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A New Algorithm for Finding a Fuzzy Optimal Solution for Fuzzy Transportation Problems

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Abstract

A new algorithm namely, fuzzy zero point method is proposed for finding a fuzzy optimal solution for a fuzzy transportation problem where the transportation cost, supply and demand are trapezoidal fuzzy numbers. The optimal solution for the fuzzy transportation problem by the fuzzy zero point method is a trapezoidal fuzzy number. The solution procedure is illustrated with numerical example.

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1 Introduction

The transportation problem is one of the earliest applications of linear programming problems. Transportation models have wide applications in logistics and supply chain for reducing the cost. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [2] and Zadeh [19] introduced the notion of fuzziness.

A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. Since the transportation problem is essentially a linear programming problem, one straight forward idea is to apply the existing fuzzy linear programming techniques [3,4,9,10,12,14,15,17] to solve the fuzzy transportation problem. Unfortunately, most of the existing techniques [3,4,9,12,15,17] provide only crisp solutions for the fuzzy transportation problem. Julien [11] and Parra et al. [14] proposed a method for solving fuzzy transportation problems and also, to find the possibility distribution of the objective value of the transportation problem provided all the inequality constraints are of ‘‘≤’’ types or ‘‘≥’’ types. Chanas et al. [6] developed a method for solving transportation problems with fuzzy supplies and demands via the parametric programming technique using the Bellman–Zadeh criterion [2]. Chanas and Kuchta [5] introduced a method for solving a transportation problem with fuzzy cost coefficients by transforming the given problem to a bicriterial transportation problem with crisp objective function which provides only crisp solution to the given transportation problem. Liu and Kao [16] developed a solution procedure for computing the fuzzy objective value of the fuzzy transportation problem, where at least one of the parameters are fuzzy numbers using the Zadeh's extension principle [18, 19,20]. Nagoor Gani and Abdul Razak [13] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers using a parametric approach.

In this paper, we propose a new algorithm namely, fuzzy zero point method for finding a fuzzy optimal solution for a fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers. The optimal solution for the fuzzy transportation problem by the fuzzy zero point method is a trapezoidal fuzzy number. The solution procedure is illustrated with numerical example.

When we use the fuzzy zero point method for finding an optimal solution for a fuzzy transportation problem, we have the following advantages.

- We do not use linear programming techniques.
- We do not use goal and parametric programming techniques.
- The optimal solution is a fuzzy number and
- The proposed method is very easy to understand and to apply.

2 Fuzzy number and Fuzzy transportation problem

We need the following mathematical orientated definitions of fuzzy set, fuzzy number and membership function which can be found in Zadeh [16] .

Definition 2.1 Let A be a classical set and $\mu_A(x)$ be a function from A to [0,1]. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined by

$$A^* = \{ (x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1] \}.$$

Definition 2.2 A real fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is a fuzzy subset from the real line R with the membership function $\mu_{\tilde{a}}(a)$ satisfying the following conditions:

(i) $\mu_{\tilde{a}}(a)$ is a continuous mapping from R to the closed interval [0, 1],

(ii) $\mu_{\tilde{a}}(a) = 0$ for every $a \in (-\infty, a_1]$,

(iii) $\mu_{\tilde{a}}(a)$ is strictly increasing and continuous on $[a_1, a_2]$,

(iv) $\mu_{\tilde{a}}(a) = 1$ for every $a \in [a_2, a_3]$,

(v) $\mu_{\tilde{a}}(a)$ is strictly decreasing and continuous on $[a_3, a_4]$ and

(vi) $\mu_{\tilde{a}}(a) = 0$ for every $a \in [a_4, +\infty]$.

Definition 2.3 A fuzzy number $\tilde{a}$ is a trapezoidal fuzzy number denoted by (a_1, a_2, a_3, a_4) where a_1, a_2, a_3 and a_4 are real numbers and its member ship function $\mu_{\tilde{a}}(x)$ is given below.

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & \text{for } x \le a_1 \\ (x - a_1)/(a_2 - a_1) & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } a_2 \le x \le a_3 \\ (a_4 - x)/(a_4 - a_3) & \text{for } a_3 \le x \le a_4 \\ 0 & \text{for } x \ge a_4 \end{cases}$$

We need the following definitions of the basic arithmetic operators on fuzzy trapezoidal numbers based on the function principle which can be found in [6,7].

Definition 2.4 Let (a_1, a_2, a_3, a_4) and (b_1, b_2, b_3, b_4) be two trapezoidal fuzzy numbers. Then

(i) $(a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$.

(ii) $(a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$.

(iii) $k(a_1, a_2, a_3, a_4) = (ka_1, ka_2, ka_3, ka_4)$, for $k \ge 0$.

(iv) $k(a_1, a_2, a_3, a_4) = (ka_4, ka_3, ka_2, ka_1)$, for $k < 0$.

(v) $(a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (t_1, t_2 \cdot t_3, t_4)$

where $t_1 = \text{minimum}\{ a_1b_1, a_1b_4, a_4b_1, a_4b_4 \}$;

$t_2 = \text{minimum}\{ a_2b_2, a_2b_3, a_3b_2, a_3b_3 \}$;

$t_3 = \text{maximum}\{ a_2b_2, a_2b_3, a_3b_2, a_3b_3 \}$ and

$t_4 = \text{maximum}\{ a_1b_1, a_1b_4, a_4b_1, a_4b_4 \}$.

We need the following definition of the defuzzified value of a fuzzy number based on graded mean integration method which can be found in [7].

Definition 2.5 If $\tilde{a} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number, then the defuzzified value or the ordinary (crisp) number of $\tilde{a}$, a is given below.

$$a = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}.$$

We need the following definitions of ordering on the set of the fuzzy numbers based on the magnitude of a fuzzy number which can be found in [1].

Definition 2.6 The magnitude of the trapezoidal fuzzy number $\tilde{u} = (x_\circ - \sigma, x_\circ, y_\circ, y_\circ + \beta)$ with parametric form $\tilde{u} = (\underline{u}(r), \overline{u}(r))$ where $\underline{u}(r) = x_\circ - \sigma + \sigma r$ and $\overline{u}(r) = y_\circ + \beta - \beta r$ is defined as

$$Mag(\tilde{u}) = \frac{1}{2}\left(\int_0^1 (\underline{u}(r) + \overline{u}(r) + x_\circ + y_\circ) r dr \right),$$

where $r \in [0,1]$.

Remark 2.1 The magnitude of the trapezoidal fuzzy number $\tilde{u} = (a,b,c,d)$ is given by

$$Mag(\tilde{u}) = \frac{a + 5b + 5c + d}{12}.$$

Definition 2.7 Let $\tilde{u}$ and $\tilde{v}$ be two trapezoidal fuzzy numbers. The ranking of $\tilde{u}$ and $\tilde{v}$ by the $Mag(.)$ on E, the set of trapezoidal fuzzy numbers is defined as follows:

(i) $Mag(\tilde{u}) > Mag(\tilde{v})$ if and only if $\tilde{u} \succ \tilde{v}$;

(ii) $Mag(\tilde{u}) < Mag(\tilde{v})$ if and only if $\tilde{u} \prec \tilde{v}$ and

(iii) $Mag(\tilde{u}) = Mag(\tilde{v})$ if and only if $\tilde{u} \approx \tilde{v}$.

Definition 2.8 The ordering $\succeq$ and $\preceq$ between any two trapezoidal fuzzy numbers $\tilde{u}$ and $\tilde{v}$ are defined as follows:

(i) $\tilde{u} \succeq \tilde{v}$ if and only if $\tilde{u} \succ \tilde{v}$ or $\tilde{u} \approx \tilde{v}$ and

(ii) u $\preceq$ v if and only if $\tilde{u} \prec \tilde{v}$ or $\tilde{u} \approx \tilde{v}$.

Note 2.1 (i) $\tilde{u} = (a,b,c,d) \approx \tilde{0}$ if and only if $Mag(\tilde{u}) = 0$;

(ii) $\tilde{u} = (a,b,c,d) \succeq \tilde{0}$ if and only if $Mag(\tilde{u}) \geq 0$ and

(iii) $\tilde{u} = (a,b,c,d) \preceq \tilde{0}$ if and only if $Mag(\tilde{u}) \leq 0$.

Definition 2.9 Let $\{ \tilde{a}_i, i = 1,2,...,n \}$ be a set of trapezoidal fuzzy numbers. If $Mag(\tilde{a}_k) \leq Mag(\tilde{a}_i)$, for all i, then the fuzzy number $\tilde{a}_k$ is the minimum of $\{ \tilde{a}_i, i = 1,2,...,n \}$.

Definition 2.10 Let $\{\ \tilde{a}_i, i = 1,2,...,n\ \}$ be a set of trapezoidal fuzzy numbers. If $Mag(\tilde{a}_t) \geq Mag(\tilde{a}_i)$, for all i, then the fuzzy number $\tilde{a}_t$ is the maximum of $\{\ \tilde{a}_i, i = 1,2,...,n\ \}$.

Consider the following fuzzy transportation problem (FTP) having fuzzy costs , fuzzy sources and fuzzy demands,

(FTP) Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i \text{ , for i = 1,2,…,m} \quad (1)$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j \text{ , for j = 1,2,…,n} \quad (2)$$

$$\tilde{x}_{ij} \succeq 0, \text{ for i = 1,2,…,m and j = 1,2,…,n,} \quad (3)$$

where

m = the number of supply points;

n = the number of demand points;

$\tilde{x}_{ij} \approx (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4)$ is the uncertain number of units shipped from supply point i to demand point j;

$\tilde{c}_{ij} \approx (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4)$ is the uncertain cost of shipping one unit from supply point i to the demand point j ;

$\tilde{a}_i \approx (a_i^1, a_i^2, a_i^3, a_i^4)$ is the uncertain supply at supply point i and

$\tilde{b}_j \approx (b_j^1, b_j^2, b_j^3, b_j^4)$ is the uncertain demand at demand point j.

The above problem can put in a table namely, fuzzy transportation table given below.

				Supply
	$\tilde{c}_{11}$	$\cdots$	$\tilde{c}_{1n}$	$\tilde{b}_1$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\tilde{c}_{m1}$	$\cdots$	$\tilde{c}_{mn}$	$\tilde{b}_n$
Demand	$\tilde{a}_1$	$\cdots$	$\tilde{a}_n$	

3 Fuzzy zero point method

We, now introduce a new algorithm called the fuzzy zero point method for finding a fuzzy optimal solution for fuzzy transportation problems in single stage.

The zero point method proceeds as follows.

Step 1. Construct the fuzzy transportation table for the given fuzzy transportation problem and then, convert it into a balanced one, if it is not.

Step 2. Subtract each row entries of the fuzzy transportation table from the row minimum.

Step 3. Subtract each column entries of the resulting fuzzy transportation table after using the Step 2. from the column minimum.

Step 4. Check if each column fuzzy demand is less than to the sum of the fuzzy supplies whose reduced costs in that column are fuzzy zero. Also, check if each row fuzzy supply is less than to sum of the column fuzzy demands whose reduced costs in that row are fuzzy zero. If so, go to Step 7. (Such reduced table is called the allotment table).
If not, go to Step 5.

Step 5. Draw the minimum number of horizontal lines and vertical lines to cover all the fuzzy zeros of the reduced fuzzy transportation table such that some entries of row(s) or / and column(s) which do not satisfy the condition of the Step4. are not covered.

Step 6: Develop the new revised reduced fuzzy transportation table as follows:
(i) Find the smallest entry of the reduced fuzzy cost matrix not covered by any lines.
(ii) Subtract this entry from all the uncovered entries and add the same to all entries lying at the intersection of any two lines .
and then, go to Step 4.

Step 7. Select a cell in the reduced fuzzy transportation table whose reduced cost is the maximum cost. Say (α,β) . If there are more than one, then select anyone.

Step 8. Select a cell in the α -row or/ and $\beta-$column of the reduced fuzzy transportation table which is the only cell whose reduced cost is fuzzy zero and then, allot the maximum possible to that cell. If such cell does not occur for the maximum value, find the next maximum so that such a cell occurs. If such cell does not occur for any value, we select any cell in the reduced fuzzy transportation table whose reduced cost is fuzzy zero.

Step 9. Reform the reduced fuzzy transportation table after deleting the fully used fuzzy supply points and the fully received fuzzy demand points and also, modify it to include the not fully used fuzzy supply points and the not fully received fuzzy demand points.

Step 10. Repeat Step 7 to Step 9 until all fuzzy supply points are fully used and all fuzzy demand points are fully received.

Step 11. This allotment yields a fuzzy solution to the given fuzzy transportation problem.

Now, we prove the following theorems which are used to derive the solution to a fuzzy transportation problem obtained by the fuzzy zero point method is a fuzzy optimal solution to the fuzzy transportation problem.

Theorem 3.1 Any optimal solution to the fuzzy problem (P_1) where

$$(P_1)\ \text{Minimize}\quad \tilde{z}^* = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j)\otimes\tilde{x}_{ij}$$

subject to (1) to (3) are satisfied ,

where $\tilde{u}_i$ and $\tilde{v}_j$ are some real trapezoidal fuzzy numbers , is an optimal solution to the problem (P) where

$$(P)\ \text{Minimize}\ \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\otimes\tilde{x}_{ij}$$

subject to (1) to (3) are satisfied .

Proof. Now, $\tilde{z}^* \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\otimes\tilde{x}_{ij}\ \Theta\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{u}_i\otimes\tilde{x}_{ij}\ \Theta\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{v}_j\otimes\tilde{x}_{ij}$

$$\approx \tilde{z}\Theta\sum_{i=1}^{m}\tilde{u}_i\otimes\tilde{b}_i\ \Theta\sum_{j=1}^{n}\tilde{v}_j\otimes\tilde{a}_j\,.\ (\text{ from (1) and (2) })$$

Since $\sum_{i=1}^{m}\tilde{u}_i\otimes\tilde{b}_i$ and $\sum_{j=1}^{n}\tilde{v}_j\otimes\tilde{a}_j$ are independent of $\tilde{x}_{ij}$, for all i and j, we can conclude that any optimal solution to the problem (P_1) is also a fuzzy optimal solution to the problem (P).

Hence the theorem.

Theorem 3.2 If $\{\tilde{x}^\circ_{ij}$, i = 1,2,…,m and j =1,2,…,m $\}$ is a feasible solution to the problem (P) and $(\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j)\succeq\tilde{0}$, for all i and j where $\tilde{u}_i$ and $\tilde{v}_j$ are some real trapezoidal fuzzy numbers, such that the minimum $\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j)\otimes\tilde{x}_{ij}$ subject to (1) to (3) are satisfied, is fuzzy zero, then $\{\tilde{x}^\circ_{ij}$, i = 1,2,…,m and j =1,2,…,m $\}$ is a fuzzy optimum solution to the problem (P).

Proof. From the Theorem 3.1, the result follows.

Hence the theorem.

Now, we prove that the solution to a fuzzy transportation problem obtained by the fuzzy zero point method is a fuzzy optimal solution to the fuzzy transportation problem.

Theorem 3.3 A solution obtained by the zero point method for a fuzzy transportation problem with equality constraints (P) is a fuzzy optimal solution for the fuzzy transportation problem (P).

Proof. We, now describe the fuzzy zero point method in detail.

We construct the fuzzy transportation table $[\tilde{c}_{ij}]$ for the given fuzzy transportation problem and then, convert it into a balanced one if it is not balanced.

Let $\tilde{u}_i$ be the minimum of i-th row of the table $[\tilde{c}_{ij}]$. Now, we subtract $\tilde{u}_i$ from the i-th row entries so that the resulting table is $[\tilde{c}_{ij}\Theta\tilde{u}_i]$.

Let $\tilde{v}_j$ be the minimum of j-th column of the resulting table $[\tilde{c}_{ij}\Theta\tilde{u}_i]$. Now, we subtract $\tilde{v}_j$ from the j-th column entries so that the resulting table is $[\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j]$. It may be noted that $\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j \succeq \tilde{0}$, for all i and j and each row and each column of the resulting table $[\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j]$ has atleast one fuzzy zero entry.

Each column fuzzy demand of the resulting table $[\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j]$ is less than to the sum of the fuzzy supply points whose reduced costs in the column are fuzzy zero. Further, each row fuzzy supply of the resulting table $[\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j]$ is less than to sum of the column fuzzy demand points whose reduced costs in the row are fuzzy zero (If not so, as per direction given in the Step 5 and 6 in the zero point method, we can make it that). The current resulting table is the allotment table.

We find a cell in the allotment table $[\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j]$ whose reduced cost is maximum. Say (α,β). We allot the maximum possible to a cell in the α-row or/ and $\beta-$column which is the only cell in the α-row or/ and $\beta-$column whose reduced cost is fuzzy zero. The resulting fuzzy transportation table is reformed after deleting the fully used fuzzy supply points and the fully received fuzzy demand points. Also, the not fully used fuzzy supply points and the not fully received fuzzy demand points are modified. We repeat the above said procedure till the total fuzzy supply are fully used and the total fuzzy demand are fully received.

Finally, we have a solution $\{\tilde{x}_{ij}$, i = 1,2,…,m and j = 1,2,…,n $\}$ for the reduced fuzzy transportation problem whose cost matrix is $[\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j]$ such that $\tilde{x}_{ij} \approx \tilde{0}$ for $\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j \succeq \tilde{0}$ and $\tilde{x}_{ij} \succ \tilde{0}$ for $\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j \approx \tilde{0}$.

Therefore, the minimum $\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}\Theta\tilde{u}_i\Theta\tilde{v}_j)\otimes\tilde{x}_{ij}$ subject to (1) to (3) are satisfied, is fuzzy zero. Thus, by the Theorem 3.2, the solution $\{\tilde{x}_{ij}$, i = 1,2,…,m and j = 1,2,…,n $\}$ is a fuzzy optimal solution to the fuzzy transportation problem (P). Hence the theorem.

4 Numerical Example

The proposed method called the fuzzy zero method is illustrated by the following example.

Example 4.1 Consider the following fuzzy transportation problem.

					Supply
	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Now, the total fuzzy supply, $\widetilde{S} = (6,17,21,32)$ and the total fuzzy demand, $\widetilde{D} = (8,17,21,30)$. Since $Mag(\widetilde{S}) = Mag(\widetilde{D})$, the given problem is a balanced one.

Now, using the Step 2 to the Step 3 of the fuzzy zero point method, we have the following reduced fuzzy transportation table.

					Supply
	$\widetilde{0}$	(-3,0,2,5)	(-4,1,5,10)	(-3,2,6,12)	(1,6,7,12)
	(-2,0,2,5)	$\widetilde{0}$	$\widetilde{0}$	$\widetilde{0}$	(0,1,2,3)
	$\widetilde{0}$	(-3,2,4,9)	(-5,2,6,13)	(-5,1,5,11)	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Now, using the Step 4 to the Step 6 of the fuzzy zero point method , we have the following allotment table.

					Supply
	(-23,-5,7,25)	$\widetilde{0}$	$\widetilde{0}$	(-44,-10,14,49)	(1,6,7,12)
	(-37,-6,16,48)	(-9,-1,5,13)	$\widetilde{0}$	(-23,-5,7,25)	(0,1,2,3)
	$\widetilde{0}$	(-33,-7,9,34)	$\widetilde{0}$	$\widetilde{0}$	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Now, using the allotment rules of the fuzzy zero point method, we have the allotment.

					Supply
		(1,5,6,10)	**(-9,0,2,11)**		(1,6,7,12)
			(0,1,2,3)		(0,1,2,3)
	(5,7,8,10)		**(-9,-1,3,11)**	**(1,2,3,4)**	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Therefore, the fuzzy optimal solution for the given fuzzy transportation problem is $\tilde{x}_{12} = (1,5,6,10)$, $\tilde{x}_{13} = (-9,0,2,11)$, $\tilde{x}_{23} = (0,1,2,3)$, $\tilde{x}_{31} = (5,7,8,10)$, $\tilde{x}_{23} = (-9,-1,3,11)$ and $\tilde{x}_{34} \approx (1,2,3,4)$ with the fuzzy objective value $\tilde{z} = (-274, 58, 188, 575)$ and the crisp value of the optimum fuzzy transportation cost for the problem, z is 132.17.

5 Conclusion

Some previous studies have provided an optimal solution for fuzzy transportation problems which is not fuzzy number, but it is a crisp value. If the obtained results are crisp values, then it might lose some helpful information. The fuzzy zero point method provides that the optimal value of the objective function and shipping units are fuzzy trapezoidal fuzzy numbers for the fuzzy transportation problem with the unit shipping costs, the supply quantities and the demand quantities are trapezoidal fuzzy numbers. This method is a systematic procedure, both easy to understand and to apply and also, it can serve as an important tool for the decision makers when they are handling various types of logistic problems having fuzzy parameters.

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