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Results on Existence and Uniqueness of Solution of Impulsive Retarded Integro-Differential System

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Abstract

The wide application of impulsive retarded integro-differential equations in the modelling of dynamic systems is on the increase. Therefore, the analysis of the theory of the model equation in the Banach space is of great importance in mathematical sciences. In this work, theorems on the existence and uniqueness of the solution of system equation in the Banach space are formulated, and the proves are provided using a defined compact semigroup $S(.)$ of uniformly bounded linear operators on the Banach space X, generated by an infinitesimal generator $A: D(A) \rightarrow X$. Results obtained satisfies Krasnolsel'skii theorem on existence of solution on X and the Gronwall's inequality on the uniqueness of the exist solution.

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1. Introduction

Impulsive retarded integro-differential equation is an equation which involves both the integral and the derivative of the unknown function, with time lag incorporated in the state of the system, and a couple difference equations (defining the impulsive term) which are satisfied at certain fixed or variable impulse times. The application of this equation in the analysis of dynamic system arises in many fields like biological models, fluid dynamics, and chemical sciences.

The analysis of the theory of the impulsive retarded integro-differential equations in the Banach space has attracted wide interest in mathematical science, and many significant results abound in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

However, among the previous researches on retarded integro-differential systems, few are concerned with the analysis of the solution of impulsive retarded integro-differential equations of the form

$$\begin{aligned} \dot{x}(t) - Ax(t) &= g(t, x_t) + \int_{t_0}^{t} f(s, x(s)ds,\ t \in [t_0, T] \\ \Delta x|_{t=t_k} &= I_k(x(t_k^-)) \qquad (1.0) \\ x(t_0) &= x_0. \end{aligned}$$

Therefore, motivated by the work of Benchohra [5], the aim of this research is to formulate theorems and proves on the existence and uniqueness of the solution of system (1.0) in the Banach space X , for $A: D(A) \to X$ being an infinitesimal generator of a compact semigroup $S(.)$ of uniformly bounded linear operators in X .

2. Preliminary Results

Consider a piecewise continuous compact linear space $PC(J, R^n)$, such that $x(t) \in PC(J, R^n)$, for $J(t_0, T) \subset R_+$ being a compact interval in R, and $t \in J(t_0, T)$. Let $x_t \in PC([t_0 - r, t], R^n)$, *for* $x_t = x(t - r)$ defining the delay function, with a delay constant $r > 0$ Let $X \subset R^n$ be a Banach space with norm $\|.\|$ such that $\sup_{t_0 - r < \tau < t} \|x(\tau)\|_r \leq x(t)$. Again, assume there exists $I : PC(J, X) \to R^n$, for $\Delta x|_{t=t_k} = I_k(x(t_k^-)); k = 1, \ldots, m$ defining an impulsive term experienced by $x(t)$ on $J[t_0, T]$. The following definitions hold.

Definition 2.1

The resolvent set $\ell(A)$ is the collection of values (λ) such that $\lambda I - A$ maps $D(A)$ one- to-one onto X . The spectrum of A, denoted by $\partial(A)$ is the complement of the resolvent set.

Definition 2.2

If $\lambda \in \ell(A)$, then $\lambda I - A$ is invertible, with its inverse $R(\lambda) = (\lambda I - A)^{-1}$ which is a closed linear map.

Definition 2.3

Let A be an infinitesimal generator of a strongly continuous compact semigroup $S(.)$ such that

i. $S(t): D(A) \to D(A)$, and A commutes with $S(t)$

ii. for each $n \geq 0$, A^n is densely defined

iii. $\|S(t)\| \leq M$, for $M \geq 1$.

Then, there exists a strongly continuous exponentially bounded family $S(t), t \geq 0$ of bounded operators such that for $S(0) = 0$, the resolvent of A is the Laplace transform of $S(t)$ written

$$R(\lambda, A) = (\lambda I - A)^{-1} = \int_0^\infty e^{-\lambda t} S(t)dt,\ \lambda > w,\ w \in R, \qquad (2.0)$$

for $(w, \infty) \subset \ell(A)$.

Definition 2.4

The function $x(t) \in PC(J, X) \cap (t_k, t_{k+1})$ is said to be a solution of (1.0) if

i. $x(t)$ is continuous at each $t \in J(t_0, T) \subset R_+$

ii. the derivative of $x(t)$ exists and satisfies equation (1.0) for $t \neq t_k$, $\Delta x|_{t=t_k} = I_k(x(t_k^-)),\ k = 1, \ldots, m$,

so that

$$x(t) = I(t_0)x_0 + \int_{t_0}^{t} R(t-\tau)[g(\tau, x_\tau) + K(x)(\tau)]d\tau \quad + \sum_{k=1}^{n} I(t_k x(t_k^-)), \quad (2.1)$$

where $I(t_0)x_0 = \int_0^{t_0} S(t)x_0 dt$, $R(t) = \int_0^\infty \eta(t)S(t)dt$, $K(x)(t) = \int_{t_0}^{t} f(\tau, x(\tau))d\tau$,

and by definition (2.2), $\|R(t)\| \leq \dfrac{M}{\lambda}$.

The following Krasnolsel'skii theorem on existence of solution on X will be useful later.

Theorem 2.0

Let $Y \subset X$ be a close convex non empty subset. Let Q, R be two operators such that

i. $Qx + Py \in Y$, for $x, y \in Y$

ii. Q is a contraction mapping

iii. P is compact and continuous.

Then, there exists a $z \in Y$ such that $Qz + Pz = z$

Main Result

Assume the following hypothesis hold:

A1 . The functions $f, g : J \to X$ are continuous, and there exist $K_1, K_2 > 0$ such that

$\|g(t, x_t) - g(t, y_t)\| \le K_1$, and $\|f(t, x) - f(t, y)\| \le K_2$, for

$f, g \in PC([t_0, T], X),\ t \le T$.

A2. The function $Lr : J \to R_+$ satisfies $Lr = \frac{M}{\lambda} b(K_1 + K_2 b_T) \le \alpha < 1$ for $t \in J$

Theorem 2.1

Let A be an infinitesimal generator of a strongly continuous compact semigroup $\{S(t)\}$, such that $\|S(t)\| \le M,\ t \ge 0$, and $f,\ g$ satisfy A1 , L_r satisfy A2. Then system (1.0) has a unique solution for every $x_0 \in X$ if

$$L = (K_1 + K_2 b_T) \le \alpha \left(\frac{M}{\lambda} b \right)^{-1},\ 0 < \alpha < 1. \qquad (2.2)$$

Proof

Consider the operator $T : PC(J, X) \to PC(J, X)$, such that

$$(Tx)(t) = I(t_0)x_0 + \int_{t_0}^{t} R(t-\tau)[g(\tau, x_\tau) + K(x)(\tau)]d\tau + \sum_{k=1}^{n} I(t_k, (x_k^-)) . \qquad (2.3)$$

Assuming $\sup_{t \in J} \|f(t,\ 0)\| = M_1$, $\sup_{t \in [t_0 - r, T]} \|g(t,\ 0)\| = M_2$, $\sup_{t = t_k.} \|I(t_k, x(t_k^-))\| \le N_k$ and the exists

$$\varepsilon \ge \frac{1}{1-\alpha} \left[\frac{M}{\lambda} \left(b[(K_1 + M_1) + (K_2 + M_2)] + \|x_0\| \right) + N_k \right], \qquad (2.4)$$

such that $\Gamma_\varepsilon = \{x(t) \in C(J) : |t - t_0| \le b,\ \|x\|_\infty \le \varepsilon\}$ defines a close nonempty and convex set.

Therefore, for any $x \in \Gamma_\varepsilon$,

$$\begin{aligned} \|(Tx)(t)\| &\le M\|x_0\| + \frac{M}{\lambda} \int_{t_0}^{t} \|g(\tau, x_\tau) + K(x)(\tau)\| d\tau + \|I(t_k, x(t_k^-))\| \\ &\le M\|x_0\| + \frac{M}{\lambda} \int_{t_0}^{t} [\|g(\tau, x_\tau - g(\tau, 0)\| + \|g(\tau, 0)\|] d\tau + \frac{M}{\lambda} \int_{t_0}^{t} K(x)(\tau) d\tau + N_k \\ &\le M\|x_0\| + \frac{M}{\lambda} b[K_1 + M_1] + \frac{M}{\lambda} \int_{t_0}^{t} K(x)(\tau) d\tau + N_k . \end{aligned}$$

But $K(x)=\int_{\tau_0}^{\tau} f(s,x(s))ds$, such that

$$\|K(x)(\tau)\| \le \int_{t_0}^{t} [\| f(s,x(s)) - f(s,0)\| + \| f(s,0)\|]ds$$
$$\le [K_2 + M_2]b_T .$$

Hence,

$$\|(Tx)(t)\| \le \frac{M}{\lambda}\left(b[(K_1+M_1)(K_2+M_2)b_T] + \|x_0\|\right) + N_k .$$
$$\le \varepsilon$$

Again, consider $x,\ y \in PC(J,\ X)$, so that

$$\|(Tx)(t)-(Ty)(t)\| \le \frac{M}{\lambda}\int_{t_0}^{t} \|g(\tau,x_\tau) - g(\tau,y_\tau) + K(x)(\tau) - K(y)(\tau)\| d\tau + N_k$$
$$\le \frac{M}{\lambda}\int_{t_0}^{t}\left[\left\|[g(\tau,x_\tau)-g(\tau,y_\tau)] + \int_{\tau_0}^{\tau}[f(s,x(s)) - f(s,y(s))]ds\right\|\right] d\tau + N_k$$
$$\le \frac{M}{\lambda}\int_{t_0}^{t}\|g(\tau,x_\tau)-g(\tau,y_\tau)\|d\tau + \frac{M}{\lambda}\int_{t_0}^{t}\left[\int_{\tau_0}^{\tau}\| f(s,x(s)) - f(s,y(s))\|ds\right]d\tau + N_k$$
$$\le \frac{M}{\lambda} b[K_1 + K_2 b_T]\|x-y\| + N_K$$
$$\le Lr\|x-y\| + N_k$$

Therefore

$$\|(Tx)(t)-(Ty)(t)\| \le \alpha\|x-y\| + N_k . \tag{2.5}$$

This result is immediately followed by theorem (2.2), by making the following assumptions:

A3. The function $g: J\times X \to X$ is continuous, and there exists a continuous positive non decreasing function $\psi:(0,\infty)\to(0,\infty)$, and $p\in L^1(J,R_+)$ such that $\|g(t,x_t)\| \le p(t)\gamma(\|x(\tau)\|_X)$, for $\mu = \|x(\tau)\|,\ t_0 \le \tau \le t,\ t\in[t_0,T]$ and $\int_{t_0}^{t} p(\tau)d\tau \le M_p$, satisfying,

$$\nu_k(t) \le \zeta_k^{-1}\left(\frac{M}{\lambda}\int_{t_{k-1}}^{t_k} p(t)dt\right)^{-1} = \eta_k,\ \ k = 1,\ 2,\ 3,\ ... \tag{2.6}$$

Theorem 2.2

If the hypothesis of theorem (2.1) holds such that f, g satisfy (A_1), (A_3) and

$$L = K_2 b_T \le \delta\left(\frac{M}{\lambda}b\right)^{-1},\ 0<\delta<1, \tag{2.7}$$

then system (1.0) has a unique solution.

Proof

By theorem (2.0), let

$$(\varphi x)(t) = \int_{t_0}^{t} R(t-\tau)K(x)(\tau)d\tau + \sum_{k=1}^{n} I(t_k, x(t_k^-)), \tag{2.8}$$

$$(\psi x)(t) = I(t_0)x_0 + \int_{t_0}^{t} R(t-\tau)g(\tau, x_\tau)(\tau)d\tau \quad . \tag{2.9}$$

Defining

$$\varepsilon \geq \frac{1}{1-\delta}\left[\frac{M}{\lambda}\left(b[(K_2 + M_2)b_T] + \psi\|\mu\|M_p + x_0\right) + N_k\right], \tag{2.10}$$

such that $\Gamma_\varepsilon = \{x(t) \in C(J,X) : |t-t_0| \leq b,\ \|x\|_\infty \leq \varepsilon\}$ is a close nonempty subset of $PC(J,X)$.

Then, for $x, y \in \Gamma_\varepsilon$,

$$\begin{aligned}
\|(\varphi x)(t) + (\psi x)(t)\| &\leq \int_{t_0}^{t} \|R(t-\tau)K(x)(\tau)\|d\tau + \|I(t_k, x(t_k^-))\| + \|I(t_0)x_0\| + \int_{t_0}^{t} \|R(t-\tau)g(\tau, x_\tau)(\tau)\|d\tau \\
&\leq \frac{M}{\lambda}\int_{t_0}^{t} \|K(x)(\tau)\|d\tau + N_k + \frac{M}{\lambda}x_0 + \frac{M}{\lambda}\int_{t_0}^{t} \|g(\tau, x_\tau)\|d\tau \\
&\leq \frac{M}{\lambda}b[(K_2 + M_2)b_T] + N_k + \frac{M}{\lambda}x_0 + \frac{M}{\lambda}\int_{t_0}^{t} p(\tau)\psi\|x\|d\tau \\
&\leq \frac{M}{\lambda}b[(K_2 + M_2)b_T] + N_k + \frac{M}{\lambda}x_0 + \frac{M}{\lambda}\psi(\mu)\int_{t_0}^{t} p(\tau)d\tau \\
&\leq \frac{1}{1-\delta}\left[\frac{M}{\lambda}\left(b[(K_2 + M_2)b_T] + \psi\|\mu\|M_p + x_0\right) + N_k\right] \\
&\leq \varepsilon
\end{aligned}$$

and so $(\varphi x)(t) + (\psi x)(t) \in \Gamma_\varepsilon$.

Again, using equation (2.8), for $x, y \in X$, then by hypothesis A_1,

$$\begin{aligned}
\|(\varphi x)(t) - (\varphi y)(t)\| &\leq \left\|\left[\int_{t_0}^{t} R(t-\tau)K(x)(\tau)d\tau + I(x_k(t_k^-))\right] - \left[\int_{t_0}^{t} R(t-\tau)K(y)(\tau)d\tau + I(y_k(t_k^-))\right]\right\| \\
&\leq \frac{M}{\lambda}b\int_{t_0}^{t} \|K(x)(\tau) - K(y)(\tau)\|d\tau - \|I(x_k(t_k^-))\| \\
&\leq \frac{M}{\lambda}b[K_2 b_T]\|x-y\| + N_k \\
&\leq \delta\|x-y\| + N_k
\end{aligned}$$

Therefore, $(\varphi x)(t)$ is a contraction mapping.

Also by equation (2.9) and hypothesis A3

$$\|(\psi x)(t)\| \le Mx_0 + \frac{M}{\lambda}\int_{t_0}^{t} p(\tau)\psi\|x(\tau)\|d\tau . \tag{2.11}$$

Consider the function $\|x(\tau)\| \le \mu,\ t_0 \le \tau \le t,\ t \in [t_0, T]$, then

$$(\psi v)(t) \le M\mu_0 + \frac{M}{\lambda}\int_{t_0}^{t} p(\tau)\psi(\mu)d\tau ,$$

for $C = M\mu_0 = v(0),\ v(t) \ge \mu(t),\ \ t \in [t_0, t_1]$,

$$(\psi \dot{v})(t) = \frac{M}{\lambda} p(t)\psi(\mu(t)) \le \frac{M}{\lambda} p(t)\gamma(v(t))$$

$$\int_{v(0)}^{v(t)} \frac{dv}{\psi(v)} \le \frac{M}{\lambda}\int_{t_0}^{t_1} p(t)dt$$

$$(\psi v_1)(t) \le \zeta_1^{-1}\left(\frac{M}{\lambda}\int_{t_0}^{t_1} p(t)dt\right)^{-1} = \eta_1 ,$$

and so for $\|x_1(\tau)\|_{t_0 \le \tau \le t} \le \mu_1,\ \ t \in [t_0, t_1]$,

$$\sup\|(\psi x)(t)\| \le \eta_1 . \tag{2.12}$$

Continuing this process for $t \in [t_n, T],\ n = 0, 1, 2, ...$

$$(\psi x_n)(t) \le M\mu_0 + \frac{M}{\lambda}\int_{t_0}^{t_n} p(t)dt, \tag{2.13}$$

so that for any constant η_n , $\sup_{t \in [t_n - T]} \|(\psi x)(t)\| \le \eta_n$,

$$\|(\psi x_n)(t)\| \le v_n \le \zeta_n^{-1}\left(\frac{M}{\lambda}\int_{t_0}^{t_n} p(t)dt\right)^{-1} = \eta_n$$

and

$$\|(\psi x(t))\| \le \{Mx_0, \eta_n,\ n = 1, 2, 3, ...\} = \eta^* . \tag{2.14}$$

Hence, $(\psi x)(t) \in Y \subset X$, and $\psi : Y \to Y$.
Showing that $(\psi x)(t)$ is relatively compact:
Since $Y \subset X$ is bounded, then $(\psi x)(t) \in Y$ is bounded and equicontinuous. Indeed, let $t_1, t_1 \in J$, such that for any $\in > 0,\ \in < t_1 < t_2$, and the following implies.

$$\|(\psi x)(t_2)-(\psi x)(t_1)\| = \left\| \int_{t_0}^{t_1} R(t_1-\tau)g(\tau,x_\tau)d\tau - \int_{t_0}^{t_2} R(t_2-\tau)g(\tau,x_\tau)d\tau \right\|$$
$$\leq \int_{t_0}^{t_1-\in} \|R(t_2-\tau)-R(t_1-\tau)\| \|g(\tau,x_\tau)\| d\tau$$
$$+\int_{t_1}^{t_1-\in} \|R(t_2-\tau)-R(t_1-\tau)\| \|g(\tau,x_\tau)\| d\tau$$
$$+\int_{t_1}^{t_2} \|R(t_2-\tau)\| \|g(\tau,x_\tau)\| d\tau$$

The right hand side will tend to zero as $t_2 \to t_1$, for $\in$ sufficiently small, as a consequence of the continuity of $S(t)$, in the uniform operator topology for $t > 0$ and the compactness of $S(t)$. Thus $\|(\psi x)(t_2)-(\psi x)(t_1)\| \to 0$ as $t_2 \to t_1$, which is independent of x_t. As a consequence of Arzela- Ascoli theorem, it suffix to show that ψ maps Y into a precompact set X. Let $\in > 0$ be any value satisfying $0 < \in < t$. Then

$$(\psi_\in x)(t) = I(t_0)x_0 + R(\in)\int_{t_0}^{t-\in} R(t-\tau-\in)g(\tau,x_\tau)d\tau \,. \tag{2.15}$$

Since $S(t)$ is a compact operator for each $t \in J$, the set $U_\in = \{(\psi_\in x)(t) \mid x \in \Gamma_\varepsilon\}$ is relatively compact in X for each $\in$, $0 < \in < t$. And so

$$\|(\psi x)(t)-(\psi_\in x)(t)\| = \left\| \int_{t_0}^{t} R(t-\tau)g(\tau,x_\tau)d\tau - R(\in)\int_{t_0}^{t-\in} R(t-\in-\tau)g(\tau,x_\tau)d\tau \right\|$$
$$\leq \frac{M}{\lambda} M_\in \int_{t-\in}^{t} \|g(\tau,x_\tau)\| d\tau$$
$$\leq \frac{M}{\lambda} M_\in \psi(u) \int_{t-\tau}^{t} p(\tau)d\tau$$
$$\leq \frac{M}{\lambda} M_\in \psi(u) M_p,$$

which implies that $U_\in$ is relatively compact in X.

Showing that ψ is continuous:

Let $\{x_n\}$ be a sequence in Γ_ε such that $x_n \to x$ for $x \in X$, and by the continuity of g in $J \times X$, then $g(t,x_{nt}) \to g(t,x_t)$, *for* $n \to \infty$. So

$$\|(\psi x_n)(t)-(\psi x)(t)\| = \left\| \int_{t_0}^{t} R(t-\tau)[g(\tau,x_{n\tau})-g(\tau,x_\tau)]d\tau \right\|$$
$$\leq \frac{M}{\lambda} \int_{t_0}^{t} \|g(\tau,x_{n\tau})-g(\tau,x_\tau)\| d\tau$$
$$\leq \frac{M}{\lambda} b \|g(\tau,x_{n\tau})-g(\tau,x_\tau)\| \to 0, \ as \ n \to \infty,$$

which implies

$$\lim_{n\to\infty}\|(\psi x_n)(t)-(\psi x)(t)\|=0. \quad (2.16)$$

Therefore ψ is continuous, and Krasnoselskii hypothesis of theorem (2.1) hold. By inference, system (1.0) has a solution $x(t)$, defined by equation (2.1).
Proving uniqueness:

Let $y(t)$ be another solution of (1.0), then

$$\|x(t)-y(t)\|\le\int_{t_0}^{t}\|R(t-\tau)\|\left[\|g(\tau,x_\tau)-g(\tau,y_\tau)\|+\|K(x)(\tau)-K(y)(\tau)\|\right]d\tau$$
$$\le\frac{M}{\lambda}\int_{t_0}^{t}[K_1+K_2b_T]\|x(\tau)-y(\tau)\|d\tau. \quad (2.17)$$

Defining $\theta(t)=\|x(t)-y(t)\|$, then (2.17) implies

$$\theta(t)\le\frac{M}{\lambda}\int[K_1+K_2b_T]\theta(\tau)d\tau, \quad (2.18)$$

and Gronwall's inequality holds by equation (2.18), which implies uniqueness of $x(t)$.

3. Conclusion

An impulsive retarded integro-differential system was considered. Theorems on existence and uniqueness of the system solution in the Banach space were formulated using a defined compact semigroup $S(.)$ of uniformly bounded linear operators on the Banach space X, generated by an infinitesimal generator $A:D(A)\to X$. The formulated theorems were proves to satisfies the Krasnolsel'skii theorem on existence of solution on X, and the Gronwall's inequality on the uniqueness of the exist solution. Results obtained are improvement on the qualitative analysis of impulsive retarded integro-differential systems.

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