. (9), the temperature is graphed versus y at x=0, δ=0.02, E=1, P_r=2, W_i=0.04, a=0.5, b=0.7, θ=1, R_e=50, $\phi=\frac{\pi}{4}$ and (d=2, 1.9, 1.8). We note that the temperature distribution θ increase as the channel width d increases.

Fig.(10) is the graph of temperature distribution versus y for different values of phase difference ϕ and at x=0, δ=0.02, P_r=1, E=1, W_i=0.03, a=0.3, b=0.4, d=0.9, θ=1, and R_e=10. It can be noticed that an increase in the phase difference ϕ result increase in the magnitude of the temperature distribution.

Variations of the heat transfer coefficient (Z) at wall have been presented in Table(1), (a)-(d). The results reveal that the heat transfer coefficient (Z) increases with increasing upper wave amplitude a, Eckert number E, Reynolds number R_e and Weissenberg number W_i.

Concluding Remarks

In this paper we presented a theoretical approach to study the effect of heat transfer on peristaltic flow of viscoelastic fluid in an asymmetry channel. The governing equations of motion and energy are solved analytically using perturbation expansion on wave as a parameter. Furthermore, The effect of various values of parameters of interest on axial velocity, temperature and heat transfer coefficient are discussed numerically and explained graphically through figures (1-10). Moreover, the heat transfer coefficient is discussed through table(1). The main results can be summarized as follows :

•. There is no appreciable effect of Weissenberg number W_i on the axial velocity u.

•. The magnitude of the axial velocity increases with increasing wave number δ, Reynolds number R_e, phase difference ϕ and channel width d.

•. The temperature θ increases with increasing Reynolds number R_e, Prandtl number P_r, Eckert number E and channel width d .

•. The magnitude of the temperature increases with an increase in phase difference ϕ.

•. The heat transfer coefficient (Z) increases with increasing upper wave amplitude a, Eckert number E, Reynolds number R_e and Weissenberg number W_i.

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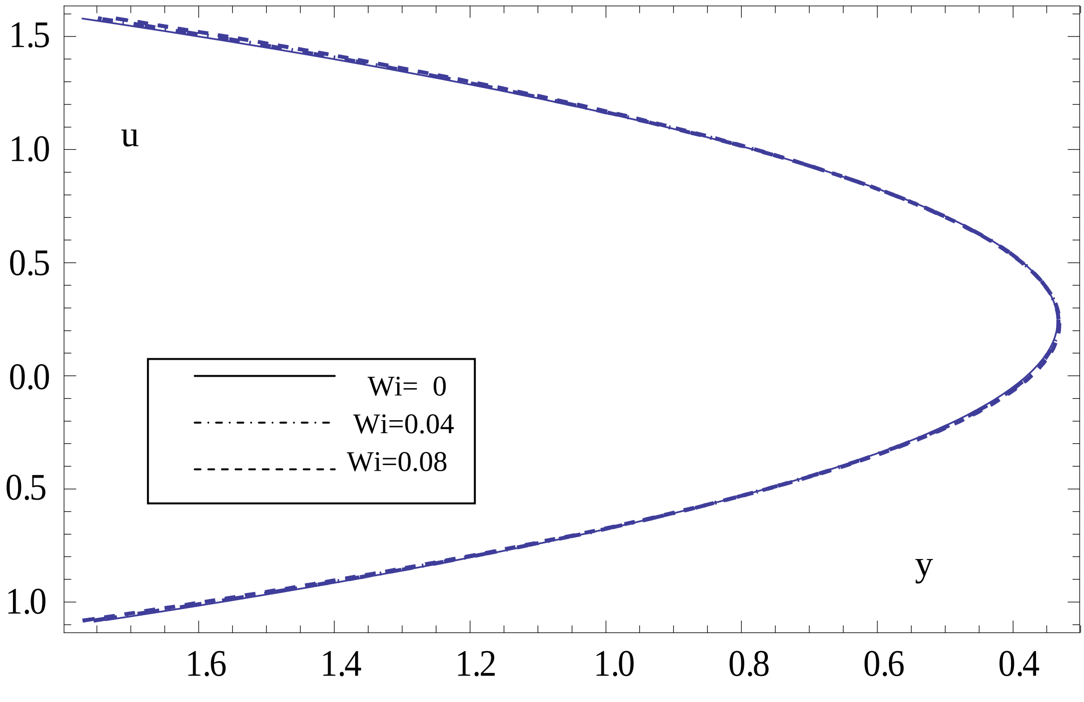

Fig.1. Axial velocity versus y at x=0.2, δ =0.02,E=1, P_r =1, R_e =10,a=0.5,b=0.7,d=1, θ =1, $\phi = \frac{\pi}{4}$.

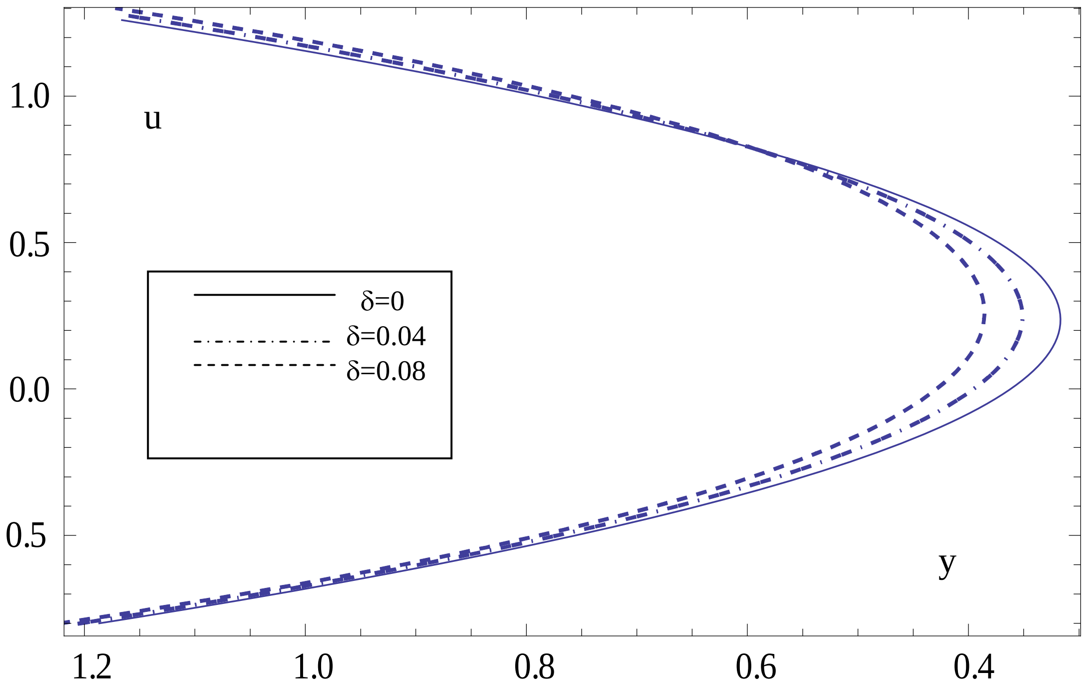

Fig.2.Axial velocity versus y at x =0.2 ,E=1, P_r =1,W=0.01, R_e =10,a=0.5,b=0.7,d=1, θ =1, $\phi = \frac{\pi}{4}$.

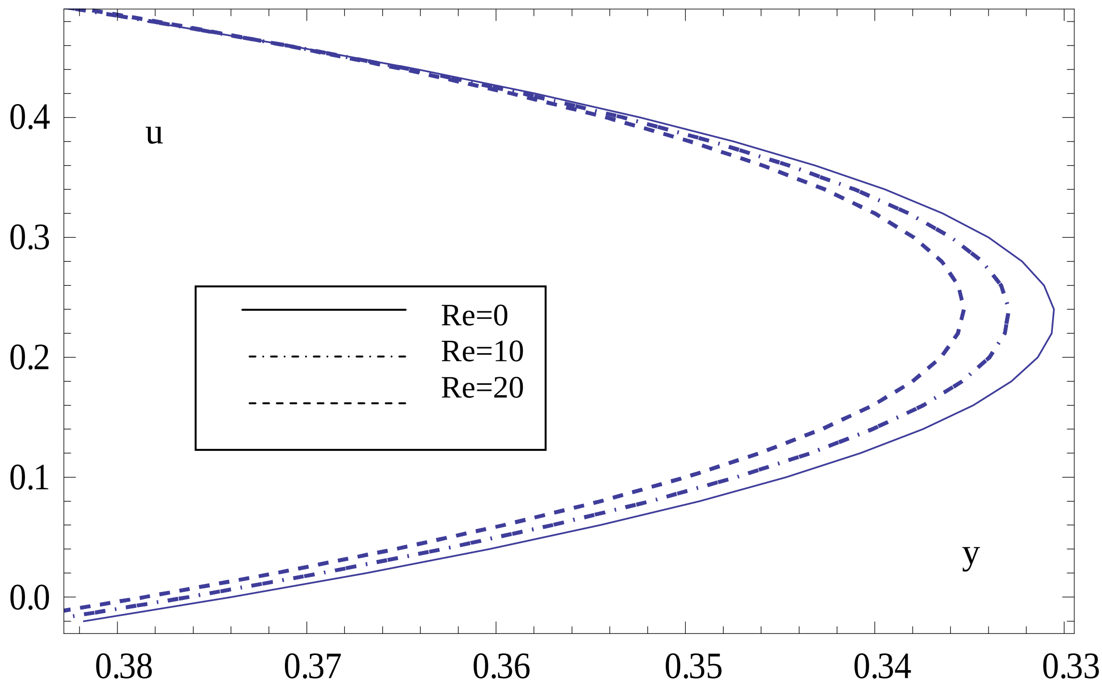

Fig.3.Axial velocity versus y at x =0.2 ,E=1, δ =0.02, P_r =1,W=0.04,a=0.5,b=0.7,d=1, θ =1, $\phi = \frac{\pi}{4}$.

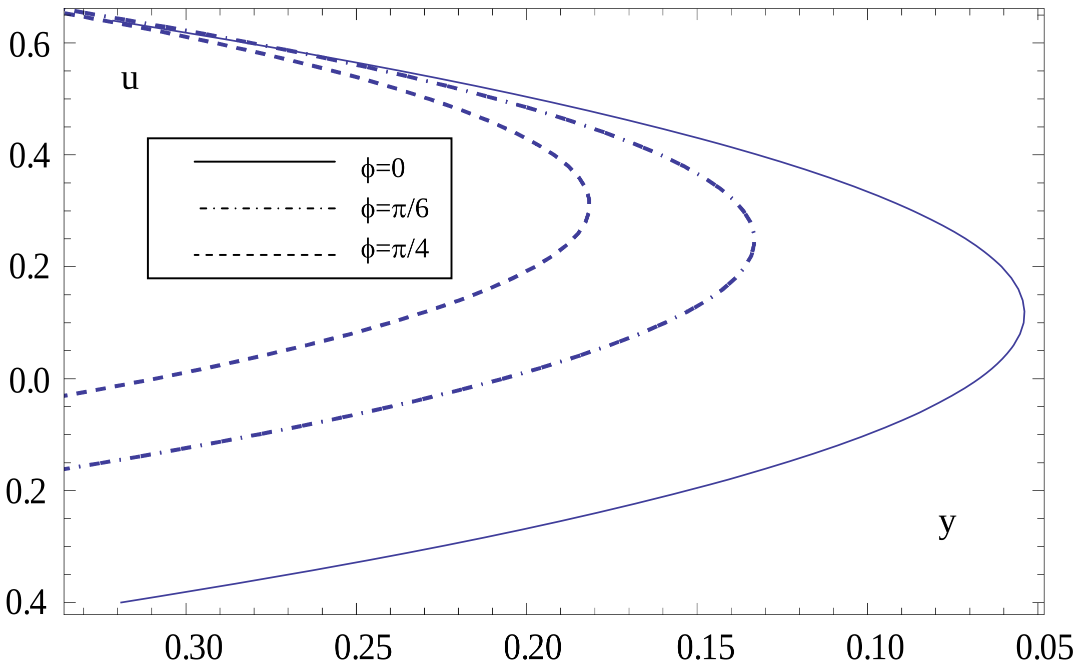

Fig.4.Axial velocity versus y at x =0.2 ,E=1, δ =0.02, P_r =1,W=0.04, R_e =10,a=0.3,b=0.5,d=0.7, θ =1.

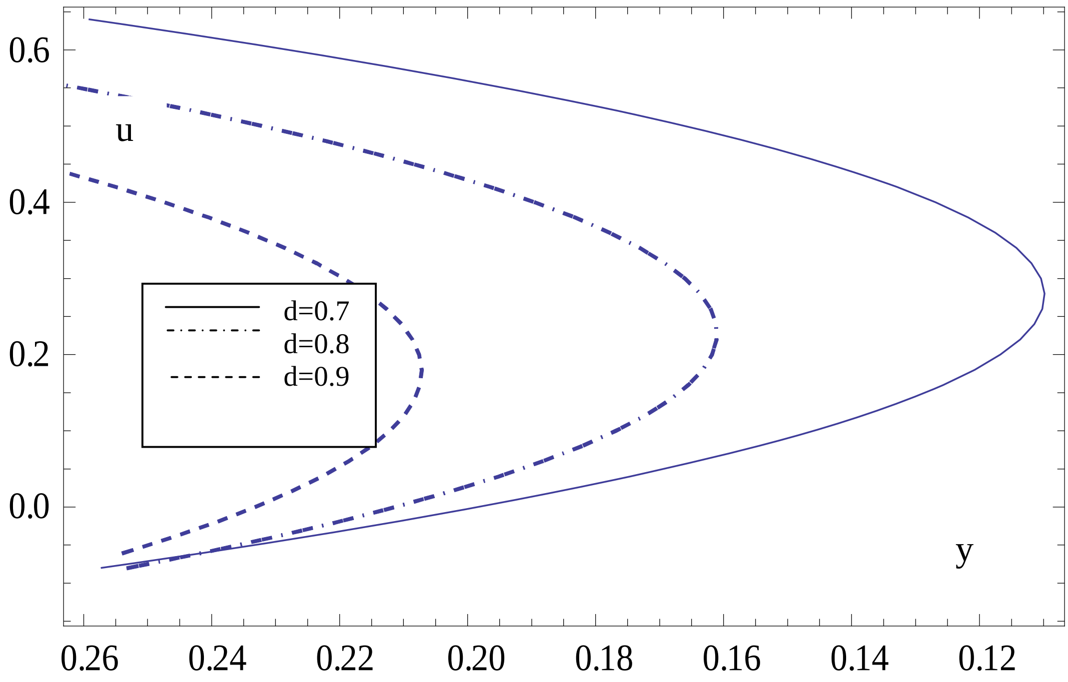

Fig.5.Axial velocity versus y at x =0.,E=1, δ =0.02, P_r =1,W=0.04,a=0.5,b=0.5, R_e =10, θ =1, $\phi = \frac{\pi}{4}$.

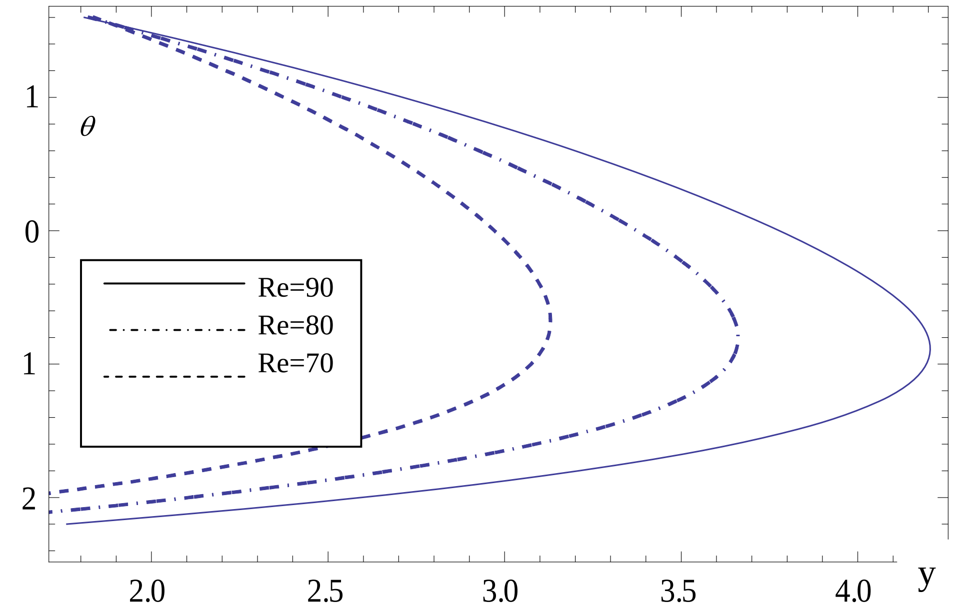

Fig.6.Temperature distribution y at x =0 ,E=1, δ =0.02, P_r =1,W=0.04,a=0.7,b=1.2,d=2, , θ =1, $\phi = \frac{\pi}{4}$

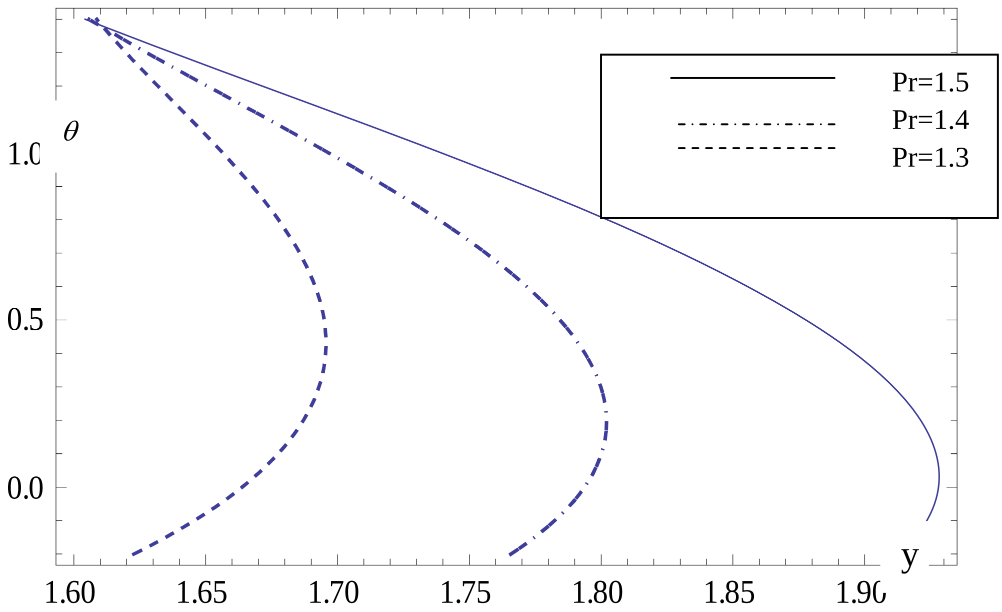

Fig.7.Temperature distribution y at x =0 ,E=1, δ =0.02, R_e =50,W=0.04,a=0.5,b=0.7,d=2, , θ =1, $\phi = \frac{\pi}{4}$

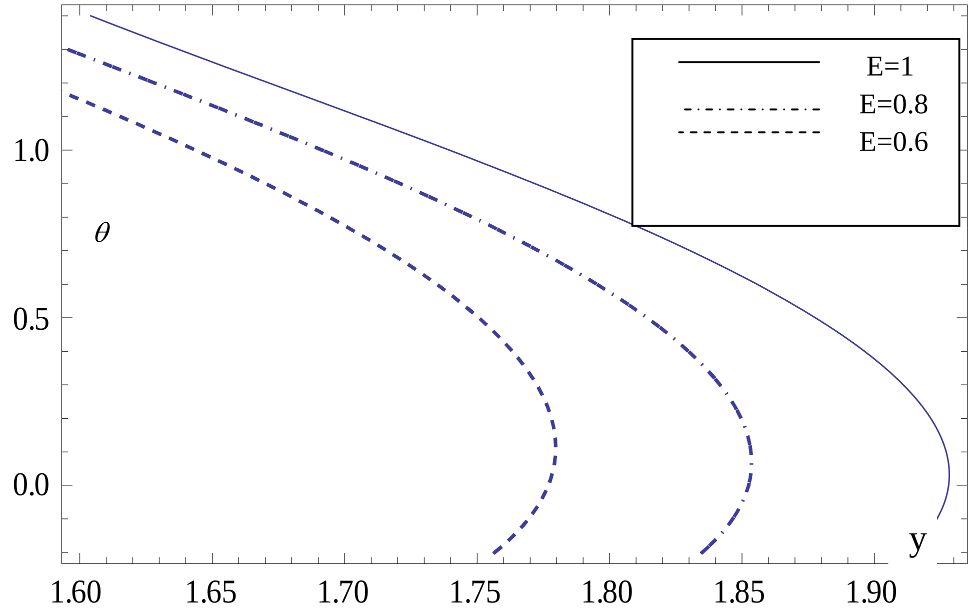

Fig .8.Temperature distribution y at x =0 , δ =0.02, P_r =1.5,W=0.04,a=0.5,b=0.7,d=2, R_e =50,

θ =1, $\phi = \dfrac{\pi}{4}$.

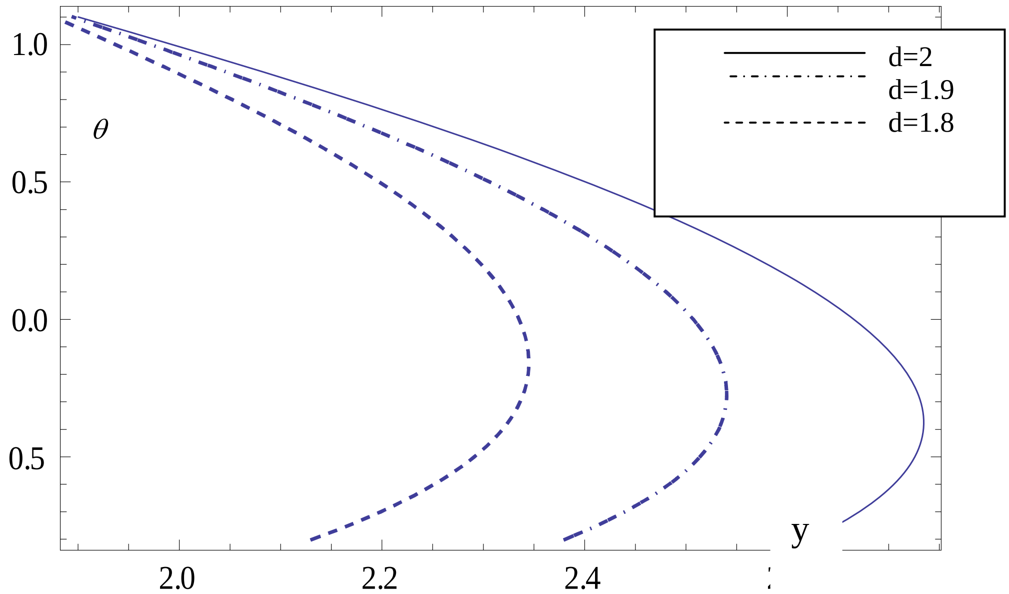

Fig .9.Temperature distribution y at x =0 ,E=1, δ =0.02, P_r =2,W=0.04,a=0.5,b=0.7, R_e =50,

θ =1, $\phi = \dfrac{\pi}{4}$

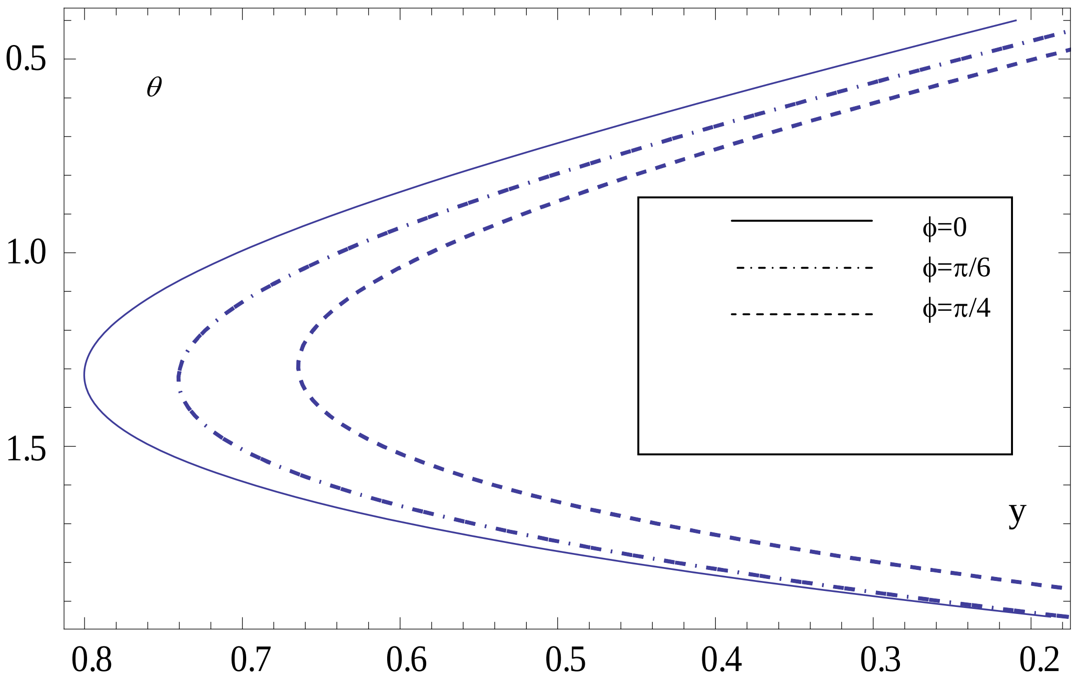

Fig.10.Temperature distribution y at x =0 ,E=1, δ =0.02, P_r =1,W=0.03,a=0.3,b=0.4,d=0.9, R_e =10, θ =1.

Table 1

Variation of heat transfer coefficient

(a): δ =0.02,E=1, P_r =1, W_i =0.04, R_e =10,b=1.2,d=1.5, θ =1, $\phi = \frac{\pi}{4}$

x	a			
	0.5	0.7	0.9	1.1
0.1	1.36228	1.38401	1.42048	1.47375
0.2	1.64078	1.67317	1.6991	1.71983
0.3	2.15405	2.72392	3.81008	5.80024

(b): δ =0.02, P_r =1,W=0.04, R_e =10,a=0.5,b=1.2,d=2, θ =1, $\phi = \frac{\pi}{4}$

x	E			
	1	2	3	4
0.1	1.21571	1.235	1.25429	1.27358
0.2	1.36437	1.41494	1.46551	1.51609
0.3	1.52971	1.57427	1.61882	1.66338

(c): δ =0.02, P_r =1,E=1,W=0.04 ,a=0.5,b=1.2,d=1.5, θ =1, $\phi = \frac{\pi}{4}$

x	R_e			
	10	2 0	3 0	40
0.1	1.36228	1.40381	1.44535	1.48689
0.2	1.64078	1.75637	1.87196	1.98755
0.3	2.15405	2.61615	3.07824	3.54034

(d): δ =0.02, P_r =1,E=1, R_e =10 ,a=0.7,b=1.2,d=1.5, θ =1, $\phi = \frac{\pi}{4}$

x	W			
	0.02	0.04	0.08	0.16
0.1	1.38303	1.38401	1.38599	1.38993
0.2	1.67104	1.67317	1.67741	1.6859
0.3	2.71924	2.72392	2.73297	2.75106

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