Unknown Input Observer Design for a Class of Nonlinear Descriptor Systems: A Takagi-Sugeno Approach with Lipschitz Constraints

Abdellah Louzimi\textsuperscript{1,2}, Abdellatif El Assoudi\textsuperscript{1,2}, Jalal Soulami\textsuperscript{1,2*} and El Hassane El Yaagoubi\textsuperscript{1,2}

\textsuperscript{1}Laboratory of High Energy Physics and Condensed Matter, Faculty of Science Hassan II University of Casablanca, B.P 5366, Maarif, Casablanca, Morocco
\textsuperscript{2}ECPI, Department of Electrical Engineering, ENSEM Hassan II University of Casablanca, B.P 8118, Oasis, Casablanca Morocco
*Corresponding author

Copyright © 2016 Abdellah Louzimi, Abdellatif El Assoudi, Jalal Soulami and El Hassane El Yaagoubi. This article is distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

This paper deals with the problem of the unknown inputs observer (UIO) design for nonlinear descriptor systems (NDS) described by Takagi-Sugeno (T-S) structure with unmeasurable premise variables satisfying Lipschitz conditions. The unknown inputs affect both state and output of the system. Indeed, the T-S fuzzy observer is synthesized in explicit form to estimate simultaneously the system state and the unknown inputs. The main idea of the proposed result is based on the separation between dynamic and static relations in the T-S descriptor model. Firstly, the method used to separate the dynamic equations from the algebraic equation is developed. Secondly, the fuzzy UIO design satisfying Lipschitz conditions and permitting the estimation of the unknown states and unknown inputs is proposed. The developed approach for the observer design is based on the synthesis of the augmented fuzzy models which regroups the differential variables and unknown inputs. The convergence of the state estimation error is studied by using the Lyapunov theory and the stability condition is given in term of only one Linear Matrix Inequality (LMI). Finally, an application to a descriptor model
of a single-link flexible joint robot is presented in order to illustrate the validity and applicability of the theoretical development.

**Key words**: Nonlinear descriptor system, Takagi-Sugeno descriptor model, Unknown input observer, Lipschitz condition, LMI

1 **Introduction**

In the last two decades, T-S fuzzy approach have been widely and successfully used in the nonlinear processes modeling to describe the behavior of many chemical and physical processes. The idea of this approach is to apprehend the global behavior of a process by a set of local models [1], [2]. The advantage of such approach relies on the fact that once the T-S fuzzy models are obtained, some analysis and design tools developed in the linear case can be used, which facilitates observer or/and controller synthesis for complex nonlinear systems see for example [3], [4] and the references therein.

In the present work, the aim is to design a fuzzy UIO for continuous time NDS described by T-S structure with unmeasurable premise variables satisfying Lipschitz conditions. Recall that, descriptor systems variously called implicit systems or singular systems or differential-algebraic equations (DAEs) have been widely used in the modeling of dynamic processes to describe the behavior of many chemical and physical processes see for example [5], [6], [7], [8] and references therein. This formulation includes both dynamic and static relations. The numerical simulation of implicit models usually combines a resolution routine of an ordinary differential equation together with an optimization algorithm.

However, the UIO design problem widely used in the area of fault detection and design of fault tolerant control strategy has received considerable attention and is still an active area of research. Indeed, many works on fuzzy UIO and its application to fault detection for T-S systems described by ordinary differential equations exist in the literature. We may cite [9], [10], [11], [12], [13], [14], [15], [16]. Likewise, for T-S fuzzy descriptor systems submitted to unknown inputs, several developments exist in the literature. More precisely, various works dealing with UIO design and application to fault diagnosis in explicit form were also proposed for implicit T-S models see for example [17], [18], [19], [20], [21], [22], [23], [24] and many references therein. Notice that, generally an interesting way to solve the various fuzzy UIO problems raised previously is to write the convergence conditions on the LMI form [25].

In this work, the main contribution consists to develop a new design methodology based on the separation between the dynamic and static relations in the T-S descriptor model. More precisely, UIO design for a class of continuous time T-S descriptor models with unmeasurable premise variables satisfying
the Lipschitz conditions allowing the simultaneous estimation of the unknown states and unknown inputs is proposed. The exponential stability of the state estimation error is studied by using the Lyapunov theory and the stability condition is given in term of only one LMI. Besides, the proposed fuzzy UIO is synthesized by only an explicit structure.

The outline of the paper is as follows. The class of NDS described by T-S descriptor models with unknown input and unmeasurable premise variables is presented in section 2. The main result about fuzzy UIO design which satisfying Lipschitz conditions permitting to estimate unknown states and unknown inputs is stated in section 3. Finally, in section 4, we illustrate the performance of the developed result in simulation through a descriptor model of the one-link flexible joint robot.

Throughout the paper, some notations used are fair standard. For example, $X > 0$ means the matrix $X$ is symmetric and positive definite. $X^T$ denotes the transpose of $X$. The symbol $I$ (or 0) represents the identity matrix (or zero matrix) with appropriate dimension.

### 2 Takagi-Sugeno descriptor systems

In this paper, the aim consists to considering the problem of UIO design for a class of NDS described by T-S structure with unmeasurable premise variables. For this objective, the following NDS with unknown inputs is considered:

\[
\begin{align*}
E \dot{x} &= A(x)x + B(x)u + F(x)d \\
y &= Cx + Du + Gd
\end{align*}
\] (1)

where $x = [X_1^T X_2^T]^T \in \mathbb{R}^n$ is the state vector with $X_1 \in \mathbb{R}^{n_1}$ is the vector of differential variables, $X_2 \in \mathbb{R}^{n_2}$ is the vector of algebraic variables with $n_1 + n_2 = n$, $u \in \mathbb{R}^m$ is the control input, $d \in \mathbb{R}^\sigma$ is the unknown control input, $y \in \mathbb{R}^p$ is the measured output. $A(x) \in \mathbb{R}^{n \times n}$, $B(x) \in \mathbb{R}^{n \times m}$, $F(x) \in \mathbb{R}^{n \times \sigma}$ are nonlinear matrices functions. $E \in \mathbb{R}^{n \times n}$ with $rank(E) = n_1$, $C \in \mathbb{R}^{p \times n}$, $D \in \mathbb{R}^{p \times m}$, $G \in \mathbb{R}^{p \times \sigma}$ are real known constant matrices with:

\[
E = \begin{pmatrix}
I & 0 \\
0 & 0
\end{pmatrix}; \quad C = \begin{pmatrix}
C_1 & 0
\end{pmatrix}
\] (2)

To design a T-S fuzzy observer, we need a T-S fuzzy model for the NDS (1). In general, there are two approaches for constructing fuzzy models: identification (fuzzy modeling) using input-output data and derivation from given nonlinear system equations. In this paper, we use the second approach which derives a fuzzy model from given NDS (1). By the sector nonlinearity approach [3],

...
system (1) can be exactly represented by the T-S descriptor systems:

\[
\begin{cases}
E \dot{x} = \sum_{i=1}^{q} \lambda_i(x)(A_i x + B_i u + F_i d) \\
y = C x + D u + G d
\end{cases}
\] (3)

where \( A_i \in \mathbb{R}^{n \times n} \), \( B_i \in \mathbb{R}^{n \times m} \), \( F_i \in \mathbb{R}^{p \times \sigma} \) are real known constant matrices with:

\[
A_i = \begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix}; \quad B_i = \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix}; \quad F_i = \begin{pmatrix} F_{1i} \\ F_{2i} \end{pmatrix}
\] (4)

where constant matrices \( A_{22i} \) are supposed invertible. \( q \) is the number of sub-models. The \( \lambda_i(x(t)) \) \( (i = 1, \ldots, q) \) are the weighting functions that ensure the transition between the contribution of each sub model:

\[
\begin{cases}
E \dot{x} = A_i x + B_i u + F_i d \\
y = C x + D u + G d
\end{cases} \] (5)

They verify the so-called convex sum properties:

\[
\begin{cases}
\sum_{i=1}^{q} \lambda_i(x) = 1 \\
0 \leq \lambda_i(x) \leq 1 \quad i = 1, \ldots, q
\end{cases}
\] (6)

Before giving the main result, let us make the following assumption [5], [20]:

**Assumption 2.1**: Suppose that:

- \((E, A_i)\) is regular, i.e. \(\text{det}(sE - A_i) \neq 0 \forall s \in \mathbb{C}\)
- All sub-models (5) are impulse observable and detectable.

In order to investigate the UIO design for T-S descriptor system (3), the approach is based on the separation between differential and algebraic equations in each sub-model (5) and the global fuzzy model is obtained by aggregation of the resulting sub-models.

So, using (2) and (4), system (5) can be rewritten as follows:

\[
\begin{cases}
\dot{X}_1 = A_{11i} X_1 + A_{12i} X_2 + B_{1i} u + F_{1i} d \\
0 = A_{21i} X_1 + A_{22i} X_2 + B_{2i} u + F_{2i} d \\
y = C_1 X_1 + D u + G d
\end{cases}
\] (7)

The form (7) for system (5) is also known as the second equivalent form [5].

From (7) and using the fact that \( A_{22i}^{-1} \) exists, the algebraic equations can be solved directly for algebraic variables, to obtain:

\[
X_2 = -A_{22i}^{-1} A_{21i} X_1 - A_{22i}^{-1} B_{2i} u - A_{22i}^{-1} F_{2i} d
\] (8)
Substitution of the resulting expression for $X_2$ in (7) yields the following model:

$$
\begin{align*}
X_1 &= M_i X_1 + N_i u + P_i d \\
X_2 &= J_i X_1 + K_i u + L_i d \\
y &= C_1 X_1 + D u + G d
\end{align*}
$$

(9)

where

$$\begin{align*}
M_i &= A_{11i} - A_{12i} A_{22i}^{-1} A_{21i} \\
N_i &= B_{1i} - A_{12i} A_{22i}^{-1} B_{2i} \\
P_i &= F_{1i} - A_{12i} A_{22i}^{-1} F_{2i} \\
J_i &= -A_{22i}^{-1} A_{21i} \\
K_i &= -A_{22i}^{-1} B_{2i} \\
L_i &= -A_{22i}^{-1} F_{2i}
\end{align*}
$$

(10)

The weighting functions $\lambda_i(x)$, $i = 1, \ldots, q$ can be rewritten as

$$\lambda_i(x) = \lambda_i(X_1, X_2 = J_i X_1 + K_i u + L_i d) = \lambda_i(X_1, u, d) = \lambda_i(\eta)$$

(11)

with $\eta = [X_1^T \ u^T \ d^T]^T$.

In descriptor form, system (9) can be rewritten as follows:

$$\begin{align*}
E \dot{x} &= M_i x + N_i u + P_i d \\
y &= C x + D u + G d
\end{align*}
$$

(12)

where

$$\begin{align*}
M_i &= \begin{pmatrix} M_i & 0 \\ J_i & -I \end{pmatrix} \\
N_i &= \begin{pmatrix} N_i \\ K_i \end{pmatrix} \\
P_i &= \begin{pmatrix} P_i \\ L_i \end{pmatrix}
\end{align*}
$$

(13)

Then, fuzzy descriptor system (3) takes the following equivalent form:

$$\begin{align*}
E \dot{x} &= \sum_{i=1}^{q} \lambda_i(\eta)(M_i x + N_i u + P_i d) \\
y &= C x + D u + G d
\end{align*}
$$

(14)

which can be rewritten in the following model:

$$\begin{align*}
\dot{X}_1 &= \sum_{i=1}^{q} \lambda_i(\eta)(M_i X_1 + N_i u + P_i d) \\
X_2 &= \sum_{i=1}^{q} \lambda_i(\eta)(J_i X_1 + K_i u + L_i d) \\
y &= C_1 X_1 + D u + G d
\end{align*}
$$

(15)

Assumption 2.2: Suppose that $d$ is considered as an unknown control input that may slow variation:

$$\dot{d} = 0$$

(16)

with $d(0)$ unknown.
Let us define the augmented state vector \( Z_1 = [X_1^T \ d^T]^T \) and \( Z_2 = X_2 \).
Thus, the system (15) can be represented as:

\[
\begin{align*}
\dot{Z}_1 &= \sum_{i=1}^{q} \lambda_i(\eta)(\tilde{M}_i Z_1 + \tilde{N}_i u) \\
Z_2 &= \sum_{i=1}^{q} \lambda_i(\eta)(\tilde{J}_i Z_1 + K_i u) \\
y &= R Z_1 + D u
\end{align*}
\]

(17)

where

\[
\begin{align*}
\tilde{M}_i &= \begin{pmatrix} M_i & P_i \\ 0 & 0 \end{pmatrix} \\
\tilde{N}_i &= \begin{pmatrix} N_i \\ 0 \end{pmatrix} \\
\tilde{J}_i &= \begin{pmatrix} J_i \\ L_i \end{pmatrix} \\
R &= \begin{pmatrix} C_1 \\ G \end{pmatrix}
\end{align*}
\]

(18)

In order to take an observer for system (17), we introduce the following matrices:

\[
W_0 = \frac{1}{q} \sum_{i=1}^{q} W_i \\
\bar{W}_i = W_i - W_0
\]

(19)

where \( W_0 = M_0, \ N_0, \ J_0, \ K_0 \) and \( \bar{W}_i = \tilde{M}_i, \ \tilde{N}_i, \ \tilde{Q}_i, \ \tilde{R}_i \) with \( W_i = M_i, \ \tilde{N}_i, \ \tilde{J}_i, \ K_i \).
By using (19), (17) takes the following equivalent form:

\[
\begin{align*}
\dot{\hat{Z}}_1 &= M_0 Z_1 + N_0 u + \sum_{i=1}^{q} \lambda_i(\eta)(\tilde{M}_i Z_1 + \tilde{N}_i u) \\
\dot{\hat{Z}}_2 &= J_0 Z_1 + K_0 u + \sum_{i=1}^{q} \lambda_i(\eta)(\tilde{J}_i Z_1 + \tilde{K}_i u) \\
\hat{y} &= R \hat{Z}_1 + D u
\end{align*}
\]

(20)

\section{Main result}

In this section, the aim is to suggest a new method for the UIO design of T-S descriptor model (1) with unmeasurable premise variables. Based on the transformation of the T-S descriptor system (1) into the equivalent form (15), the proposed UIO is given by the following equations:

\[
\begin{align*}
\dot{\hat{Z}}_1 &= M_0 \hat{Z}_1 + N_0 u - H(\hat{y} - y) + \sum_{i=1}^{q} \lambda_i(\eta)(\tilde{M}_i \hat{Z}_1 + \tilde{N}_i u) \\
\dot{\hat{Z}}_2 &= J_0 \hat{Z}_1 + K_0 u + \sum_{i=1}^{q} \lambda_i(\eta)(\tilde{J}_i \hat{X}_1 + \tilde{K}_i u) \\
\hat{y} &= R \hat{Z}_1 + D u
\end{align*}
\]

(21)
where \((\hat{Z}_1, \hat{Z}_2), \hat{y} \) and \(\hat{\eta} \) denote the estimated augmented state vector, the output vector and the decision variable vector respectively. \(H \) is the gain of UIO which is determined such that \((\hat{Z}_1, \hat{Z}_2)\) asymptotically converges to \((Z_1, Z_2)\).

In order to establish the conditions for the asymptotic convergence of the observer (21), we define the state estimation error:

\[
e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} \hat{Z}_1 - Z_1 \\ \hat{Z}_2 - Z_2 \end{pmatrix}
\]  

From systems (20) and (21), the dynamics of the estimation error is given by:

\[
\begin{cases}
\dot{e}_1 = \Omega_0 e_1 + \delta \\
\dot{e}_2 = J_0 e_1 + \sum_{i=1}^{q} \lambda_i(\hat{\eta}) \tilde{J}_i e_1 + \sum_{i=1}^{q} (\lambda_i(\hat{\eta}) - \lambda_i(\eta))(\tilde{J}_i Z_1 + \tilde{K}_i u)
\end{cases}
\]  

where

\[
\begin{cases}
\Omega_0 = M_0 - HR \\
\delta = \sum_{i=1}^{q} (\tilde{M}_i \delta_1 + \delta_2 u)
\end{cases}
\]  

with

\[
\begin{cases}
\delta_1 = \lambda_i(\hat{\eta}) \tilde{Z}_1 - \lambda_i(\eta) Z_1 \\
\delta_2 = \tilde{N}_i (\lambda_i(\hat{\eta}) - \lambda_i(\eta))
\end{cases}
\]  

**Assumption 3.1**: Assume that the following conditions hold:

\[
\begin{cases}
|\delta_1| < \mu_i |e_1| \\
|\delta_2| < \nu_i |e_1| \\
|u| < \rho
\end{cases}
\]  

where \(\mu_i, \nu_i\) are positives scalars Lipschitz constants and \(\rho > 0\).

Using the Assumption 3.1, the term \(\delta\) can be bounded as follows:

\[|\delta| < \alpha |e_1|\]

where

\[
\alpha = \sum_{i=1}^{q} (\sigma(\tilde{M}_i) \mu_i + \nu_i \rho)
\]  

where \(\sigma(\tilde{M}_i)\) denote the maximum singular value of the matrix \(\tilde{M}_i\).

Note that to prove the convergence of the estimation error \(e\) toward zero, it suffice to prove from (23), that \(e_1\) converges toward zero. Then, the following results can be stated.
Theorem 3.2: Under above Assumption 3.1, the system (23) is globally exponentially stable if given $\gamma > 0$ there exists matrices $P > 0$, $Q > 0$ and $U$ verifying the following LMI:

\[
\begin{pmatrix}
M_0^T P + PM_0 - RT^TU - UR + \alpha^2 Q + 2\gamma P & P \\
P & -Q
\end{pmatrix} < 0
\] (29)

The gain $H$ of the observer (21) is computed by:

\[H = P^{-1}U \] (30)

Proof of Theorem 3.2: Consider the following candidate of quadratic function:

\[V = e_1^T Pe_1 \quad P > 0 \] (31)

The time derivative of the Lyapunov function (31) along the trajectories of the system (23) is obtained as:

\[\dot{V} = e_1^T (\Omega_0^T P + P\Omega_0)e_1 + \delta^T Pe_1 + e_1^T P\delta \] (32)

Lemma 3.3: For any matrices $X$ and $Y$ with appropriate dimensions, the following property holds for any invertible matrix $J$:

\[X^TY + Y^TX \leq X^TJ^{-1}X + Y^TJY \] (33)

For $Q > 0$, by applying Lemma 3.3, (32) becomes:

\[\dot{V} < e_1^T (\Omega_0^T P + P\Omega_0 + PQ^{-1}P)e_1 + \delta^T Q\delta \] (34)

Taking into account (27), (34) becomes:

\[\dot{V} < e_1^T (\Omega_0^T P + P\Omega_0 + PQ^{-1}P + \alpha^2 Q)e_1 \] (35)

Thus, estimation error convergence is exponentially ensured if the following condition is guaranteed:

\[\dot{V} < e_1^T (\Omega_0^T P + P\Omega_0 + PQ^{-1}P + \alpha^2 Q)e_1 < -2\gamma V(e_1) \quad \gamma > 0 \] (36)

which is equivalent to the following condition:

\[\Omega_0^T P + P\Omega_0 + PQ^{-1}P + \alpha^2 Q + 2\gamma P < 0 \] (37)

Replacing $\Omega_0$ from (24) into (37), we can establish the LMI condition (29) of Theorem 3.2 by using the Schur complement [25] and the following change of variables:

\[U = PH \] (38)

Thus, from the Lyapunov stability theory, if the LMI condition (29) is satisfied, the system (23) is exponentially asymptotically stable. This completes the proof of Theorem 3.2.
4 Application to one-link flexible joint robot

In order to illustrate the efficiency and applicability of the proposed algorithm (21), let us consider the one-link flexible joint robot described by the dynamical model given in [26] which can be described by the following NDS:

\[
\begin{align*}
\dot{E}x &= A(x)x + Bu + Fd \\
y &= Cx \\
\end{align*}
\]  

(39)

where \(x = (x_1, x_2, x_3, x_4, x_5, x_6)^T\) is the state vector with \(x_1\) and \(x_2\) are the angles of rotations of the motor and the link respectively. \(x_3\) and \(x_4\) are their angular velocities. \(x_5\) and \(x_6\) are their angular accelerations. \(u\) is the control variable, \(y\) is the output measurement vector and \(d\) is the unknown input variable.

\[
A(x) = \begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-\theta_1 & \theta_1 & \theta_2 & 0 & 0 & 1 \\
\theta_1 & -\theta_2 & 0 & 0 & 0 & 1 \\
\eta & 0 & 0 & 0 & -1 \\
\end{pmatrix}, \quad B = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1 \\
\end{pmatrix}, \quad F = \begin{pmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\]

\[
E = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}, \quad C = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
\end{pmatrix}
\]

(41)

where

\[
\eta = -\frac{\theta_1}{\theta_2} - \frac{mb \sin(x_2)}{x_2}
\]

(40)

To express the model (39) process as a T-S model with unmeasurable premise variable, we consider the sector of nonlinearities of the term \(\eta \in [\eta_{\min}, \eta_{\max}]\) of the matrix \(A(x)\). Then, we can transform the nonlinear term under the following shape:

\[
\eta = \Lambda_1 \eta_{\max} + \Lambda_2 \eta_{\min}
\]

(41)

where

\[
\begin{align*}
\Lambda_1 &= \frac{\eta - \eta_{\min}}{\eta_{\max} - \eta_{\min}} \\
\Lambda_2 &= \frac{\eta_{\max} - \eta}{\eta_{\max} - \eta_{\min}}
\end{align*}
\]  

(42)
Hence, the global T-S fuzzy model which is a particular case of the system (3) is inferred as:

\[
\begin{align*}
E \dot{x} &= \sum_{i=1}^{2} \lambda_i(\eta)(A_i x + Bu + F_d) \\
y &= C x
\end{align*}
\]  

(43)

with

\[
A_1 = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-k_1 J & k_1 J & -\beta J & 0 & -1 & 0 \\
k_1 J & -k_1 J & 0 & -1 & 0
\end{bmatrix}, \quad \quad A_2 = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-k_1 J & k_1 J & -\beta J & 0 & -1 & 0 \\
k_1 J & -k_1 J & 0 & -1 & 0
\end{bmatrix}
\]

The weighting functions are given by:

\[
\begin{align*}
\lambda_1(\eta) &= \Lambda_1 \\
\lambda_2(\eta) &= \Lambda_2
\end{align*}
\]  

(44)

The physical parameters definition and their numerical values are given in table 1 and the expression of unknown input signal is defined as in Figure 1. Notice that, to apply the proposed UIO (21) for the one-link flexible joint robot, it suffices to rewritten the model (43) in the its equivalent form (20). Thus, by Theorem 3.2 with \( \gamma = 8 \) the following observer gain \( H \) is obtained:

\[
H = \begin{bmatrix}
28.20 & -4.30 \\
16.37 & 30.55 \\
-16.53 & 59.24 \\
247.49 & 151.75 \\
288.41 & -69.88
\end{bmatrix}
\]  

(45)

Simulation results with initial conditions:

\[
\begin{align*}
Z_1(0) &= [0 \ 0.31 \ 0 \ 0 \ 5]^T, \quad Z_2(0) = [16.37 \ -16.69]^T \\
\hat{Z}_1(0) &= [0 \ 0.63 \ 0.1 \ 3]^T, \quad \hat{Z}_2(0) = [31.65 \ -32.34]^T
\end{align*}
\]

are given in Figures 1, 2, 3 and 4. These simulation results show the performances of the proposed UIO (21) with the gain \( H \) where the dashed lines denote the state variables and unknown input estimated by the fuzzy observer. They show that the observer gives a good estimation of unknown states and unknown input of the considered robot.
UIO design for a class of NDS: a T-S approach with Lipschitz constraints

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_m$</td>
<td>Motor inertia</td>
<td>$0.0037 \text{ kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>$J_L$</td>
<td>Link inertia</td>
<td>$0.0093 \text{ kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>$m$</td>
<td>Link mass</td>
<td>$0.21 \text{ kg}$</td>
</tr>
<tr>
<td>$b$</td>
<td>Center of mass</td>
<td>$0.15 \text{ m}$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Elastic constant</td>
<td>$0.18 \text{ N} \cdot \text{m} / \text{rad}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Viscous friction coefficient</td>
<td>$0.046 \text{ kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>$k_2$</td>
<td>Amplifier gain</td>
<td>$0.08 \text{ N} \cdot \text{m} / \text{V}$</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to Gravity</td>
<td>$9.81 \text{ m/s}^2$</td>
</tr>
<tr>
<td>$u$</td>
<td>control variable</td>
<td>$0.05 \text{ N} \cdot \text{m}$</td>
</tr>
</tbody>
</table>

Table 1: Model parameters of one-link flexible joint robot

![Figure 1: Unknown input $d$ and its estimate](image.png)
Figure 2: State variables $x_1$, $x_3$ and their estimates
Figure 3: State variables $x_2$, $x_4$ and their estimates
Figure 4: State variables $x_5$, $x_6$ and their estimates
5 Conclusion

In this paper, a new fuzzy UIO design approach based on explicit structure is proposed in order to estimate the state and unknown inputs for a class of NDS described by T-S fuzzy models with unmeasurable premise variables satisfying Lipschitz conditions. The main idea is based on the separation between dynamic and static relations in the T-S descriptor model. The exponential convergence of the state estimation error is studied using the Lyapunov theory and the existence of condition ensuring this convergence is expressed in term of only one LMI. To demonstrate the good performance of the proposed UIO design, a T-S descriptor model of the one-link flexible joint robot is considered. The effectiveness of the proposed UIO for the on-line estimation of unknown states and unknown inputs of the used model is verified by numerical simulation.

References


Received: November 27, 2016; Published: February 24, 2017