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Forecasting Stock Returns Volatility on Uganda Securities Exchange Using TSK Fuzzy-GARCH and GARCH Models

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Abstract

In finance, accurately forecasting volatility of any financial asset is very important due to its usefulness in areas such as option pricing, decision making, and risk management. However, this is still a major challenge despite the many methodologies introduced to solve this problem. One of such methods include the hybrid models that combine Fuzzy Inference system(FIS) and the classical GARCH model which has registered improvement on the forecasting accuracy. There is however limited literature on the applicability of these methods to emerging African markets despite their reported outstanding performance. This study therefore seeks to forecast stock returns volatility of daily closing prices of the Uganda Securities Exchange(USE) using TSK Fuzzy-GARCH model and three GARCH family models; GARCH(1,1), EGARCH(1,1) and TGARCH(1,1) models. Mean square error(MSE) and mean absolute error(MAE) were used to determine the forecasting performance of the models under study. Results obtained show that the TSK Fuzzy-GARCH model gives the best forecasting results.

Keywords: Forecasting, volatility, TSK Fuzzy- GARCH, Uganda Securities Exchange, stock returns

1 Introduction

In finance, volatility plays a great role in areas such as risk management, option pricing and decision making and as such a lot of methodologies have been developed with a major aim of obtaining better forecasts. However, this remains a major challenge despite the tireless efforts by researchers and stake holders in this field.

According to [1], volatility is defined as a statistical measure of the dispersion of returns for a given security or market index and it can either be measured using the standard deviation or variance between returns from that same security or market index.

There are major stylized characteristics usually exhibited by financial time series. Firstly, it was observed by [2] that financial returns displayed volatility clustering meaning that large changes in the price of an asset are often followed by other large changes, and small changes are often followed by other small changes. Secondly, [3] demonstrated that financial data are leptokurtic meaning that the distribution of the returns is heavy-tailed. Thirdly, [4] introduced the leverage effect meaning that volatility is higher after negative shocks than after positive shocks of the same magnitude. Thus, the choice of the model should be based on its ability to account for these stylized characteristics.

Many models have been developed to try to understand the market behavior with the aim of managing and if possible mitigating risk. These range from the Effective Market Hypothesis (EMH), the simple Random Walk model, Brownian Motion, AutoRegressive (AR), AutoRegressive Moving Average (ARMA), Generalised AutoRegressive Conditional Heteroscedasticity (GARCH) family to the Artificial Intelligence (AI) to which FIS is a subset. In literature, the most successful models that have been widely used are the GARCH family models and now the AIs which are becoming an interesting and fast growing research area in the field of finance.

The ARMA approach introduced by [5] on stationary time-series assumes that variance of the disturbance is constant and also that the data can be modeled as a linear process which is not always the case. According to [6], the assumption that variance is constant through time is inefficient and inconsistent from the statistical point of view. This is because variance changes with time in real life financial data, a phenomenon called heteroscedasticity.

This prompted the introduction of models that can account for the varying variance.

The ARCH model proposed by [7] and its extension, GARCH model by [8], were the first models to be introduced into the literature. These models have made it possible to estimate the variance of a series at a particular point in time, which accounts for their wide use and popularity. Additionally, there is introduction of many more of their extensions like GARCH-M by [9], EGARCH by [10], TGARCH by [11], AGARCH by [12] and many others. These extensions are aimed at improving the GARCH model in capturing the stylized characteristics exhibited by financial data.

Despite these many GARCH family variants, no agreement has been reached on which model is best in capturing volatility basing on previous studies. Some show preferable results using simple GARCH(p,q) models whereas others show that extensions of GARCH models perform better. The performance of these models varies across markets and time period and is affected by many factors one of them being whether the market is emerging or developed, the estimation method and many others [13]. Additionally, financial time series data is complex and non-linear in nature and yet GARCH models are not able to effectively capture all these characteristics. To solve this, other methodologies such as fuzzy inference system(FIS) and their hybrids have been developed.

According to [14], FIS is defined as universal approximations that can estimate nonlinear continuous functions uniformly with arbitrary accuracy. They are comprised of IF-THEN rules defined explicitly for linguistic variables [15]. The concept of fuzzy logic has yielded fruitful results in finance such as stocks, exchange rates [16].

Although FIS has received utmost attention and application in FTS forecasting, it does not capture all the stylized facts of the data such as volatility clustering and asymmetries present in the data. To solve this problem, different hybrid models such as hybrid fuzzy-GARCH have been developed. This yields better results compared to using a single model in as far as forecasting is concerned as seen in [17], [18], and [19]. However, the applicability of these methods is still limited to African markets. This study therefore uses a TSK Fuzzy-GARCH model for forecasting stock returns volatility of Uganda Securities Exchange(USE) data from from 04/01/2005 to 31/07/2014 comprising of 1571 observations. The performance of the TSK Fuzzy-GARCH model is compared with that of three other models using MSE and MAE. The models include; GARCH(1,1), EGARCH(1,1) and TGARCH(1,1) models.

2 Method

In this section, a brief description of the models relevant to our study is given.

2.1 Fuzzy Inference System (FIS)

Fuzzy set theory is a generalization of the classical set theory, [20]. The elements to a given fuzzy set may partially belong to that set. It has been developed for modeling complex systems in uncertain and imprecise environment. Fuzzy logic is based on the theory of fuzzy sets. A fuzzy logic model is a logical- mathematical procedure that allows the reproduction of the human way of thinking in a computational manner. Thus, FIS can be defined as the universal approximations that can estimate nonlinear continuous functions uniformly with arbitrary accuracy [21]. They are comprised of IF-THEN rules defined explicitly for linguistic variables [15]. The FIS architecture is shown in Figure 1.

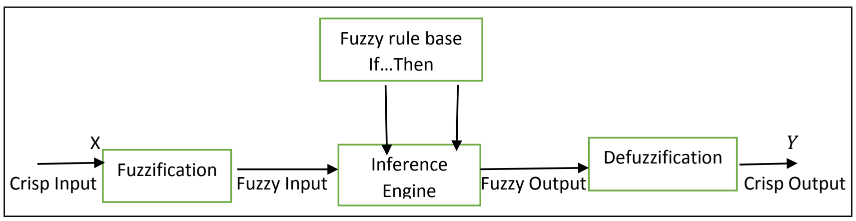


Figure 1: FIS architecture

There are four major steps followed to implement fuzzy logic to real application.

1. Fuzzification: Here, classical data or crisp data is converted into fuzzy data of MFs. This process involves computing values of Membership functions (MFs) of fuzzy sets for given values of base variables.

2. Fuzzy rules: At this stage, the IF-THEN logic system as fuzzy rules links the input to the output variables.

3. Fuzzy inference Process: MFs are combined with the control rules to derive the fuzzy output.

4. Defuzzification: In fuzzy logic, fuzzy rules produce fuzzy output, which is in contrast with classical logic rules. It can be a set of values of the MF values or a linguistic term. MFs are used to retranslate the fuzzy output into a crisp value; a process known as defuzzification.

The general form of a fuzzy IF- THEN rule is written as;

$$\text{Rule}: \quad \textbf{IF} \quad x \quad \text{is} \quad A \quad \textbf{THEN} \quad Y \quad \text{is} \quad B$$

The commonly used methods for developing fuzzy rule systems are those proposed by [22] and [23]. The similarity of these two systems is that the fuzzification of the inputs and application of the fuzzy operator is exactly the same in both cases. The difference is that in the Mamdani method, the output is a linguistic label while in the Sugeno method, the output is either a constant statement or a linear statement. This study used the Sugeno method in developing the fuzzy rules as discussed below.

The Takagi-Sugeno-Kang (TSK) FIS belongs to a broader class of Quasi-nonlinear fuzzy models. A first-order TSK model with K rules is expressed as;

$$\text{Rule}^k: \ \textbf{IF} \ x_1 \ \text{is} \ A_i^k \ \textbf{AND} \ldots x_n \ \text{is} \ A_n^k \ \ \textbf{THEN} \ Y^k = P_0^k + \sum_{i=1}^{n} P_i^k x_i,$$

where $x_i, \ i = 1, 2, \ldots, n$ and y are the input and output linguistic variables respectively. $A_i^k, \ i = 1, 2, \ldots, n$ represent the fuzzy sets; P_0^k and $P_i^k, \ i = 1, 2, \ldots, n$ denote the parameters to be estimated. Note that the antecedent parts of the rules are the same as that of the traditional fuzzy IF-THEN rules, while the consequent parts are the linear combinations of input variables plus a constant term and the final output is the weighted average of each rule's output.

2.2 GARCH(p, q) Model

This is an extension of the ARCH model by [7] and was introduced by [8]. Let $\{Z_t\}_{t\in\mathbb{Z}}$ be a sequence of independent and identically distributed(iid) random variables such that $Z_t \sim N(0, 1)$.

Let also P_t and P_{t-1} denote the closing market index at the current time, (t) and previous day $(t-1)$, respectively. The returns at any time are given by $r_t = log\left(\frac{P_t}{P_{t-1}}\right)$. The GARCH$(p, q)$ model can be written as;

$$r_t = \mu + \varepsilon_t \tag{1}$$

$$\begin{aligned} \varepsilon_t &= \sigma_t Z_t; \\ \sigma_t^2 &= \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \end{aligned} \tag{2}$$

where μ is the mean return, p and q are the orders of the process ω, α_i and β_j are the parameters to be estimated. In order for the variance to be positive the necessary condition is that $\omega > 0, \alpha_i \geq 0$ (for $i = 1, \ldots, p$) and $\beta_j \geq 0$ (for $j = 1, \ldots, q$).

One of the shortcomings of GARCH(p,q) model is its inability to capture the leverage effects and yet these are believed to be present in most financial data. One such solution to this is using asymmetric GARCH models which can capture the leverage effects. In this study, we employ TGARCH and EGARCH models as discussed in the next subsection.

2.3 The Exponential GARCH Model

This model was developed by [10] and is defined by;

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \frac{\mid \varepsilon_{t-i} \mid + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j \ln \sigma_{t-j}^2, \tag{3}$$

where γ is the asymmetric response parameter or leverage parameter. In most empirical cases, γ is expected to be negative so that a negative shock increases future volatility or uncertainty while a positive shock eases the effect on future uncertainty, [1].

For $p = q = 1$, we have the EGARCH(1,1) model given by

$$\ln\left(\sigma_t^2\right) = \omega + \beta_1 \ln\left(\sigma_{t-1}^2\right) + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}. \tag{4}$$

2.4 The Threshold GARCH Model

This model was developed by [11] and is a special case of APARCH model by [24]. Below is its specification;

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i d_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta \sigma_{t-j}^2 \tag{5}$$

where d_{t-i} is an indicator for negative ε_t, *i.e.*

$$d_{t-i} = \begin{cases} 1 & \text{if} \quad \varepsilon_{t-i} < 0, \quad \text{bad news} \\ 0 & \text{if} \quad \varepsilon_{t-i} \geq 0, \quad \text{good news.} \end{cases}$$

α_i, γ_i and β_j are non-negative parameters satisfying conditions similar to those of GARCH(p,q) model. γ_i is the asymmetric response parameter or leverage parameter. The model reduces to the standard GARCH form when $\gamma_i = 0$. Otherwise, when the shock is positive (*i.e.*, good news) the effect on volatility is α_i, but when the shock is negative (*i.e.*, bad news) the effect on volatility is $\alpha_i + \gamma_i$. [25], assert that when γ_i is significant and positive, negative shocks have a larger effect on σ_t^2 than positive shocks.

When $p = q = 1$, we obtain the TGARCH(1,1) model as shown below.

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{6}$$

The dummy variable d_{t-1} is given by

$$d_{t-1} = \begin{cases} 1 & \text{if} \quad \varepsilon_{t-1} < 0, \quad \text{bad news} \\ 0 & \text{if} \quad \varepsilon_{t-1} \geq 0, \quad \text{good news.} \end{cases}$$

Despite the success of the GARCH models in modeling and forecasting volatility, they can not explain the dynamical complexities and nonlinearities in the data. The use of fuzzy-GARCH is one of the solutions to these shortcomings with an aim of obtaining better forecast values.

2.5 The TSK Fuzzy- GARCH Model

Fuzzy-GARCH model was suggested by [21]. It is described by a collection of fuzzy rules in the form of IF-THEN statements in order to describe the stock market fluctuations via a GARCH model. The antecedent of each rule are fuzzy sets and the consequent is the GARCH model. The lth rule of the Fuzzy-GARCH(p,q) is written as:

$$\begin{aligned} \mathcal{R}_l: \quad & \text{IF } r_{t-i} \text{ is } \tilde{A}_{l,i} \text{ AND } \sigma^2_{t-j} \text{ is } \tilde{A}_{l,p+j} \\ \text{THEN} \quad & \sigma^2_{l,t} = \omega_l + \sum_{i=1}^{p} \alpha_{l,i} r^2_{t-i} + \sum_{j=1}^{q} \beta_{l,j} \sigma^2_{t-j}, \text{ for } l = 1,2,\dots,L \end{aligned} \tag{7}$$

where σ_t^2 is the output of the system, $\tilde{A}_{lk}$ for $k = 1,2,\dots,q+p$ is the fuzzy set. L is the number of fuzzy IF-THEN rules, r_{t-i} and σ^2_{t-j} are the previous value of the stock market's returns and volatility respectively defined for $i = 1,2,\dots,p$ and $j = 1,2,\dots,q$. This study used Adaptive Neural Fuzzy Inference System (ANFIS) to determine the parameters of the TSK Fuzzy-GARCH(1,1) model. The ANFIS identifies the relationship between the input and output data, and determines the optimal distribution of membership function through a hybrid learning rule combining the back-propagation gradient descent and the least squares method.

ANFIS was introduced by [26]. It is a multilayer feed forward network which uses neural network(NN) learning algorithms and fuzzy reasoning to map inputs into an output. The ANFIS architecture has five layers; fuzzy layer, product layer, normalized layer, defuzzification layer and total output layer [26]. ANFIS gives the advantages of the mixture of neural network and fuzzy logic.

2.6 Forecasting ability of the Models

The forecasting performance of the models under study was evaluated using MSE and MAE defined below.

Let h be the number of lead steps, S the sample size, $\hat{\sigma}_t^2$ is the forecasted variance and σ^2 is the actual variance.

$$MSE = \frac{1}{h+1} \sum_{t=s}^{s+h} \left(\hat{\sigma}_t^2 - \sigma^2\right)^2 \tag{8}$$

Another alternative measure is the mean absolute error (MAE) by [27] defined by

$$MAE = \frac{1}{h+1} \sum_{t=s}^{s+h} \mid \hat{\sigma}_t^2 - \sigma^2 \mid \tag{9}$$

Over all, the best model is one that minimizes the error functions.

3 Results and Discussion

3.1 Data

Daily closing prices of Uganda Securities Exchange (USE) All share index data from 04/01/2005 to 31/07/2014 with 1571 observations were used. The USE is the major stock market index in Uganda. The data can be accessed from http://www.use.or.ug.

Let P_t and P_{t-1} denote the closing market index of USE at the current day (t) and previous day ($t-1$), respectively. The USE All Share returns (log returns or continuously compounded returns) at any time are given by:

$$r_t = log\Big(\frac{P_t}{P_{t-1}}\Big) \tag{10}$$

In order to understand the behavior of the USE return series, summary statistics together with its distribution are reported below;

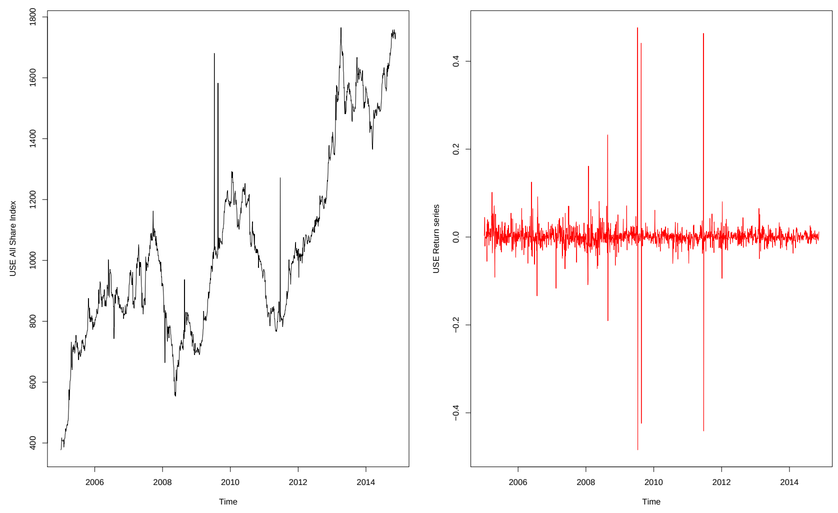
Table 1: Descriptive statistics of USE returns Series

Mean	0.0010
median	0.0003
Maximum	0.4766
minimum	-0.4844
Std.Dev	0.0349
Skewness	0.3309
Excess Kurtosis	112.3108
Jarque-Bera	827337.2164
JB probability	<0.001
No. of observations	1570

Sample: Jan 04, 2005 to July 31, 2014

Descriptive statistics for the USE ALSI return series are shown in Table 1. As is expected for a time series of returns, the mean is close to zero. The return series are positively skewed an indication that the USE ALSI has non-symmetric returns. The excess kurtosis is positive indicating that the underlying distribution of the returns are leptokurtic or heavy tailed. The series is non-normal based on the JB test which rejects normality at the 1% significance level.

Figure 2a shows the distribution of the USE ALSI and Figure 2b shows the return series distribution. From the graphs, the stock prices are non stationary while the return series are mean-stationary with a mean return of zero. There is also evidenced volatility clustering in the return series. This is analogous to other studied stock exchanges see ([28], [1]). During the years of 2009 and



(a) USE All share Index distribution (b) USE returns distribution

Figure 2: USE Daily prices and returns distributions(Jan.2005-July.2014)

2011, volatility was very high. This could be as a result of macro economic factors like inflation, politics, policies, exchange rate, exports and imports.

Before using the GARCH models under study, several tests were carried out on the return series. The series were found to be stationary, ARCH effects were present in the residual series. Using Akaike Information criterion (AIC) by [29] and Bayesian Information criterion(BIC) by [30], the best specification for the models was $p = 1$ and $q = 1$, that is; (1,1). The GARCH models under student -t distribution were estimated using quasi maximum likelihood. The estimation results of the GARCH models under study are indicated in Table 2.

From Table 2, except for the mean return which is not significantly different from zero for both models in the mean equation, the moving average (MA) and autoregressive(AR) coefficients are statistically significant at all levels. In the ARMA(1,1)-GARCH(1,1) model, all the parameters in the variance equation are significant at 5% level.

In the ARMA(1,1)-TGARCH(1,1) model, except α and the shape parameter, the rest of the parameters are not statistically significant in the variance equation. The leverage parameter, γ in the ARMA(1,1)-TGARCH(1,1) model is not significant which shows probable absence of leverage effects.

In the ARMA(1,1)-EGARCH(1,1) model,all other parameters are statistically significant at 5% level except α. The leverage parameter, γ in the ARMA(1,1)-EGARCH(1,1) model is positive and significant which means that negative shocks have a larger effect on σ_t^2 than positive shocks.

Table 2: Estimation results of the GARCH Models for USE returns

Parameter	GARCH	TGARCH	EGARCH
	Mean Equation		
μ	6.019e-06(0.569)	0.0006(0.273)	0.0006(0.033)
ar1	9.827e-01($<$ 0.01)	0.977(0)	-0.421704(0)
ma1	-9.725e-01($<$ 0.01)	-0.964(0)	0.437906(0)
	Variance Equation		
ω	3.428e-04(0.023)	0.0003(0.064)	-2.556(0.008)
α	1.000(0.020)	0.924(0.007)	-0.108(0.192)
β	2.433e-01(0.0004)	0.232(0.1)	0.636(0.000054)
γ	–	-0.314(0.422)	0.617(0.008)
shape	2.303($<$ 0.01)	2.393(0)	2.250(0)
	Model Performance		
JB test	2748893(0)	841560($<$ 0.01)	842400($<$ 0.01)
Ljung-Box Test	0.201(1)	2.310(0.9672)	1.06604(0.9997)
ARCH-LM Test	0.153(1)	1.543(0.9999)	0.05199(0.9999)
LL	4264.745	4264.434	4264.880
AIC	-5.424	-5.422	-5.423
BIC	-5.400	-5.395	-5.396

Note: Values in parenthesis indicate P-Values

The model performance results show that the ARMA(1,1)-GARCH(1,1) model out performs the ARMA(1,1)-TGARCH(1,1) model in modeling volatility of USE returns since it has the smallest AIC and BIC. This result is similar to that of [31] where they found out that GARCH(1,1) outperforms other models in modeling volatility of USE returns. The ARCH-LM tests show that ARCH effects are absent in the standardized residuals. Also the Ljung-Box test accept absence of serial correlation in the standardized residuals. This means that except for normality tests, all the three models fit the data well.

The estimation results of the TSK-Fuzzy-GARCH(1,1) model are also reported. The results obtained are for 5 linguistic terms with 10 fuzzy IF-THEN rules as shown in Table 3.

From Table 3, all the consequent parameters are positive except the constant parameter for rule 1. Next, the forecast performance of the models under study is reported in Table 4.

From Table 4, the TSK Fuzzy-GARCH model performs better than the ARMA(1,1)-GARCH(1,1)and ARMA(1,1)-TGARCH(1,1) models. The forecast accuracy of the classical GARCH models is not significantly different. Therefore, the hybrid Fuzzy-GARCH family models perform better than classical GARCH family models which is in agreement with the results of [32], [33] and many others.

Table 3: Estimation results of the TSK-Fuzzy- GARCH(1,1) model for 5 linguistic terms.

Rule, l	$\alpha_{l,1}$	$\beta_{l,1}$	ω_l
1	0.312	0.544	-0.39
2	0.393	0.063	0.821
3	0.406	0.875	0.069
4	0.771	0.528	0.371
5	0.025	0.430	0.067
6	0.142	0.394	0.344
7	0.437	0.226	0.390
8	0.969	0.144	0.142
9	0.461	0.252	0.495
10	0.820	0.198	0.073

Table 4: USE volatility forecast accuracy.

Model	MSE	MAE
TSK Fuzzy -GARCH(1,1)	4.6080e-08	9.974e-05
ARMA(1,1)-GARCH(1,1)	1.47738e-04	8.5552e-03
ARMA(1,1)-TGARCH(1,1)	1.47736e-04	8.5554e-03
ARMA(1,1)-EGARCH(1,1)	1.4796e-04	8.5554e-03

4 Conclusion

This study forecasts stock returns volatility of USE using TSK Fuzzy-GARCH(1,1), ARMA(1,1)-GARCH(1,1) and ARMA(1,1)-TGARCH(1,1) models. The forecasting performance of the models is determined using MSE and MAE as the two statistical error measures. Results obtained indicate that the TSK Fuzzy-GARCH(1,1) models gives best forecast results.

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