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Weak Resolvable Spaces and Decomposition of Continuity

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Abstract

Notions of somewhat open, somewhat continuous and R−continuous, functions introduced and investigated by Chhanda Bandyopadhyay and Chandann Chattopadhya [2]. In this paper we apply the notions of ω −open, pre− ω −open, and pre−open sets in topological space to present and study new classes of functions as new generalizations of somewhat open function, somewhat continuity and R−continuity.

Keywords: ω − open set, ω −resolvability, pre-dense, R− ω −continuous

1. Introduction, basic definitions and preliminaries

A topological space (X,T) is called *resolvable* [2] if it has a pair of disjoint dense subsets, otherwise *irresolvable*. Resolvable spaces have been studied in a paper of Hewitt [5] in 1943. Each open subspace of a resolvable space is also resolvable. " "and" cl " denote interior and closure respectively. Let (X,T) and (Y,σ) be topological spaces, recall that a function $f:(X,T)\to(Y,\sigma)$ is called *somewhat open* [2] if for each nonempty open set U, $int\ f(U)\neq\emptyset$, and we call it *somewhat continuous* [2] if for each open set V with $f^{-1}(V)\neq\emptyset$, $int\ f^{-1}(V)\neq\emptyset$. We shall call a function $f:(X,T)\to(Y,\sigma)$ to be *weakly surjective* [2] if for each non empty open set V in (Y,σ), $f^{-1}(V)\neq\emptyset$. Note that a surjective function is weakly surjective but not conversely.

A point x in X is called *condensation* point of A if for each U in T with x in U, the set $U\cap A$ is un countable [4]. In 1982 the ω −closed set was first introduced by H. Z. Hdeib in [4], and he defined it as: A is ω −*closed* if it contains all its condensation points and the ω −*open* set is the complement of the ω −closed set. Equivalently, a sub set W of a space (X,T), is ωω −open if and only if for each $x \in W$, there exists $U \in T$ such that $x \in U$and $U \backslash W$ is countable. In 1982 in [6] A. S. Mashhour, M. E. Abd El- Monsef and S. N. El-Deeb, introduced the pre –open set and they defined it as: The set A in the space X is called *pre –open* if and only if $A \subseteq int(cl(A))$. In 2009 in [7] T. Noiri, A. Al-Omari, M. S. M. Noorani introduced pre −ω −open, set which is weaker than ω −open set, and defined as: A subset A of a space X is called *pre −ω −open* if $A \subseteq int_{\omega}(cl(A))$. A space (X,T) is called a *door space* [7] if every subset of X is either open or closed. In our work we make use of some conditions, to define it we need the following primary definitions: [7] A subset A of a space X is called An $\omega - t$ −*set*, if $int(A) = int_{\omega}(cl(A))$. An $\omega - B$ −*set* if $A = U \cap V$, where U is an open set and V is an $\omega - t$ −set. An ω −*set* if $A = U \cap V$, where U is an open set and $int(V) = int_{\omega}(V)$. Let (X,T) be topological space. It said to be satisfy: The ω −*condition* [3] if every ω −open set is ω −set. The $\omega - B$ −*condition* [3] if every pre − ω −open is ω − B −set. Now let us introduce the following lemma from [7].

Lemma 1.3. [7] For any subset A of a space X, We have

1. A is open if and only if A is ω −open and ω −set.

2. A is open if and only if A is pre − ω −open and ω − B −set.

Definition 1.4. [7]. A topological space (X,T) is called *anti-locally countable* if every non empty open set is uncountable.

Lemma 1.5. [7]. Let (X,T) be an anti-locally countable topological space and A a subset of X. If A is pre − ω −open, then it is pre −open.

Lemma 1.6. [7] If (X,T) is a door space, then every pre − ω −open set is ω −open.

2. Generalizations of somewhat open and somewhat continuous functions

In this article we generalize the concept of resolvability by mean of generalized open sets.

Definition 2.1. A sub set A of (X,T) is called ω −*dense* (*pre-dense,* and *pre-* ω −*dense*) in X, if and only if $int_{\omega}(cl(A)) = X$, $(int_{pre}(cl(A)) = X$, and $int_{pre-\omega}(cl(A)) = X$).

Proposition.2.2. Let (X,T) be a topological space, then

1. If A is dense set in X, then it is ω −dense in X.

2. If A is ω −dense set in X, then it is pre−ω −dense in X.

3. If A is pre−dense set in X, then it is pre−ω −dense in X.
4. If A is dense set in X, then it is pre−dense in X.

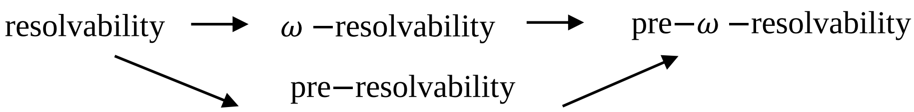
Proof. Easily using the facts

$$int(A) \subseteq int_{\omega}(A) \subseteq int_{pre-\omega}(A)$$
$$int(A) \subseteq int_{\text{pre}}(A) \subseteq int_{pre-\omega}(A).$$

Definition 2.3. A space X is called ω −resolvable (pre-resolvable, and pre− ω −resolvable) if and only if it contains two disjoint ω −dense (pre−dense, and pre− ω −dense) subset of X. Otherwise it is ω −irresolvable (pre−iresolvable, and pre− ω −irresolvable).
Note that every open subspace of resolvable space is resolvable [2]. Also a product space is resolvable if one factor is resolvable [5].

Proposition 2.4. Any resolvable space is pre−ω −resolvable.

Proof. Let X be resolvable space, so there exist two disjoint sets A_1, dense (so pre−ω −dense) in X and A_2 dense (so pre−ω −dense) in X, this implies X is pre−ω −resolvable.



Definition 2.5. A function $f:(X,T) \to (Y,\sigma)$ is called *somewhat* ω *−open (somewhat pre−open* and *somewhat pre−*ω *−open*) if and only if for each nonempty open set U, $(int_{\omega}(f(U)) \neq \emptyset$,$(int_{pre}(f(U)) \neq \emptyset$ and $int_{pre-\omega}(f(U)) \neq \emptyset$).

Remark 2.6. Any somewhat open function is somewhat *pre*− ω −open. In fact: let $f:(X,T) \to (Y,\sigma)$ be a somewhat open function and let U be a nonempty open set in X. We have $\emptyset \neq int(f(U)) \subseteq int_{pre-\omega}(f(U))$, this implies $int_{pre-\omega}(f(U)) \neq \emptyset$ and f is somewhat pre− ω −open function.

Remark 2.7. As the above remark the following statements are true:
1. Any somewhat pre-open function is somewhat pre− ω −open.
2. Any somewhat open function is somewhat ω −open.
3. Any somewhat ω −open function is somewhat pre−ω −open.

Proposition 2.8. Let (X,T) and (Y,σ) be two topological spaces, and $f:(X,T) \to (Y,\sigma)$ is a function, then

1. If (X,T) is anti-locally countable topological space, then any somewhat pre$-\omega-$open function is somewhat pre$-$open.
2. If (X,T) is door space, then any somewhat pre$-\omega-$open function is somewhat $\omega-$open.
3. If (X,T) satisfy the $\omega - B-$condition then any somewhat pre$-\omega-$open function is somewhat open.
Proof. Easy.

Definition 2.9. A function $f:(X,T) \to (Y,\sigma)$ is called *somewhat $\omega-$continuous (pre$-$continuous, pre$-\omega-$continuous)* if for each open set V with $f^{-1}(V) \neq \emptyset$, $int_{\omega}\left(f^{-1}(V)\right) \neq \emptyset$ ($int_{pre}\left(f^{-1}(V)\right) \neq \emptyset$ and $int_{pre-\omega}\left(f^{-1}(V)\right) \neq \emptyset$)).

Remark 2.10. We can restate Remark 2.6, Remark 2.7 and Proposition 2.8 for similar relations among somewhat pre$-\omega-$continuity, somewhat $\omega-$continuity, and somewhat pre$-$continuity.

The following theorems from [2] give characterizations of somewhat continuous, somewhat open and weakly surjective functions in terms of dense sets:

Theorem 2.11. [2] Let (X,T) and (Y,σ) be two topological spaces. For a function $f:(X,T) \to (Y,\sigma)$ the following are equivalent:
1. f is somewhat continuous and weakly surjective.
2. $f(D)$ is dense set in (Y,σ) for each dense set D in (X,T).

Theorem 2.12. [2] Let (X,T) and (Y,σ) be two topological spaces. For a function $f:(X,T) \to (Y,\sigma)$ the following are equivalent:
1. f is somewhat open.
2. $f^{-1}(D)$ is dense set in (X,T) for each dense set D in (Y,σ).

Let us now generalize the above theorems using the concepts $\omega-$open, pre$-\omega-$open, and pre$-$open sets.

Theorem 2.13. Let (X,T) and (Y,σ) be two topological spaces. For a bijective function $f:(X,T) \to (Y,\sigma)$ the following are equivalent:
1. f is somewhat $\omega-$open.
2. $f^{-1}(D)$ is $\omega-$dense set in (X,T) for each $\omega-$dense set D in (Y,σ).

Proof. (1)$\Rightarrow$(2). Let D be an $\omega-$dense in Y. Assume $f^{-1}(D)$ is not $\omega-$dense in X. Since $X \backslash cl(f^{-1}(D))$ is nonempty open set in X, so that $int_{\omega}(f(X\backslash cl(f^{-1}(D)))) \neq \emptyset$. This implies $int_{\omega}(f(X)\backslash f\left(clf^{-1}(D)\right)) \neq \emptyset$. Then since $int_{\omega}(f(X))\backslash int_{\omega}(fclf^{-1}(D)) \supset int_{\omega}(f(X)\backslash f(clf^{-1}(D))) \neq \emptyset$.

Then $Y\setminus int_{\omega}(clff^{-1}(D)) \neq \emptyset$. This means D is not ω −dense in Y, which is a contradicts our hypothesis, so $f^{-1}(D)$ is ω −dense set in (X,T).
(2)⟹(1). Let U be a nonempty open subset of X. $D = X\setminus U$ is closed and nonempty subset of X. Then $cl(X\setminus U) = X\setminus U$. This implies $int_{\omega}cl(X\setminus U) = int_{\omega}(X) \cap int_{\omega}(U^{c}) \neq X$. So D is not ω −dense in X. Then $f^{-1}(f(D))$ not ω −dense in X. (2) implies $f(D)$ is not ω −dense in Y, so that $Y\setminus int_{\omega}clf(D) \neq \emptyset$. Therefore $\emptyset \neq Y\setminus int_{\omega}clf(D) \subseteq cl_{\omega}(Y\setminus clf(D)) \subseteq Y\setminus cl_{\omega}(f(D)) = int_{\omega}(Y\setminus f(D))$. Hence $int_{\omega}(Y\setminus f(D)) \neq \emptyset$.

Theorem 2.14. Let (X,T) and (Y,σ) be two topological spaces. For a bijective function $f:(X,T) \to (Y,\sigma)$ the following are equivalent:
1. f is somewhat pre−ω −open (pre−open).
2. $f^{-1}(D)$ is pre−ω −dense (pre−dense) set in (X,T) for each pre−ω −dense (pre−dense) set D in (Y,σ).

Proof. As the proof of Theorem 2.13.

Theorem 2.15. Let (X,T) and (Y,σ) be two topological spaces. For a bijective function $f:(X,T) \to (Y,\sigma)$ the following are equivalent:
1. f is somewhat ω −continuous .
2. $f(D)$ is ω −dense set in (Y,σ) for each ω −dense set D in (X,T).

Proof. (1)⟹(2). Let D be ω −dense in X. Assume $f(D)$ is not ω −dense in Y. Since $X\setminus cl(f(D))$ is nonempty open set in X, so that $int_{\omega}(f(Y\setminus cl(f(D)))) \neq \emptyset$. This implies $int_{\omega}(X\setminus f(clf^{-1}(D))) \neq \emptyset$. Then since

$$int_{\omega}(X\setminus int_{\omega}(f^{-1}clf(D))) \supset int_{\omega}(X\setminus f^{-1}(clf(D))) \neq \emptyset.$$

Then $X\setminus int_{\omega}(clf^{-1}f(D)) \neq \emptyset$. This means D is not ω −dense in X, which is a contradicts our hypothesis, so $f^{-1}(D)$ is ω −dense set in (X,T).
(2)⟹(1). Let V be a nonempty open subset of Y. $D = Y\setminus U$ is closed and nonempty subset of Y. Then $cl(Y\setminus V) = Y\setminus V$ This implies $int_{\omega}cl(Y\setminus V) = int_{\omega}(Y) \cap int_{\omega}(V^{c}) \neq Y$. So D is not ω −dense in X. Then $f(f^{-1}(D))$ not ω −dense in Y. (2) implies $f^{-1}(D)$ is not ω −dense in X, so that $X\setminus int_{\omega}clf^{-1}(D) \neq \emptyset$. Therefore $\emptyset \neq X\setminus int_{\omega}clf^{-1}(D) \subseteq cl_{\omega}(X\setminus clf^{-1}(D)) \subseteq X\setminus cl_{\omega}(f^{-1}(D)) = int_{\omega}(X\setminus f^{-1}(D))$. Hence $int_{\omega}(X\setminus f^{-1}(D)) \neq \emptyset$. Hence f is somewhat ω −continuous.

Theorem 2.16. Let (X,T) and (Y,σ) be two topological spaces. For a bijective function $f:(X,T) \to (Y,\sigma)$ the following are equivalent:
1. f is somewhat pre−ω −continuous (pre−continuous).

2. $f(D)$ is pre$-\omega-$dense (pre$-$dense) set in (Y,σ) for each pre$-\omega-$dense (pre$-$dense) set D in (X,T).

Proof. As the proof of Theorem2.14.

Definition 2.17. A function $f:(X,T)\to(Y,\sigma)$ is called *weak somewhat* $\omega-$*open* (*weak somewhat pre*$-\omega-$*open and weak somewhat pre*$-$*open*) if for each $\omega-$dense (pre$-\omega-$dense *and* pre$-$dense) in (Y,σ) with $int_{\omega}(D)\neq\emptyset$ ($int_{pre-\omega}(D)\neq\emptyset$ and $int_{pre}(D)\neq\emptyset$) $f^{-1}(D)$ is $\omega-$dense (pre$-\omega-$dense *and* pre$-$dense) set in (X,T).

Remark 2.18. Let (X,T) and (Y,σ) be two topological spaces:

1. If (X,T) and (Y,σ) satisfy the $\omega-B-$condition, then $f:(X,T)\to(Y,\sigma)$ is weak somewhat open if and only if it is somewhat $\omega-$open.

2. If (X,T) and (Y,σ) satisfy the $\omega-B-$condition, then $f:(X,T)\to(Y,\sigma)$ is weak somewhat open if and only if it is somewhat pre$-\omega-$open.

Theorem 2.19. The following statements are equivalent for a space (Y,σ):

1. (Y,σ) is $\omega-$irresolvable,

2. For every space (X,T), every weak somewhat $\omega-$open function $f:(X,T)\to(Y,\sigma)$ is somewhat $\omega-$open.

Proof. (1)⟹(2). Follows from Theorem 2.13 and Definition 2.17.
(2)⟹(1). Assume (Y,σ) is $\omega-$resolvable, then Y contains disjoint $\omega-$dense sets D_1 and D_2. If $X=D_1\cup\{x_0\}$, $x_0\in D_2$, $T=\{\emptyset,X,D_1\}$ and $f:(X,T)\to(Y,\sigma)$ defined by $f(x)=x, x\in X$. This implies $f^{-1}(D_2)=\{x_0\}$ which is not dense set in (X,T), this leads f is not somewhat $\omega-$open. Also if D is $\omega-$dense set in (Y,σ) such that $int_{\omega}(D)\neq\emptyset$, then $D\cap D_1\neq\emptyset$ and $f^{-1}(D\cap D_1)=D\cap D_1$ $\omega-$dense in (X,T), so $f^{-1}(D)$ is $\omega-$dense in (X,T). Hence f is weak somewhat $\omega-$open which contradicts our hypothesis. Thus (Y,σ) is $\omega-$irresolvable.

As the proof above one can obtain the following result:

Theorem 2.20. The following statements are equivalent for a space (Y,σ):

1. (Y,σ) is pre$-\omega-$irresolvable (pre$-$irresolvable).

2.For every space (X,T) every weak somewhat pre$-\omega-$open (weak somewhat pre$-$open) function $f:(X,T)\to(Y,\sigma)$, is somewhat pre$-\omega-$open (somewhat pre$-$open).

To introduce our next theorem we need to define the graph function.

Definition 2.21. [2] Let $f:X\to Y$ be a function, then the *graph function* of f is the function $g_f:X\to X\times Y$ defined by $g_f(x)=(x,f(x))$ for each $x\in X$.

Theorem 2.22. Let (X,T) and (Y,σ) are two topological spaces and $f:(X,T) \to (Y,\sigma)$ is a function. If the graph function of f is weak somewhat $\omega -$open (weak somewhat pre$-$ $\omega -$open and weak somewhat pre$-$open) then so is f.
Proof. Using the same lines of Theorem 3.5 in [2] with simple modification we can prove this theorem.

3. Some types of weak R-continuity

In this article we shall consider a function which is weaker than continuous function.
First we need the following definition from [7].

Definition 3.1. [7] Let (X,T) and (Y,σ) are two topological spaces. A function $f:(X,T) \to (Y,\sigma)$, is $\omega -$*continuous* if for each open set U of Y, $f^{-1}(U)$ is $\omega -$open in X.
On the light of this definition we can introduce the following definition

Definition 3.2. Let (X,T) and (Y,σ) are two topological spaces. A function $f:(X,T) \to (Y,\sigma)$ is *pre-continuous* (*pre*$-$ $\omega -$*continuous*), if for each open set U of Y, $f^{-1}(U)$ is pre$-$open (pre$-$ $\omega -$open) in X.

Definition 3.3. A function $f:(X,T) \to (Y,\sigma)$ is said to be $R-$*continuous* [2] ($R-$ $\omega -$*continuous, $R-$pre$-$ $\omega -$continuous, and $R-$ pre$-$continuous*) if for each resolvable ($\omega -$resolvable, pre$-$ $\omega -$resolvable, and pre$-$resolvable) open subset V in (Y,σ), $f^{-1}(V)$ is open ($\omega -$open, pre$-$ $\omega -$open, and pre$-$open) in (X,T).

Theorem 3.4. Let (X,T) be any topological spaces, (Y,σ) **is** resolvable topological space and let $f:(X,T) \to (Y,\sigma)$ be a function. If f is R$-$ $\omega -$continuous then it is $\omega -$continuous.

Proof. Let U be an open subset of $Y,$ since Y is resolvable then U is also resolvable, so it is $\omega -$resolvable, the R$-$ $\omega -$continuity of f implies $f^{-1}(U)$ is $\omega -$open. Therefore f is $\omega -$continuous.

Remark 3.5. As the proof of Theorem 3.4. One can prove the following statements:
Let (X,T) be any topological spaces, (Y,σ) i**s** resolvable topological space and let $f:(X,T) \to (Y,\sigma)$ be a function.
1. If f is R$-$ pre$-$continuous function, then it is pre$-$continuous.
2. If f is R$-$ pre$-\omega$ $-$continuous function then it is pre$-$ $\omega -$continuous.

Remark 3.6. **1.** Any $\omega -$continuous function is R$-\omega$ $-$continuous.
2. pre$-$ continuous function is R$-$pre$-$continuous.

3. Any pre$-\omega-$ continuous function is R$-$pre$-\omega-$continuous.

Proof of (1). Let (X,T) and (Y,σ) be two topological spaces, and $f:(X,T)\to(Y,\sigma)$ be an $\omega-$continuous function. Assume U be an open $\omega-$resolvable subset of Y, then $f^{-1}(U)$ is $\omega-$open in X, so that f is R$-\omega-$continuous. Similarly we can prove the other cases.

Theorem 3.7. Let (X,T) and (Y,σ) are two topological spaces, such that (X,T) satisfies the $\omega-B-$condition and (Y,σ) is door space. If $f:(X,T)\to(Y,\sigma)$ is R$-\omega-$continuous then f is R$-$pre$-$continuous.

Proof. Let f be an R$-\omega-$continuous function and let U be an open, pre-resolvable subset of Y. Since Y is door space, so that U is $\omega-$resolvable. By the R$-\omega-$continuity of f, we obtain $f^{-1}(U)$ is $\omega-$open. Then since X satisfies $\omega-B-$condition so $f^{-1}(U)$ is pre-open in X and hence f is R$-$pre$-$continuous.

Theorem 3.8. If (X,T) is door space and $f:(X,T)\to(Y,\sigma)$ is R-pre-continuous, then f is R$-\omega-$continuous.

Proof. Let f be an R$-$pre$-$continuous function and let U be an open $\omega-$resolvable set in Y, so that it is also pre$-$resolvable. Then R$-$pre$-$continuity of f implies $f^{-1}(U)$ is pre$-$open set in X. Since X is door space so that $f^{-1}(U)$ is $\omega-$open and f is R$-\omega-$continuous.

Theorem 3.9. If (X,T) is anti-locally countable topological space and (Y,σ) is any topological space. If $f:(X,T)\to(Y,\sigma)$ is R$-$pre$-\omega-$continuous, then f is R$-$pre$-$continuous.

Proof. Let f be an R$-$pre$-\omega-$continuous function and let U be an open pre-resolvable set in Y, then U is also pre$-\omega-$resolvable. The R$-$pre$-\omega-$continuity of f implies $f^{-1}(U)$ is pre$-\omega-$open in X. Then since X anti-locally countable topological space so that $f^{-1}(U)$ is pre$-$open and so that f is R$-$pre$-$continuous.

Theorem 3.10. If (Y,σ) is anti-locally countable topological space and (X,T) is any topological space. If $f:(X,T)\to(Y,\sigma)$ is R$-$pre$-$ continuous, then f is R$-$pre$-\omega-$continuous.

Proof. Let U be an open pre$-\omega-$resolvable set in Y. Since Y is anti-locally countable space, so U is pre-resolvable set in Y and by the R$-$pre$-$ continuity of f we obtain $f^{-1}(U)$ is pre$-$open so it is pre$-\omega-$open set in X and therefore f is R$-$pre$-\omega-$continuous.

Theorem 3.11. If (X,T) is door space and $(Y,\sigma)$is any topological space. If $f:(X,T)\to(Y,\sigma)$ is R$-$pre$-\omega-$continuous, then f is R$-\omega-$continuous.

Proof. Let U be an open $\omega-$resolvable set in Y, so it is also open pre$-\omega-$resolvable. R$-$pre$-\omega-$continuity of f implies $f^{-1}(U)$ is pre$-\omega-$open in X and since X is door space so that $f^{-1}(U)$ is $\omega-$resolvable in X. This implies f is R$-\omega-$continuous.

Theorem 3.12. Let (Y,σ) is a topological space satisfies $\omega-$condition, (X,T) is any topological space and $f:(X,T)\to(Y,\sigma)$. If g_f is R$-\omega-$continuous then f is R$-\omega-$continuous.
Proof. Let U be an open $\omega-$resolvable set in Y, (which is also resolvable since Y satisfies $\omega-$condition), then $X\times U$ is open and by $\omega-$condition it is also $\omega-$open set in $X\times U$, since a product space is resolvable if one factor is resolvable. Then $X\times U$ is $\omega-$resolvable. Now since g_f is R$-\omega-$continuous so that $g_f^{-1}(X\times U)=f^{-1}(V)$ is $\omega-$open in X. Hence f is R$-\omega-$continuous.

Theorem 3.13. If (Y,σ) is a topological space satisfies $\omega-B-$condition, (X,T) is any topological space, and $f:(X,T)\to(Y,\sigma)$. If g_f is R$-$ pre$-$continuous then f is R$-$ pre$-$continuous.
Proof. Using the two facts: any open set is pre$-$open and any resolvable set is pre$-$resolvable and the same lines of the proof of the above theorem we can prove this theorem.

Theorem 3.14. If (Y,σ) is a topological space satisfies $\omega-B-$condition, (X,T) is any topological space, and $f:(X,T)\to(Y,\sigma)$. If g_f is R$-$ pre$-\omega-$continuous then f is R$-$ pre$-\omega-$continuous.
Proof. Following the same lines of Theorem 3.12. with the two facts : any open set is pre$-\omega-$open and any resolvable set is pre$-\omega-$resolvable, we can prove this theorem.

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