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Quasiparticle-Phonon Coupling Plus Rotor Model for Low-Lying States of the Neutron-Rich Sr, Zr, Mo and Ru Isotopes

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Abstract

The systematic of low-lying states for odd nuclei of the A~100 mass of transitional region is studied using the quasiparticle-phonon coupling plus rotor model (QPRM). The individual excitation is retained in a deformed average field of Nilsson and a monopole pairing interaction. The collectif vibrational motion is represented by including the quadrupole phonon term given from the Tamm-Dancoff Approximation. The two effects of recoil and Coriolis forces are included with the assumption of a symmetric rotation motion. To determine the intrinsic states of an odd-nucleus we adopted an exact diagonalization in the basis of both 1-quasiparticle and quasiparticle-phonon states. The theoretical level schemes of the neutron-rich Sr, Zr, Mo and Ru isotopes is established and compared with the existing experimental data.

Keywords: neutron-rich nuclei; quasiparticle states; low-lying levels

Introduction

The region of the neutron-rich nuclei near A~100 is distinctive for its sudden change in the ground state properties of nuclei. It has been shown that the ground

states of Sr, Zr, Mo and Ru isotopes with *N* ranging from the magic number *N* = 50 up to *N* ~ 60 are weakly deformed, but they undergo a shape transition from nearly spherical to well deformed prolate (or oblate) deformations as *N* = 60 is approached and crossed [15]. A good knowledge of the spectroscopic properties of these nuclei should allow a better understanding of the origin of the deformation and the nature of the shape-coexistence phenomena in this mass region. In the transitional region A~100, the nuclear shape is known to be soft spherical-deformed, which theoretically has the reason to not use a rigid triaxiality. It is then better to treat this spherical-deformed shape by the coupling between rotation and vibration mode. We had used a microscopic description of the spectroscopy of neutron-rich odd-A = 105 and 133 nuclei [5-6]. In the present work, we study the nuclear structure of the neutron-rich Sr, Zr, Mo and Ru isotopes within the quasiparticle-phonon coupling plus rotor model (QPRM model). We give the band-heads states for the band in window less than 1-MeV. In this sense, we can therefore compare the experimental data with our broadened theoretical shape.

Theoritical Formalism

The QPRM model use the champ moyen of Nilsson, the pairing correlation and vibration mode are treated by BCS and TDA formalisms. The total Hamiltonien is separated in three terms, the intrinsic Hamiltonien H_{int}, the rotational Hamiltonien H_I and the Coriolis force H_C which couples rotational and intrinsic motions.

$$H = H_{\text{int}} + H_I + H_C \tag{1}$$

The intrinsic Hamiltonien is separated into four terms. The first term, H_{sp}, describe the Nilsson deformed mean field. The term H_P describes the monopole pairing interaction witch describes the superfluidity of nuclear matter [5-6]. The term H_Q is a quadrupole-quadrupole force giving account of the nuclear vibration. The last term H_J represents the recoil force resulting from the hypothesis of an axial symmetric rotational motion.

Where

$$\begin{aligned} H_{\text{int}} &= H_{sp} + H_P + H_Q + H_J \\ H_I &= A_R (I^2 - I_3^2) \\ H_C &= -A_R (I_+ J_- + I_- J_+) \\ H_J &= A_R (J^2 - J_3^2) \end{aligned} \tag{2}$$

The inclusion of the Coriolis force H_C requires the matrix Hamiltonien H to be built and diagonalized in the space of the symmetrized function [1].

$$\left|IMK_{\rho}\right\rangle = \sqrt{\frac{2I+1}{16\pi^2}}\left\{D^{I}_{MK}\left|K_{\rho}\right\rangle + (-)^{I+K} D^{I}_{M-K}\left|\overline{K_{\rho}}\right\rangle\right\}, \tag{3}$$

where $\left|\overline{\mathrm{K}}_{\rho}\right\rangle$ is the time reversal of an intrinsic state $\left|\mathrm{K}_{\rho}\right\rangle$ which can be obtained by resolution of the secular problem.

$$\begin{aligned} H_{\mathrm{int}}\left|K_{\rho}\right\rangle &= (H_{sp} + H_{P} + H_{Q} + H_{J})\left|K_{\rho}\right\rangle \\ &= E^{\mathrm{int}}_{K_{p}}\left|K_{\rho}\right\rangle. \end{aligned} \tag{4}$$

D^{I}_{MK} is the rotational matrix and is an eigen-function of I^2 and I^3 with respective eigenvalue $I(I+1)$ and K. Diagonalizing H within the basis states, the Eq. (2) requires therefore and essentially to determine the matrix element of Coriolis term H_C ,

$$\left\langle IMK^{'}_{\rho'}\left|H_{C}\right|IMK_{\rho}\right\rangle = -A_{R}\left((-)^{I+\frac{1}{2}}(I+\frac{1}{2})\left\langle K^{'}_{\rho'}\left|J_{+}\right|\overline{K_{\rho}}\right\rangle\delta_{K'\frac{1}{2}}\delta_{K\frac{1}{2}} + \sqrt{(I\mp K)(I\pm K+1)}\left\langle K^{'}_{\rho'}\left|J_{\pm}\right|\overline{K_{\rho}}\right\rangle\delta_{K',K\pm1}\right) \tag{5}$$

Discussion

For the Nilsson calculations, the even-even core structure is reproduced using conjointly the deformation parameter ε_2 from Möller and Meyer data, and the K = 0.068 and μ = 0.35 parameters of deformed average Nilsson field [7-8]. The gap is fixed for protons and neutrons by the well-known phenomenological relation $\Delta_p=\Delta_n=12/A^{1/2}$ MeV [9]. For the TDA calculations, the parameter of quadrupole force χ is fitted from the experimental energy of quadrupole vibrational core using the experimental data from Refs [3-4]. We take for ^{102}Mo, ^{104}Mo and ^{106}Mo, $E(2^+)$ =295 keV, $E(2^+)$ =192 keV and $E(2^+)$ =171 keV, respectively. The inertia parameters are determined semi-empirically by Grodzing relation using the energy of first excited state $\varepsilon_2^2 \approx 1176(A^{7/3}E(2^+))^{-1}$ [2-14].

We have shown in figure1, the comparison between theoretical and experimental energy levels of the collective bands of ^{103}Mo is investigated by our QPRM calculations, in respect to the deformation parameters given by Möller [8], and compared to the existing experimental data [4]. The ground state perfectly corresponds to $3/2^+[411]$ both experimentally and theoretically. The states characterized by the same asymptotic quantum numbers Ω^{π} [N, n_z, Λ], where Ω is the quantum number that corresponds to the third component of the angular momentum in the intrinsic frame, π and N being its parity and the principal quantum number of the major oscillator shell, n_z is the number of quanta associated with the wave function moving along the z-direction and Λ is the projection of the orbital angular momentum onto the z axis (symmetry axis).

We note that the odd-neutron wave function for the $\Omega^{\pi} = 5/2^{-}$ isomer is calculated to have the following asymptotic Nilsson components:

$|5/2^{-}\rangle$ = -0.064[503]+ 0.273 [512]- 0.395 [523]+ 0.875 [532], while that of the $\Omega^{\pi} = 3/2^{+}$ ground state has components:

$|3/2^{+}\rangle$ = -0.174[402]+ 0.894[411]- 0.398[422]- 0.106000 [431].

Those components in the wave functions whose squared amplitudes are less than 1% are not listed. Through analyzing the intrinsic structure of positive and negative parity side bands, we conclude that the low-excitation deformation bands are attributed to the high-j intruder states $1g_{7/2}$ and $1h_{11/2}$ in the $N = 4, 5$ shells. In addition, we conclude from the discussion of quasi-particle bands that the quasi-particles in the orbit ν3/2+[411] and ν5/2−[532] play an important role in the deformation of ^{103}Mo.

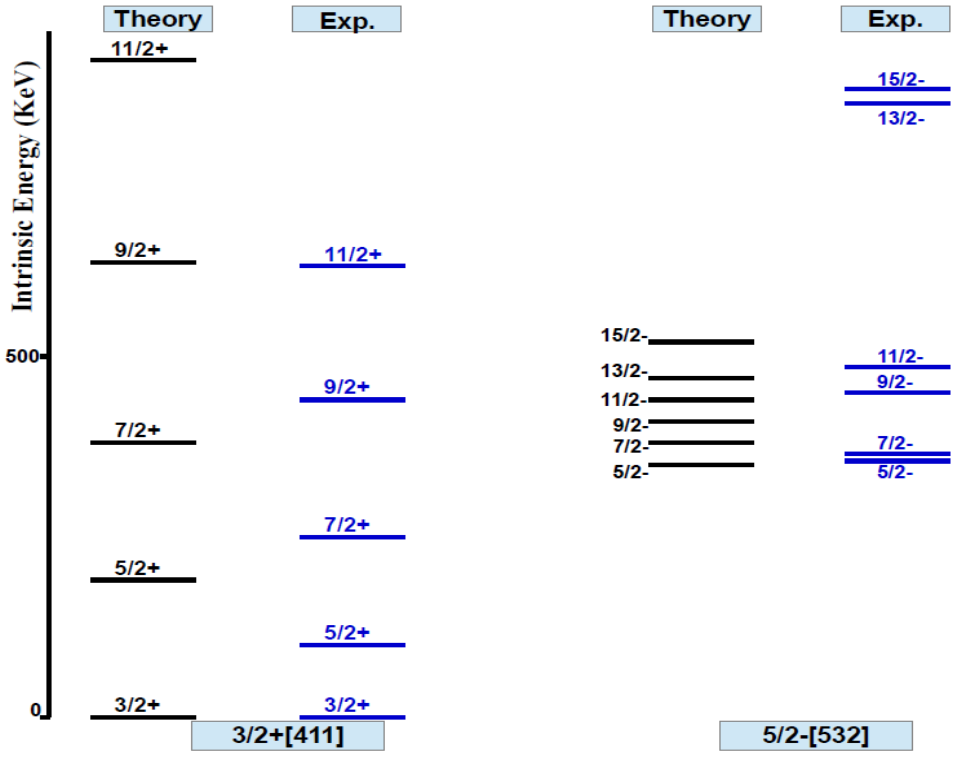

Fig.1: Comparison between experimental and theoretical values of the ground and rotational band for ^{103}Mo using QPRM Method.

In figure 2, the isotonic chain for N = 61 (^{99}Sr, ^{101}Zr, ^{103}Mo and ^{105}Ru) is investigated by our QPRM calculations, in respect to the deformation parameters given by Möller [8], and compared to the existing experimental data [10, 11, 12, 13]. The ground state perfectly corresponds to $3/2^{+}$[411] both experimentally and theoretically. In Refs. [11, 12, 13], the level scheme has been adopted for ^{99}Sr, ^{101}Zr and ^{105}Ru. There was suggested an assignment of $5/2^{-}$[532] from the systematic of neighbouring nuclei. It is based at 423.3 keV excitation for ^{99}Sr, 216.67 and 673.52 keV for ^{101}Zr, and at 246.37, 587.0 and 644.03 keV for ^{105}Ru.

Within the framework of our calculations, we localize this state at 626, 558, 349 and 661 keV for ^{99}Sr, ^{101}Zr, ^{103}Mo and ^{105}Ru, respectively. We have shown that the observed spectroscopic properties have been adjusted from a competition between a quadrupole and pairing forces. The originality of these results for the isotonic chain helps us to further study the systematic of the Mo isotopes (^{103,105,107}Mo).

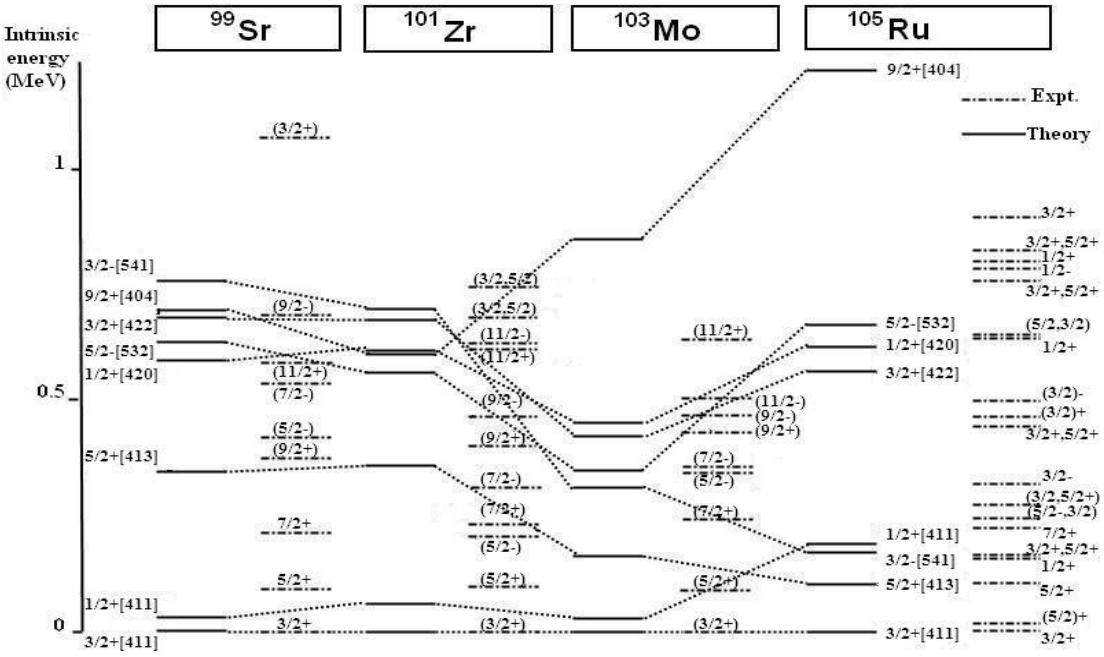

Fig.2: Experimental excitation energies and spin-parity assignments of the non-collective states compared with the QPRM results for isotonic chain N = 61.

Finally, as we can see from Figure 3, the systematic of low-lying states for ^{103,105,107}Mo is studied within the framework of QPRM model and the intrinsic states $3/2^+[411]$ and $5/2^+[413]$, except the case of ^{105}Mo where the ground state is $5/2^-[532]$, are well reproduced for ^{103}Mo and ^{107}Mo, respectively. They are emanating from the spherical shell $\nu g_{7/2}$ for $3/2^+[411]$ as well as from $\nu d_{5/2}$ for $5/2^+[413]$. Otherwise, for the case of negative parity, our calculations predict the existence of two intrinsic states, $3/2^-[541]$ and $5/2^-[532]$, originating from $\nu h_{11/2}$ spherical orbital. Their systematics indicate an abrupt change in position at N = 63. For the nucleus ^{103}Mo, the state $3/2^-[541]$ has a lower energy than the $5/2^-[532]$. This situation is reversed for the isotopes ^{105}Mo and ^{107}Mo, where the energy gap between these two states was growing up. This effect is explained in terms of deformation parameter which varies from $\varepsilon_2 = 0.3$ for ^{103}Mo to $\varepsilon_2 = 0.317$ for ^{105}Mo and 0.325 for ^{107}Mo. Moreover, our calculation results are completely interchanged in ^{105}Mo (see fig.3). The performer could be explained in terms of our approach to only diagonalize the total Hamiltonian for low spin orbitals.

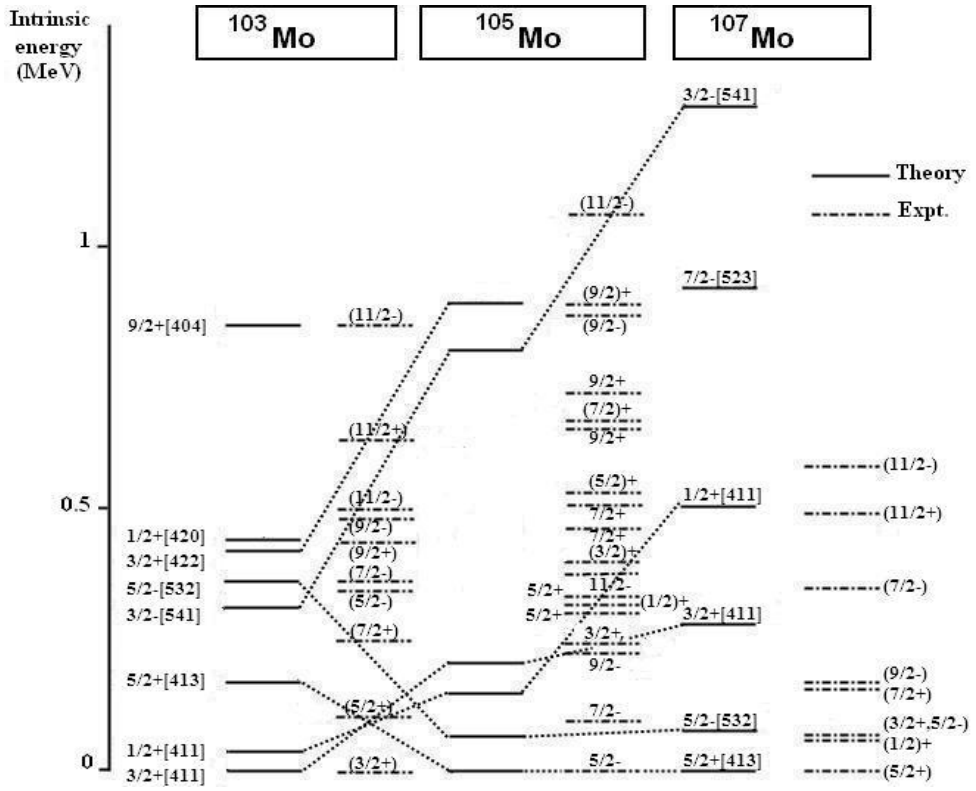

Fig.3: Systematic of one –quasiparticle states in ^{103,105,107}Mo isotopes, compared to the existing data.

Conclusion

In this work, an alternative theoretical method based on the quasiparticle-phonon coupling, inspired by MQPM of Soloviev, is developed. We used TDA phonon instead of RPA phonon to simplify the calculations. We show that the residual interaction coming from rotational Hamiltonian favours the positive parity states, because the deformation alignment of the states coming from $\nu h_{11/2}$ spherical orbital. We have first used this approach to study the isotonic trends, the ground and low-lying one-quasiparticle configurations, of ^{99}Sr, ^{101}Zr, ^{103}Mo and ^{105}Ru. We find that the nuclear shape is determined by a competition between a quadrupole and pairing forces**.** The quadrupole force tends to deform the nucleus where the spherical shape is stabilized by the pairing force. When more nucleons are added to the composed spherical shape, the relative strength of the quadrupole force increases and at a certain point the transition to the deformed shape takes place. To understand such effects, we have then studied qualitatively the isotopic trends for ^{103,105,107}Mo. We reproduced the ground and low-lying states of ^{103}Mo and ^{107}Mo, which is not the case for ^{105}Mo where the ground state is not reproduced, owing to the blocking effects of the $\nu h_{11/2}$ bands, and the underlying configuration is interchanged. The results obtained for ^{103,105,107}Mo are in agreement

with the experimental data. This method confirms parity and spin-experimental assignments and allows prediction of other excited states that are not yet observed ($3/2^-(541)$, $5/2^+(413)$). We are then aware of the challenge of reproducing in detail the observed spectroscopic properties of the particular mass region considered in the present study. Nevertheless, our spectroscopic levels scheme for ^{103,105,107}Mo can be considered as a plausible step Forward to a much more detailed work, which is in progress.

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