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Analysis of Volatility Spillover Effects Using Trivariate GARCH Model

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Abstract

One of the main purposes of studying volatility spillover effects is for economic benefits. Such studies provide useful insights into how information is transmitted from one market to another. US financial crisis has an impact on international gold market by some transmission channels. Using trivariate generalized autoregressive conditional heteroscedasticity (GARCH) model, the time-varying volatility relationships between international gold market, Malaysia's gold bullion coins called *Kijang Emas* (KE) and US index are investigated. The findings are useful to investors, commercial banks and researchers as they look for measures at the policy level that can safeguard and avoid adverse impact of fluctuations in gold prices.

Keywords: trivariate generalized autoregressive conditional heteroscedasticity (GARCH) model, volatility spillover, gold market

1 Introduction

In the past, gold and silver were used as currencies. The US dollar (USD) became a true fiat currency in the early 1970s. Many foreign banks hold USD as a reserve currency while they consider gold as global currency. Gold is identified as a hedge against fluctuations in USD market. This is due to its high demand and finite supply. In the past, when the USD rates went down, the gold prices remained. Investors will buy more gold to protect their money when the USD weakens. Similarly, when the USD strengthens, investors will invest in USD and abandon gold. There is a negative relationship between gold and USD [1].

However, it would not be appropriate to conclude that the price of gold and the US dollar will always move in the opposite directions. There will be other external factors that have impacts on the prices of these two currencies. Looking back at the financial history from 1940s, the frequency of financial crises and recessions are quite high. From the US National Bureau of Economic Research data, on the average, there is about one crisis for every 58 months. The world's economy is interconnected more from the past and the US economy is the most comparative economy in the world [2].

The purpose of the current study is to analyze the relationships between international gold markets, Malaysia's gold bullion coins called *Kijang Emas* (KE), and US index. KE provides an alternative form of investment and knowledge on how information is transmitted among these markets is useful to investors, commercial banks and researchers as they look for measures at the policy level that can safeguard and avoid adverse impact of fluctuations in gold prices. The method used is trivariate BEKK model which will be described in the next section.

2 Methodology

Trivariate BEKK Model

Trivariate GARCH model is a multivariate form of the generalized univariate volatility model. It allows three series, which is to be jointly modeled as a vector ARMA process in both the first moment and second moment. Trivariate GARCH models can be estimated as a constant correlation model or estimated with time-varying correlations.

In BEKK parameterization of the trivariate GARCH model, no restriction of constant correlation among variables over time is carried out [3]. This model ensures that the *H* matrix is always positive definite by incorporating quadratic forms. The estimates of the coefficients of the conditional daily return of BEKK parameterization for trivariate GARCH model is presented as follows:

$$y = \mu + r$$
$$r \sim N(0, H)$$
$$H_t = \omega\omega' + \beta H_{t-1}\beta' + \alpha r_{t-1} r_{t-1}' \alpha'$$

with y and μ are 3×3 matrix matrices; H is a 3×3 symmetric matrix, ω is the 3×3 triangular matrix; α and β are 3×3 diagonal matrices. They can be expanded as follows:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix},\ y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix},\ H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{12} & h_{22} & h_{23} \\ h_{13} & h_{23} & h_{33} \end{bmatrix}$$

$$\omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ \omega_2 & \omega_4 & 0 \\ \omega_3 & \omega_5 & \omega_6 \end{bmatrix}, \ \alpha = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}, \ \beta = \begin{bmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 \end{bmatrix}$$

Therefore, trivariate BEKK GARCH model is as follows

$$\begin{aligned}
Var(y_1) &= h_{11,t} = \omega_1^2 + \beta_1^2 h_{11,t-1} + \alpha_1^2 r_{1,t-1}^2 \\
Var(y_2) &= h_{22,t} = \omega_2^2 + \omega_4^2 + \beta_2^2 h_{22,t-1} + \alpha_2^2 r_{2,t-1}^2 \\
Var(y_3) &= h_{33,t} = \omega_3^2 + \omega_5^2 + \omega_6^2 + \beta_3^2 h_{33,t-1} + \alpha_3^2 r_{3,t-1}^2 \\
Cov(y_1, y_2) &= h_{12,t} = \omega_1\omega_2 + \beta_1\beta_2 h_{12,t-1} + \alpha_1\alpha_2 r_{1,t-1} r_{2,t-1} \\
Cov(y_1, y_3) &= h_{13,t} = \omega_1\omega_3 + \beta_1\beta_3 h_{13,t-1} + \alpha_1\alpha_3 r_{1,t-1} r_{3,t-1} \\
Cov(y_2, y_3) &= h_{23,t} = \omega_2\omega_3 + \omega_4\omega_5 + \beta_2\beta_3 h_{23,t-1} + \alpha_2\alpha_3 r_{2,t-1} r_{3,t-1} \\
\mathrm{Cov}(y_2, y_3) &= h_{23,t} = \omega_2\omega_3 + \omega_4\omega_5 + \beta_2\beta_3 h_{23,t-1} + \alpha_2\alpha_3 r_{2,t-1} r_{3,t-1}
\end{aligned}$$

In the model, H_t is the conditional variance matrix which depends only on past values of itself and past values of r_t r_t. This indicates that the variances depend solely on past squared residuals and the covariances depend solely on past covariances. *C* is a low triangular matrix of parameters, *B* is a 3×3 matrix of parameters which depicts the extent to which current levels of conditional variances are related to past conditional variances, *A* is a 3×3 matrix of parameters where conditional variances are related with past squared errors.

The trivariate BEKK GARCH model is able to capture return and volatility spillover effects of gold and USD index. The parameters of the model are estimated by maximizing the following log-likelihood function:

$$Max \log L_T(\theta) = \sum_{t=1}^{T} l_t(\theta)$$

$$l_t = \frac{TN}{2} log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(log H_t) + \varepsilon_t' H_t \varepsilon_t$$

where θ denotes all the unknown parameters to be estimated, *N* is the number of series and *T* is the number of observations.

3 Data and Statistical Tests

In this paper, the database consists of daily log returns of KE (Y1), USD index (Y2), and international gold price (Y3) recorded from 18th July 2001 to 12th May 2015. It is divided into four sub-periods of (1) global stable from July 2001 to November 2007, (2) global financial crisis from December 2007 to June 2009, (3) global recession July 2009 to December 2012, and (4) post-crisis period from January 2013 to May 2015.

Augmented Dickey-Fuller (ADF) test was first applied to examine the stationary properties of the series. The results of the test for the four sub-periods are summarized in Table 1.

Table 1: Unit Root Test (ADF)

Period:	Y1	Y2	Y3
July 2001 - November 2007	-44.95046*	-19.10150*	-40.36239*
December 2007 - Jun 2009	-17.93261*	-6.680161*	-15.89298*
July 2009 - December 2012	-29.86720*	-11.41326*	-30.64973*
January 2013 - May 2015	-24.70290*	-10.05350*	-24.72111*

*, ** and *** denotes coefficients are significant at 1%, 5% and 10% level respectively.

From Table 1, it can be concluded that all the series are stationary during the considered periods. In Table 2, the correlation matrices for the series during global stable and global financial crisis sub-periods are presented.

Table 2: Correlation matrices for global stable and global financial crisis periods

	(1) July 2001-November 2007			(2) Dec 2007 - Jun 2009		
	Y1	**Y2**	**Y3**	**Y1**	**Y2**	**Y3**
Y1	1.000000	0.23791	0.04401	1.000000	0.78219	0.93927
Y2	0.23791	1.000000	-0.12538	0.78219	1.000000	-0.73581
Y3	0.04401	-0.12538	1.000000	0.93927	-0.73581	1.000000

Table 3 presents the correlation matrices for the series during global recession and post crisis sub-periods.

Table 3: Correlation matrices for global recession and post crisis periods

	(3) July 2009 - Dec 2012			(4) January 2013 - May 2015		
	Y1	**Y2**	**Y3**	**Y1**	**Y2**	**Y3**
Y1	1.000000	-0.26655	0.61876	1.000000	-0.20175	-0.34277
Y2	-0.26655	1.000000	-0.1501	-0.20175	1.000000	0.05193
Y3	0.61876	-0.1501	1.000000	-0.34277	0.05193	1.000000

The results of the correlation analysis in Table 2 reveal a weak significant correlation during the stable period from July 2001 to November 2007 for the three markets especially between Y1 and Y3 (0.04401). Nevertheless, a very strong correlation were recorded between Y1 and Y2 (0.78219), between Y2 and Y3 (-0.73581) and between Y1 and Y3 (0.93927) during global financial crisis. However, the correlation between Y1 and Y3 increased to 0.61876 while correlations between Y2 and Y1 (-0.26655), and between Y2 and Y3 (-0.1501)

decreased slightly during global recession. During post-crisis period, the correlations for Y1 and Y2, Y1 and Y3, Y2 and Y3 were -0.20175, -0.34277 and 0.05193 respectively.

Granger causality tests and impulse response functions were performed. The results indicated that Granger causality runs one-way from Malaysian *Kijang Emas* to US index and international gold to US index in the stable period. However, in the global financial crisis period, international gold affected Malaysian *Kijang Emas* and US index as well. At the same time, a unidirectional causality was detected from US index to Malaysian *Kijang Emas*. There was no causal relationship recorded between Malaysian *Kijang Emas* with US index and Malaysian *Kijang Emas* with international gold during the global recession period and during post-crisis period. However, international gold has an influence on the US index in these two periods.

Granger causality tests also indicated causal links between these markets. The results showed the existence of interactions between these three series and that each might react to a shock on another. Impulse response showed that Malaysian *Kijang Emas* wore off after five periods, US index wore off after nine periods while international gold wore off after six periods, each reacted positively to its own shock. It was concluded that Malaysian *Kijang Emas* has essentially no effect on US index. It was also concluded that international gold has a low negative response to US index and disappeared after six periods. Conversely, US index reacted positively and negatively to the international gold, while the shocks dissipated after seven periods.

During global financial crisis, the amplitude of US index reaction increased negatively and positively to international gold and wore off after ten periods while international gold gave a weak positive response to US index and wore off after six periods. For Malaysian *Kijang Emas* and international gold, the responses were positive and negative but with lower amplitude. This was the same with US index and Malaysian *Kijang Emas*.

During the global recession period, the amplitude between US index and Malaysian *Kijang Emas* became lower and disappeared after eight periods. The response of Malaysian *Kijang Emas* and international gold were positive and negative but with lower amplitude and dissipated after six periods. During post-crisis period, the responses between Malaysian *Kijang Emas* with US index and Malaysian *Kijang Emas* with international gold decreased slightly in their amplitudes and disappeared after seven periods. However, the amplitude of response between US index and international gold increased and the shocks lasted for seven periods.

4 Results of Analysis

Table 4 lists the estimated coefficients for variances covariances matrix of trivariate BEKK GARCH(1, 1) model.

Table 4: Estimated coefficients for variances covariances matrix of trivariate BEKK GARCH(1, 1) model

	July 2001 - November 2007		**December 2007 - Jun 2009**	
Variable	**Coeeficient**	**Significant**	**Coeeficient**	**Significant**
MU(1)	0.000649*	0.0002	0.0010*	0.0011
MU(2)	-0.0003*	0.0001	0.0002*	0.0002
MU(3)	0.0006*	0.0002	0.0004*	0.0053
OMEGA(1)	0.0028*	0.0002	0.0031*	0.0007
BETA(1)	0.9239**	0.0104	0.9448**	0.0103
ALPHA(1)	0.2678**	0.0200	0.3092**	0.0319
OMEGA(2)	0.0000*	0.0000	0.0000*	0.0012
OMEGA(4)	0.0002*	0.0001	0.0033*	0.0003
BETA(2)	0.9898*	0.0022	0.1973	0.2182
ALPHA(2)	0.1347**	0.0137	0.7405***	0.0778
OMEGA(3)	0.0000*	0.0001	0.0000*	0.0034
OMEGA(5)	-0.0002*	0.0002	0.0000*	0.0096
OMEGA(6)	0.0010*	0.0002	0.0001	0.3234
BETA(3)	0.9798*	0.0031	0.9824**	0.0130
ALPHA(3)	0.1701	0.0105	0.7318	0.2390
	July 2009 - December 2012		**January 2013 - May 2015**	
Variable	**Coeeficient**	**Significant**	**Coeeficient**	**Significant**
MU(1)	0.0004*	0.0003	0.0003*	0.0004
MU(2)	0.0000*	0.0001	0.0003*	0.0001
MU(3)	0.0007*	0.0003	-0.0004*	0.0005
OMEGA(1)	0.0017*	0.0002	0.0067*	0.0006
BETA(1)	0.9591*	0.0071	0.6208***	0.0623
ALPHA(1)	0.2297**	0.0206	0.5209**	0.0280
OMEGA(2)	0.0000*	0.0000	0.0000*	0.0001
OMEGA(4)	0.0002*	0.0001	0.0002*	0.0001
BETA(2)	0.9918*	0.0031	0.9798*	0.0046
ALPHA(2)	0.1146**	0.0165	0.2028**	0.0235
OMEGA(3)	0.0001*	0.0001	-0.0003*	0.0003
OMEGA(5)	0.0001*	0.0002	-0.0001*	0.0002
OMEGA(6)	0.0015*	0.0003	0.0010*	0.0003
BETA(3)	0.9740*	0.0058	0.9906*	0.0044
ALPHA(3)	0.1805**	0.0180	0.0939**	0.0151

*, **, *** denotes significance in 1%, 5% and 10% respectively.
Standard errors are reported in parentheses.

From Table 4, the previous shock and past volatility for all series are significant except ALPHA(3) during July 2001 - November 2007 and BETA(2) and OMEGA(6) during December 2007 - Jun 2009. ALPHA(3) equals to 0.1701 and less than beta values implies that the behaviour of variance and covariance are not much affected by the magnitude of past innovations, by value of lagged variances and covariances. The insignificance of BETA(2) which is 0.1973 implies that there is no volatility clustering. Alpha (2) in four sub-periods implies

that there is volatility spillover between international gold, US index and Malaysian *Kijang Emas*. Therefore, the statistical significance of beta values during July 2001 - November 2007 implies that the lagged volatility persistence in US index, international gold and Malaysian *Kijang Emas* has a negative effect on current volatility on each other over time. There only exists volatility spillover between US index and Malaysian *Kijang Emas*. Figure 1 plots the estimates of conditional variance-covariance of trivariate BEKK GARCH(1, 1) model.

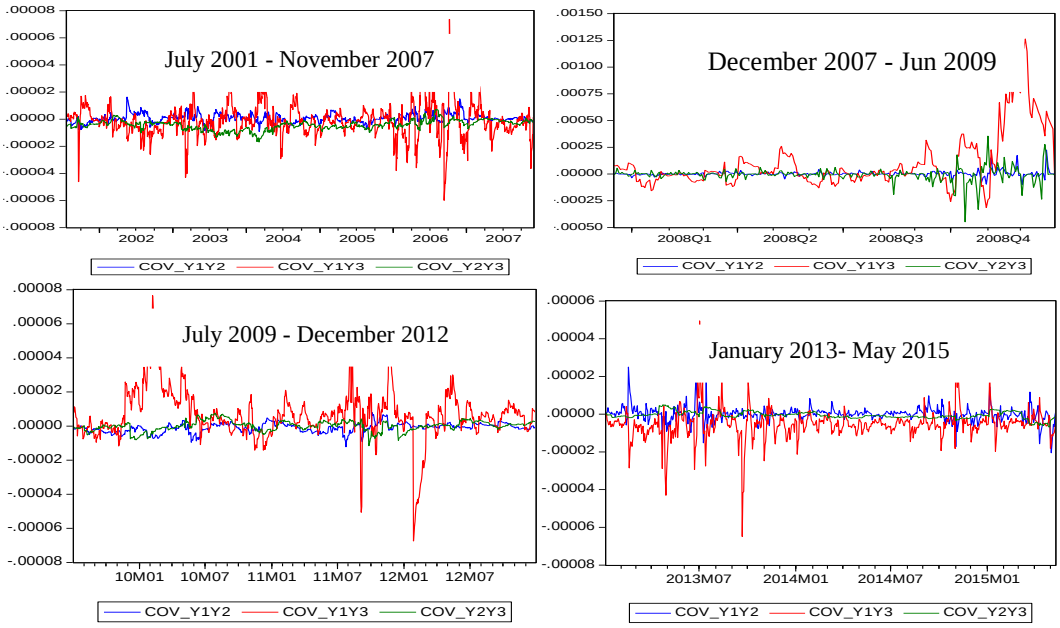

Figure 1: Estimates of conditional variance-covariance

From Figure 1, it can be concluded that Malaysian *Kijang Emas* and international gold were highly correlated during July 2001 - November 2007. There only exists low correlation between Malaysian *Kijang Emas* and international gold with US index. A high correlation was detected between Malaysian *Kijang Emas* and international gold at the end of 2008. During global recession from July 2009 - December 2012, the correlation between Malaysian *Kijang Emas* and international gold maintained at a high level until July 2010. It then stabilized in the mid-year 2011 and fluctuated from the year 2012 to 2013. It then stabilized in 2014 and increased again in 2015. The correlation between Malaysian *Kijang Emas* and international gold with US index was very low during global recession. However, their correlations increased during post-crisis period in 2013 and declined in 2014. It however rose again in 2015.

5 Concluding Remarks

In this paper, using trivariate BEKK GARCH model, linkages and volatility spillover are shown to exist between international gold, Malaysian *Kijang Emas*

and US dollar index. Low and stabilized relationships were revealed before and after financial crisis. Unilateral causality was exhibited but not two-way causal direction. For example, international gold affects Malaysian *Kijang Emas* and US index in unidirectional relationship while US index only affect Malaysian *Kijang Emas* in unidirectional relationship during global financial crisis. Stability was detected in the global recession and causal again during post-crisis period. There exists positive and negative shocks and volatility spillover during global financial crisis.

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References

[1] K.E. Craig, *Brief History of the Gold Standard in the United States*, Congressional Research Service Report, 2011.

[2] J. F. Jaffee, Gold and gold stocks as investments for institutional portfolios, *Financial Analysts Journal*, **45** (1989), 53 - 59. http://dx.doi.org/10.2469/faj.v45.n2.53

[3] Y. Baba, R.F. Engle, D. Kraft and K. Kroner, *Multivariate Simultaneous Generalized ARCH*, University of California, San Diego, 1990.

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