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A Low-Cost Calibrated Magnetometer Based on a Finite Solenoid Field Standard

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Abstract

We report the design, calibration, and experimental validation of a low-cost magnetic field measurement system based on a finite solenoid used as a magnetic field standard and a linear Hall-effect sensor. The setup enables quantitative axial magnetic field measurements using a geometry- and current-based calibration, avoiding the need for commercial reference magnetometers. The axial field generated by the solenoid is modeled using the exact finite-length expression and experimentally verified over a controlled range of currents. Linearity, repeatability, and spatial uniformity along the solenoid axis are systematically investigated. A complete uncertainty analysis is presented following standard metrological practice. Results show agreement between experiment and theory within the combined standard uncertainty, demonstrating a practical methodology for applied magnetometry in laboratory and educational contexts.

Keywords: Magnetometry; Hall-effect sensor; finite solenoid; calibration; uncertainty analysis; applied physics instrumentation

1 Introduction

Accurate and quantitatively reliable magnetic field measurements are essential in applied and educational laboratories, motivating the development of low-cost calibration methodologies based on well-defined physical field standards.

The solenoid provides a particularly attractive magnetic field source due to its well-understood electromagnetic behavior. Although the infinite-solenoid approximation is frequently used for qualitative discussions, real solenoids have finite length and exhibit significant edge effects. The exact finite-solenoid magnetic field, derived from the Biot–Savart law, provides a rigorous description suitable for quantitative experimental validation [1, 2, 9].

Hall-effect sensors offer an inexpensive and robust means of magnetic field detection and are widely used in applied magnetometry [4, 5]. However, their quantitative use requires careful calibration, offset correction, and uncertainty evaluation. In this work, a finite solenoid is used as a magnetic field standard to calibrate a linear Hall-effect sensor using only directly measurable electrical and geometrical quantities, following a traceable approach grounded in first-principles electromagnetism.

The objective of this study is twofold: to experimentally validate the finite-solenoid magnetic field model and to demonstrate a practical calibration methodology suitable for applied and educational laboratories.

2 Theoretical Model of the Magnetic Field of a Finite Solenoid

Consider a solenoid of length L, radius R, and total number of turns N, carrying a steady current I. The solenoid axis coincides with the z-axis, and the origin is located at its geometrical center. The axial magnetic field component is given by

$$B_z(z) = \frac{\mu_0 N I}{2L}\left[\frac{z+\frac{L}{2}}{\sqrt{R^2+(z+\frac{L}{2})^2}} - \frac{z-\frac{L}{2}}{\sqrt{R^2+(z-\frac{L}{2})^2}}\right]. \qquad (1)$$

At the solenoid center ($z = 0$), this expression reduces to

$$B_0 = \frac{\mu_0 N I}{2\sqrt{R^2+(L/2)^2}}. \qquad (2)$$

3 Experimental Setup

3.1 Solenoid Construction and Geometrical Characterization

The magnetic field source consists of a cylindrical air-core solenoid wound with insulated copper wire on a rigid, non-magnetic former. The solenoid geometry is defined by its total length L, mean radius R, and total number of turns N, all of which are directly measured prior to the experiments. The solenoid is powered by a stable DC current supply and operated within a current range chosen to avoid significant Joule heating, ensuring stable magnetic field conditions throughout the measurements.

3.2 Current Supply, Hall-Effect Sensor, and Measurement Protocol

A stable DC current is supplied to the solenoid using a laboratory power supply operated in current-controlled mode. The actual current value is independently monitored using either a digital multimeter or a calibrated shunt resistor, ensuring accurate current determination throughout the measurements.

The axial magnetic field component is measured using a linear Hall-effect sensor. Sensors of this type provide an output voltage proportional to the magnetic field and are well suited for low-cost magnetometry applications [4, 5]. The sensor is powered by a regulated voltage source and mounted on a non-magnetic holder that allows precise positioning along the solenoid axis. Careful mechanical adjustment is performed to ensure that the sensitive axis of the Hall sensor is aligned with the solenoid axis.

Magnetic field measurements are carried out in two stages. First, the magnetic field at the center of the solenoid is measured as a function of the applied current in order to assess linearity and perform sensor calibration. Second, the axial magnetic field profile is obtained by translating the Hall sensor along the solenoid axis while maintaining a fixed current.

4 Sensor Calibration and Data Analysis

The Hall sensor output voltage is modeled as

$$V = V_{\text{off}} + SB, \tag{3}$$

where V_{off} is the offset voltage and S is the sensitivity. Calibration is performed using the theoretical magnetic field at the solenoid center computed from the finite-solenoid model described in Section 2.

Offset suppression is achieved through current inversion:

$$V_{\text{off}} = \frac{V(+I) + V(-I)}{2}, \tag{4}$$

while the sensitivity is obtained from

$$\frac{V(+I) - V(-I)}{2} = SB_0(|I|). \tag{5}$$

To suppress offset drift and background magnetic fields, measurements are performed for both positive and negative current directions. The offset voltage is obtained from the average of the two readings, while the sensitivity is determined from the slope of the voltage versus magnetic field relation.

Once calibrated, magnetic field values are obtained from

$$B_{\text{est}} = \frac{V - V_{\text{off}}}{S}. \tag{6}$$

5 Uncertainty Budget

A comprehensive uncertainty budget is constructed by identifying all significant sources of uncertainty affecting the magnetic field estimation, following standard uncertainty analysis practice and the Guide to the Expression of Uncertainty in Measurement (GUM) [7, 8]. The analysis is based on a measurement model that relates the magnetic field to experimentally accessible electrical and geometrical quantities, allowing a systematic identification of the dominant contributions.

Uncertainty contributions include sensor voltage resolution, offset determination, sensitivity calibration, current measurement, solenoid geometry, sensor positioning, angular alignment, and environmental magnetic field fluctuations. Each contribution is expressed as a standard uncertainty and propagated to obtain the combined standard uncertainty associated with each magnetic field measurement.

The combined standard uncertainty is obtained by propagating all relevant contributions. For the voltage-to-field conversion,

$$u^2_{\text{conv}}(B) = \left(\frac{1}{S}\right)^2 u^2(V) + \left(\frac{1}{S}\right)^2 u^2(V_{\text{off}}) + \left(\frac{B_{\text{est}}}{S}\right)^2 u^2(S). \tag{7}$$

This expression accounts for the uncertainties associated with the measured Hall voltage V, the offset voltage V_{off} determined through current reversal, and the calibrated sensor sensitivity S, with the corresponding sensitivity coefficients evaluated at the nominal operating conditions.

Additional terms account for current measurement, solenoid geometry, positioning, alignment, and environmental magnetic field fluctuations. These contributions are propagated in an analogous manner using the appropriate sensitivity coefficients and are included as additive terms in the overall uncertainty budget.

Although several input quantities entering the uncertainty budget are obtained using the same measurement instruments or experimental setup, potential correlations between them are expected to be small compared to the dominant uncertainty contributions. In particular, the measured Hall voltage and the offset voltage are acquired using the same voltmeter, and the current values used for offset determination and calibration are generated by the same current source. Within the resolution and repeatability of the present measurements, no statistically significant correlations were observed. In the absence of independent evidence of non-negligible correlation effects, covariance terms are therefore neglected in the uncertainty propagation, and all input quantities are treated as uncorrelated, in accordance with standard GUM practice [7, 8].

Under these assumptions, the resulting uncertainty budget provides a consistent and conservative estimate of the combined standard uncertainty associated with the magnetic field measurements.

6 Results and Discussion

6.1 Magnetic Field as a Function of Current

The measured magnetic field at the solenoid center exhibits a clear linear dependence on the applied current, as shown in Fig. 1.

The magnetic field at the center of the solenoid was measured as a function of the applied current over the range from $-1.50\,\mathrm{A}$ to $+1.50\,\mathrm{A}$ in steps of $0.25\,\mathrm{A}$. A linear fit to the data yields

$$B = (3.42 \pm 0.03) \times 10^{-3}\,\mathrm{T\,A^{-1}}\,I,$$

with a coefficient of determination $R^2 = 0.9996$. Residuals show no systematic dependence on current magnitude or sign.

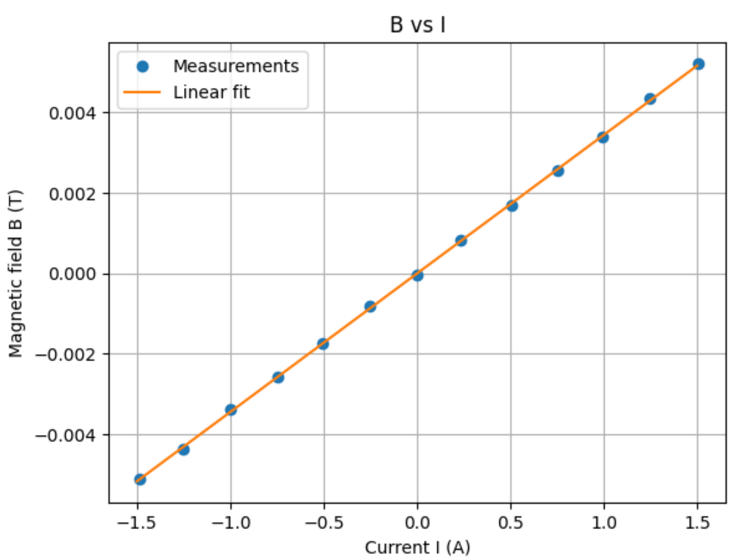

Figure 1: Magnetic field at the solenoid center as a function of current.

6.2 Repeatability

Repeatability tests performed at $I = 1.00\,\mathrm{A}$ yield a standard deviation $\sigma_B = 1.8 \times 10^{-5}\,\mathrm{T}$, corresponding to a relative repeatability of approximately $0.5\,\%$.

Repeated current sweeps demonstrate good repeatability, with short-term fluctuations significantly smaller than the combined standard uncertainty. This indicates stable sensor behavior and adequate current control during the measurements.

In metrological terms, the observed level of repeatability confirms that random short-term fluctuations of the magnetic field measurement are well controlled under steady operating conditions. The measured standard deviation is consistent with the expected noise performance of the Hall sensor and the stability of the current source, indicating that temporal drift and electronic noise play a secondary role in the present setup. Consequently, repeatability contributes only marginally to the combined standard uncertainty, supporting the validity of treating the measurements as statistically stable over the duration of each current sweep.

6.3 Axial magnetic field profile and uniformity region

The axial magnetic field profile was measured at a fixed current of $I = 1.20\,\mathrm{A}$ over the range $-80\,\mathrm{mm} \leq z \leq +80\,\mathrm{mm}$. The experimental data closely follow the theoretical prediction of the finite-solenoid model, with larger deviations near the solenoid ends, where the axial field gradient is strongest.

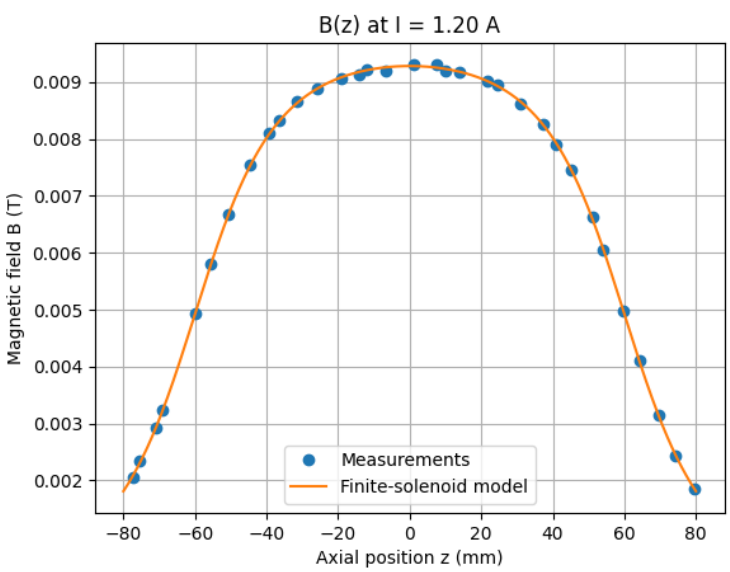

Figure 2: Axial magnetic field profile at $I = 1.20\,\mathrm{A}$ compared with the finite-solenoid model.

The axial magnetic field profile shown in Fig.2 reveals a well-defined central region where the field remains approximately uniform. Within this interval, the measured variations are small and exhibit no systematic trend with position, indicating that the residual deviations are dominated by measurement uncertainty rather than by geometric or alignment effects. This behavior confirms that the central section of the solenoid provides a suitable operating region for calibration and precision measurements, while deviations observed toward the ends are consistent with the expected finite-length effects of the coil geometry.

From the axial magnetic field profile, a central region of approximately uniform magnetic field is identified. Within this region, the relative field variation remains below 1 %, making it suitable for sensor calibration and precision magnetic field measurements. In particular, for $|z| \leq 25\,\text{mm}$ the relative field variation does not exceed 1 %.

6.4 Uncertainty Budget

The uncertainty budget summarized in Table 1 provides a quantitative breakdown of the dominant contributions affecting the magnetic field measurement at a central position of the solenoid for a supply current of $I = 1.20\,\text{A}$. The results show that the combined standard uncertainty is primarily governed by the calibration of the sensor sensitivity and by the uncertainty associated with the supplied current, while contributions arising from voltage measurement resolution, offset determination, and environmental magnetic field fluctuations remain comparatively small.

Geometrical factors, including sensor positioning along the solenoid axis and angular alignment with respect to the magnetic field direction, contribute at an intermediate level and reflect the practical limitations of mechanical alignment in the experimental setup. The relative balance of contributions indicates that further reductions in the overall uncertainty would mainly require improvements in sensitivity calibration and current control, whereas refinements in voltage resolution or environmental shielding would yield only marginal gains under the present operating conditions.

7 Conclusion

In this work, a low-cost magnetic field measurement system based on a finite solenoid and a Hall-effect sensor has been designed, calibrated, and experimentally validated. The solenoid provides a well-defined magnetic field that can be accurately modeled using first-principles electromagnetism, enabling sensor calibration without the need for commercial reference magnetometers.

Table 1: Uncertainty budget for a central-field measurement at $I = 1.20\,\mathrm{A}$.

Source	$u(B)$ (T)	Contribution (%)
Voltage measurement	5×10^{-6}	0.15
Offset voltage	6×10^{-6}	0.18
Sensitivity calibration	2.0×10^{-5}	0.59
Current measurement	1.5×10^{-5}	0.44
Positioning	1.2×10^{-5}	0.35
Angular alignment	8×10^{-6}	0.24
Environmental fields	5×10^{-6}	0.15
Combined standard uncertainty	3.2×10^{-5}	0.9

Experimental results demonstrate excellent linearity of the magnetic field with current, good repeatability, and close agreement between measured and theoretical axial field profiles within the combined standard uncertainty. A systematic uncertainty budget highlights the dominant sources of measurement uncertainty and provides clear guidance for improving accuracy.

The proposed methodology offers a practical and accessible approach to applied magnetometry and is readily adaptable to educational laboratories and low-cost instrumentation applications. Future work will focus on extending the setup to two-dimensional field mapping and time-dependent magnetic field measurements.

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