Nonlinear Analysis and Differential Equations, Vol. 4, 2016, no. 6, 295 - 304
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/nade.2016.6526

The Normal Stress Dıstribution in an Infinite Elastic Body with a Locally Curved and Hollow Fiber under Geometrical Nonlinear Statement

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Abstract

The results of the normal stress distribution in an elastic body with a locally curved and hollow fiber are expounded. The normal stress distribution is studied when the body is loaded at infinity by uniformly distributed normal forces in the fiber direction. The investigations are carried out in the framework of the piecewise homogeneous body model with the use of the three-dimensional geometrically non-linear exact equations of the theory of elasticity. The mathematical formulation of corresponding boundary-value problem was given. The boundary form perturbation method is employed for the investigation.
Numerical results related to the normal stress distribution considered and the influence of parameters on these stresses are analyzed. The corresponding numerical results are presented.

Keywords: Fibrous composite, stress distribution, geometric nonlinearity, locally curving

1 Introduction

It should be noted that in the structure of the unidirectional fibrous composites in many cases fibers have an initial curving caused by design factors or caused by action of various factors during technological processes, see Tarnopolsky, Yu, Jigun and Polyakov (1987), Guz (1990), Kelly (1998) , Akbarov and Guz (2000), Guz (2003). For this purpose, within the framework of a piecewise homogeneous body model, by using the exact three-dimensional equations of elasticity theory, in Akbarov and Guz (1985) are given a method for investigation of the stress state in unidirectional composites. In Simsek (2005) proposed a method to investigation a locally curved fiber in geometrically nonlinear statement. Alan and Akbarov (2011) considered the normal stress distribution with unidirectional locally curved covered nanofibers. Alan (2014) is analyzed the normal stress distribution in an elastic body with a locally curved and hollow nanofiber. However, the results remained within the limits of the first approximation. It is obvious that is necessary to increase the number of approximation added to the solution, so as to increase the sensitivity of the numerical results. In this paper, The Results of the normal stress distribution in an elastic body with a locally curved and hollow fiber are analyzed. The Method is developed in such way as to obtain the normal stress values up to second approximation on the fiber and matrix interface. Within the framework of the three dimensional geometrical nonlinear exact equations of the theory of elasticity, we investigate the normal stresses in the composites with unidirectional locally curved and hollow fiber. We assume that the concentration of fibers in the composite is small and any interaction between them is disregarded.

2 Formulation of the Problem

In the present study, we consider an infinite body containing a single locally curved and hollow fiber with an initial local imperfection and we determine through the Lagrange coordinates $Ox_1x_2x_3$ in the cylindrical system of coordinates $Or\theta z$ associated with the middle line of the fiber (Fig.1). Values related to matrix will be denoted by upper indices (1), to fiber by upper indices (2). We suppose that the matrix and fiber materials are homogeneous, isotopric and lineer elastic. Moreover, we assume that the fibers cross section is on the plane which is perpendicular to its axial line, is a closed circular ring of the constant inner radius R_1 and outer radius R_2 . For formulation of the problem can be seen in Akbarov, Kosker and Simsek (2005).

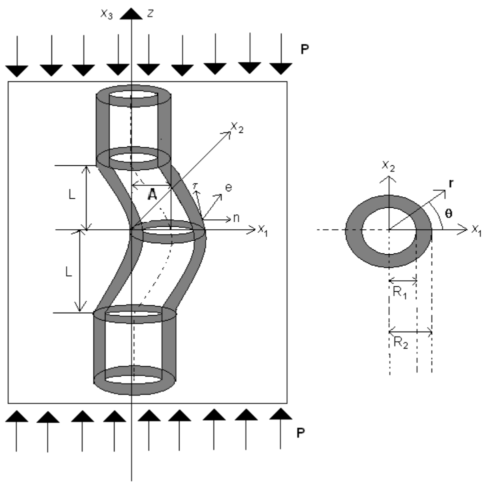

Figure 1: The geometry of structure and composite considered

3 Solution Method

Using the series presentation in Akbarov and Guz (2000) with respect to small parameter ε and doing mathematical manipulations similar that made in Akbarov and Guz (2000), we obtain the boundary-value problems for determination of the zeroth and the first approximations. For the solution method for zeroth and first approximation, see Alan (2014).

By using the solution procedure given in Kosker and Simsek (2006), we obtain the contact condition for second approximation.

$$\left[\sigma_{(i)r}\right]_{1,2}^{2,2}+f_1\left[\frac{\partial\sigma_{(i)r}}{\partial r}\right]_{1,1}^{2,1}+\phi_1\left[\frac{\partial\sigma_{(i)r}}{\partial z}\right]_{1,1}^{2,1}+f_2\left[\frac{\partial\sigma_{(i)r}}{\partial r}\right]_{1,0}^{2,0}+\phi_2\left[\frac{\partial\sigma_{(i)r}}{\partial z}\right]_{1,0}^{2,0}+$$
$$\left(f_1\right)^2\left[\frac{\partial^2\sigma_{(i)r}}{\partial r^2}\right]_{1,0}^{2,0}+f_1\phi_1\left[\frac{\partial^2\sigma_{(i)r}}{\partial r\partial z}\right]_{1,0}^{2,0}+\frac{1}{2}\left(\phi_1\right)^2\left[\frac{\partial^2\sigma_{(i)r}}{\partial z^2}\right]_{1,0}^{2,0}+\gamma_r\left[\sigma_{(i)r}\right]_{1,1}^{2,1}+$$
$$f_1\gamma_r\left[\frac{\partial\sigma_{(i)r}}{\partial r}\right]_{1,0}^{2,0}+\phi_1\gamma_r\left[\frac{\partial\sigma_{(i)r}}{\partial z}\right]_{1,0}^{2,0}+\gamma_\theta\left[\sigma_{(i)\theta}\right]_{1,1}^{2,1}+f_1\gamma_\theta\left[\frac{\partial\sigma_{(i)\theta}}{\partial r}\right]_{1,0}^{2,0}+\varphi_1\gamma_\theta\left[\frac{\partial\sigma_{(i)\theta}}{\partial z}\right]_{1,0}^{2,0}+$$
$$\gamma_z\left[\sigma_{(i)z}\right]_{1,0}^{2,0}+f_1\gamma_z\left[\frac{\partial\sigma_{(i)z}}{\partial r}\right]_{1,0}^{2,0}++\beta_r\left[\sigma_{(i)\mathrm{r}}\right]_{1,0}^{2,0}+\beta_\theta\left[\sigma_{(i)\theta}\right]_{1,0}^{2,0}+\beta_z\left[\sigma_{(i)z}\right]_{1,0}^{2,0}=0,$$

$$\left[u_{(i)}\right]_{1,2}^{2,2}+f_1\left[\frac{\partial u_{(i)}}{\partial r}\right]_{1,1}^{2,1}+\varphi_1\left[\frac{\partial u_{(i)}}{\partial z}\right]_{1,1}^{2,1}+f_2\left[\frac{\partial u_{(i)}}{\partial r}\right]_{1,0}^{2,0}+\phi_1\left[\frac{\partial u_{(i)}}{\partial z}\right]_{1,0}^{2,0}+\varphi_2\left[\frac{\partial u_{(i)}}{\partial z}\right]_{1,0}^{2,0}+$$
$$\left(f_1\right)^2\left[\frac{\partial^2 u_{(i)}}{\partial r^2}\right]_{1,0}^{2,0}+f_1\varphi_1\left[\frac{\partial^2 u_{(i)}}{\partial r\partial z}\right]_{1,0}^{2,0}+\frac{1}{2}\left(f_1\right)^2\left[\frac{\partial^2 u_{(i)}}{\partial z^2}\right]_{1,0}^{2,0}=0 \qquad (1)$$

In Eq. (1) the following notation is used:

$$\left[\mathrm{X}\right]_{1,q}^{2,q}=\mathrm{X}^{(2),q}\left(R,\theta,t_3\right)-\mathrm{X}^{(1),q}\left(R,\theta,t_3\right),\ \mathrm{q=0,1},$$

$$f_1=\delta(t_3)\cos\theta\quad,\quad\varphi_1=-R\frac{d\delta(t_3)}{dt_3}\cos\theta\quad,\quad\gamma_{\mathrm{r}}=\left(\frac{\delta(t_3)}{\mathrm{R}}-\frac{\mathrm{d}^2\delta(t_3)}{\mathrm{d}t_3^2}\mathrm{R}\right)\cos\theta\quad,$$
$$\gamma_\theta=\frac{\delta(t_3)}{R}\sin\theta,\quad\gamma_z=-\frac{d\delta(t_3)}{dt_3}\cos\theta,\qquad\delta(t_3)=\exp(\tfrac{-x_3}{L})^2\cos(m\tfrac{-x_3}{L})\qquad(2)$$

Similar contact conditions can also obtained for subsequent approximation.
In this case, an exact analytical solution is given in Akbarov and Guz (2000), Guz (2003), Akbarov and Guz (1985). According to which , at $\nu^{(1)}=\nu^{(2)}$, we have the following relations:

$$\varepsilon_{zz}^{(1),0}=\varepsilon_{zz}^{(2),0}=\frac{p}{E^{(1)}},\quad \sigma_{zz}^{(1),0}=p,\quad u_{z}^{(1),0}=u_{z}^{(2),0}=\frac{p}{E^{(1)}}z,\quad u_{r}^{(1),0}=-\nu^{(1)}\varepsilon_{zz}^{(1),0}r,$$
$$u_{r}^{(2),0}=-\nu^{(2)}\varepsilon_{zz}^{(2),0}r,\ u_{\theta}^{(1),0}=u_{\theta}^{(2),0}=0,\ \sigma_{rr}^{(1),0}=\sigma_{rr}^{(2),0}=\sigma_{\theta\theta}^{(1),0}=\sigma_{\theta\theta}^{(2),0}=0,$$
$$\sigma_{zz}^{(2),0}=p\frac{E^{(2)}}{E^{(1)}},\sigma_{\theta z}^{(1),0}=\sigma_{\theta z}^{(2),0}=\sigma_{rz}^{(1),0}=\sigma_{rz}^{(2),0}=\sigma_{r\theta}^{(1),0}=\sigma_{r\theta}^{(2),0}=0 \tag{3}$$

Where $E^{(1)}$, $E^{(2)}$ in (3) represent the elasticity modules of the matrix, fiber, respectively.
By employing the solution procedure described in Guz (1999), we obtain the following equations for the first and second approximations

$$\frac{\partial\sigma_{rr}^{(k),q}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}^{(k),q}}{\partial\theta}+\frac{\partial\sigma_{rz}^{(k),q}}{\partial z}+\frac{1}{r}\left(\sigma_{rr}^{(k),q}-\sigma_{\theta\theta}^{(k),q}\right)+\sigma_{zz}^{(k),0}\frac{\partial^2u_{r}^{(k),q}}{\partial z^2}=0,$$
$$\frac{\partial\sigma_{r\theta}^{(k),q}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}^{(k),q}}{\partial\theta}+\frac{\partial\sigma_{\theta z}^{(k),q}}{\partial z}+\frac{2}{r}\sigma_{r\theta}^{(k),q}+\sigma_{zz}^{(k),0}\frac{\partial^2u_{\theta}^{(k),q}}{\partial z^2}=0,$$
$$\frac{\partial\sigma_{rz}^{(k),q}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}^{(k),q}}{\partial\theta}+\frac{\partial\sigma_{zz}^{(k),q}}{\partial z}+\frac{1}{r}\sigma_{rz}^{(k),q}+\sigma_{zz}^{(k),0}\frac{\partial^2u_{z}^{(k),q}}{\partial z^2}=0,\qquad q=1,2 \tag{4}$$

These equations coincided with the 3-dimensional linearized elasticity equations.

We can written the strain-displacement as follows.

$$\varepsilon_{rr}^{(k),q}=\frac{\partial u_{r}^{(k),q}}{\partial r},\qquad \varepsilon_{\theta\theta}^{(k),q}=\frac{\partial u_{\theta}^{(k),q}}{r\partial\theta}+\frac{u_{r}^{(k),q}}{r},\qquad \varepsilon_{zz}^{(k),q}=\frac{\partial u_{z}^{(k),q}}{\partial z},$$
$$\varepsilon_{r\theta}^{(k),q}=\frac{1}{2}\left(\frac{\partial u_{r}^{(k),q}}{r\partial\theta}+\frac{\partial u_{\theta}^{(k),q}}{\partial r}-\frac{u_{\theta}^{(k),q}}{r}\right),\qquad \varepsilon_{\theta z}^{(k),q}=\frac{1}{2}\left(\frac{\partial u_{\theta}^{(k),q}}{\partial z}+\frac{\partial u_{z}^{(k),q}}{r\partial\theta}\right),$$
$$\varepsilon_{zr}^{(k),q}=\frac{1}{2}\left(\frac{\partial u_{z}^{(k),q}}{\partial r}+\frac{\partial u_{r}^{(k),q}}{\partial z}\right),\qquad q=1,2 \tag{5}$$

To determine the contact condition and the eqs. (4) and (5), for the second approximation, we use the representations in Guz (1999).

$$u_{r}^{(k),q}=\frac{1}{r}\frac{\partial}{\partial\theta}\psi^{(k),q}-\frac{\partial^2}{\partial r\partial z}\chi^{(k),q};\qquad u_{\theta}^{(k),q}=-\frac{\partial}{\partial r}\psi^{(k),q}-\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\chi^{(k),q};$$

$$\Delta_1^{(k)}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

$$u_z^{(k),q} = (\lambda^{(k)} + \mu^{(k)})^{-1}\left((\lambda^{(k)} + 2\mu^{(k})\Delta_1^{(k)} + (\mu^{(k)} + \sigma_{zz}^{(k),0})\frac{\partial^2}{\partial z^2} \right)\chi^{(k),q} \tag{6}$$

The functions of $\psi^{(k),q}$, $\chi^{(k),q}$ are determined by the following equations:

$$\left(\Delta_1^{(k)} + \left(\xi_1^{(k)}\right)^2 \frac{\partial^2}{\partial z^2} \right)\psi^{(k),q} = 0\,, \quad \left(\Delta_1^{(k)} + \left(\xi_2^{(k)}\right)^2 \frac{\partial^2}{\partial z^2} \right)\left(\Delta_1^{(k)} + \left(\xi_3^{(k)}\right)^2 \frac{\partial^2}{\partial z^2} \right)\chi^{(k),q} = 0 \tag{7}$$

Where, $\xi_i^{(k)}$ (k=1,2; i=1,2,3) given in the following equations:

$$\xi_1^{(k)} = \sqrt{\frac{\mu^{(k)} + \sigma_{zz}^{(k),0}}{\mu^{(k)}}}\,,\quad \xi_2^{(k)} = \sqrt{\frac{\mu^{(k)} + \sigma_{zz}^{(k),0}}{\mu^{(k)}}}\,,\quad \xi_3^{(k)} = \sqrt{\frac{\lambda^{(k)} + 2\mu^{(k)} + \sigma_{zz}^{(k),0}}{\lambda^{(k)} + 2\mu^{(k)}}}$$

We apply the following exponential Fourier transform with respect to z, i.e.,

$$\overline{f}(s) = \int_{-\infty}^{\infty} f(z)e^{-isz}dz \tag{8}$$

We select the solution to the equations (7) and employing the Fourier transform we can determine $\overline{\psi}^{(k),1}, \overline{\chi}^{(k),1}$ and $\overline{\psi}^{(k),2}, \overline{\chi}^{(k),2}$ functions,

$$\overline{\psi}^{(1),1} = \overline{A}_1^{(1)}(s)K_1(\xi_1^{(1)}s\frac{r}{L})\sin\theta\,,$$

$$\overline{\chi}^{(1),1} = i\left[\overline{A}_2^{(1)}(s)K_1(\xi_2^{(1)}s\frac{r}{L}) + \overline{A}_3^{(1)}(s)K_1(\xi_3^{(1)}s\frac{r}{L}) \right]\cos\theta \tag{9}$$

$$\overline{\psi}^{(2),1} = \left[\overline{A}_{11}^{(2)}(s)I_1(\xi_1^{(2)}s\frac{r}{L}) + \overline{A}_{12}^{(2)}(s)K_1(\xi_1^{(2)}s\frac{r}{L}) \right]\sin\theta\,,$$

$$\overline{\chi}^{(2),1} = i\left[\overline{A}_{21}^{(2)}(s)I_1(\xi_2^{(2)}s\frac{r}{L}) + \overline{A}_{22}^{(2)}(s)K_1(\xi_2^{(2)}s\frac{r}{L}) + \overline{A}_{31}^{(2)}(s)I_1(\xi_3^{(2)}s\frac{r}{L}) + \overline{A}_{32}^{(2)}(s)K_1(\xi_3^{(2)}s\frac{r}{L}) \right]\cos\theta$$

$$\overline{\psi}^{(1),2} = \overline{A}_{12}^{(1)}(s_1)K_2(\xi_1^{(1)}s_1\frac{r}{L})\sin 2\theta\,,$$

$$\overline{\chi}^{(1),2} = i[\overline{A}_{20}^{(1)}(s_1)K_0(\xi_2^{(1)}s_1\, r/L) + \overline{A}_{30}^{(1)}(s_1)K_0(\xi_3^{(1)}s_1\, r/L) + \{\overline{A}_{22}^{(1)}(s_1)K_2(\xi_2^{(1)}s_1\, r/L)$$
$$+ \overline{A}_{32}^{(1)}(s_1)K_2(\xi_3^{(1)}s_1\, r/L)\}]\cos 2\theta$$

$$\overline{\psi}^{(2),2} = \left[\overline{A}_{12}^{(2)}(s_1) I_2(\xi_1^{(2)} s_1 \frac{r}{L}) + \overline{B}_{12}^{(2)}(s_1) K_2(\xi_1^{(2)} s_1 \frac{r}{L}) \right] \sin 2\theta \qquad (10)$$

$$\overline{\chi}^{(2),2} = i \left\{ \overline{A}_{20}^{(2)}(s_1) I_0(\xi_2^{(3)} s_1 \frac{r}{L}) + \overline{B}_{20}^{(2)}(s_1) K_0(\xi_2^{(3)} s_1 \frac{r}{L}) + \overline{A}_{30}^{(2)}(s_1) I_1(\xi_3^{(3)} s_1 \frac{r}{L}) + \overline{B}_{30}^{(2)}(s_1) K_1(\xi_3^{(2)} s_1 \frac{r}{L}) + \right.$$

$$\left. \left[\overline{A}_{22}^{(2)}(s_1) I_2(\xi_2^{(2)} s_1 \frac{r}{L}) + \overline{B}_{22}^{(2)}(s_1) K_2(\xi_2^{(2)} s_1 \frac{r}{L}) + \overline{A}_{32}^{(2)}(s_1) I_2(\xi_3^{(2)} s_1 \frac{r}{L}) + + \overline{B}_{32}^{(2)}(s_1) K_2(\xi_3^{(2)} s_1 \frac{r}{L}) \right] \cos 2\theta \right\}$$

where $I_n(x)$ are Bessel functions of a purely imaginary argument and $K_n(x)$ are the Macdonald functions. Moreover, s and s_1 are Fourier transform parameter. The unknown constant entering (9) and (10) are determined from the relations (6), (7) and the contact condition equations given in Alan (2014). This gives the first and second approximations of targeted stresses.

The following inverse Fourier transform is used to find the real stress values, for example, $\sigma_{rr}^{(1),2}$ is $\sigma_{rr}^{(1),1} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{\sigma}_{rr}^{(1),1} e^{isz} ds$ (11)

The expressions for the other sought for quantities are determined likewise. Thus, by the above-described method we determine completely the values in the second approximation.

4 Numerical Results and Discussion

By using the solution procedure described in Akbarov and Guz (2000), we obtain the numerical results for first and second approximations. In Alan and Akbarov (2010) are given also these solution method and mathematical calculation for first and second approximation. See Alan and Akbarov (2010).

We will analyze some numerical results related to the normal stress $\sigma_{\tau\tau}$ which constitute the matrix and fiber intersection surfaces. In view of corresponding symmetry, we consider the distribution of these stresses only for $x_3 \geq 0$ and $0 \leq \theta \leq \pi$ (Fig. 1). If $\varepsilon = 0$ (i.e. if the curving is absent), the stresses $\sigma_{\tau\tau}$ coincide with σ_{zz}. We introduce the parameter $\kappa = R_2/L$, bk= h/R and assume that $\nu^{(1)} = \nu^{(2)} = 0.3$, $\varepsilon = 0.07$, $\theta = 0$. $\alpha = p/E^{(1)}$ shows the influence of geometrical non-linearity on the values of the normal stresses mentioned.

The Figure 2 shows dependencies of $\sigma_{\tau\tau}/|p|$ on the parameter h/R for various values of m=0, 1, 3 at $\chi_3/L=1.0$, $\kappa=0.1$, $\alpha=0.00005$ and $E^{(2)}/E^{(1)}=50$. These graphs shows also the influence of the parameter m on the distribution of the normal stresses. It follows from the graphs that maximal values of the normal stresses $\sigma_{\tau\tau}/|p|$ increase monotonically with m. We note that when the thickness of fiber is increased, it is possible to obtain the same result as those obtained in the case of the single locally curved fiber in an infinite body in the same parameter values. Note that under compression of the considered infinite body we assume that the selected values of α are smaller than those of critical values α_{cr} , which correspond to the micro-buckling of the fiber in the matrix in Guz (1990)

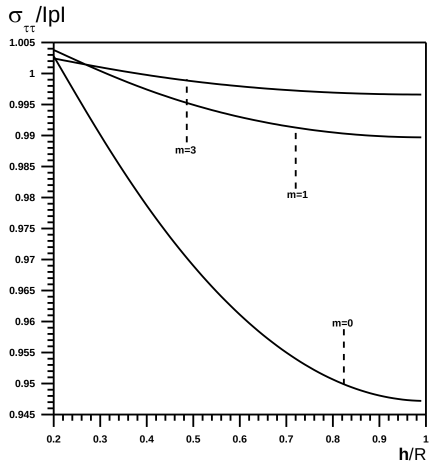

Fig 2: Distribution of the $\sigma_{\tau\tau}/p$ with respect to h /R where $E^{(2)}/E^{(1)}=50$, $\varepsilon=0.07$ and $\kappa=0.1$, under $m=0$; $m=1$; $m=3$

Figure 3 shows the graphs of the dependencies among $\sigma_{\tau\tau}/|p|$ and x_3 /L for h/R=0.5, $\kappa=0.1$ and $E^{(2)}/E^{(1)}=50$. The influence of the values m on the normal stress distribution illustrated by these graphs. It follows from these graphs that’s absolute maximal values of the normal stresses $\sigma_{\tau\tau}/|p|$ increase monotonically with m. Note that under compression of the considered infinite body we assume that the selected values of α are smaller than those of $\alpha_{cr.}$ which correspond to the micro-buckling of the fiber in the matrix in Guz (1990).

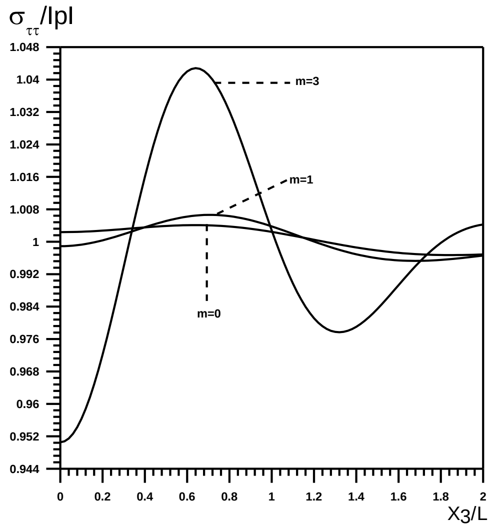

Fig 3: Distribution of the $\sigma_{\tau\tau}/p$ with respect to x_3/L where $E^{(2)}/E^{(1)}=50$, $\varepsilon = 0.07$ and $\kappa = 0.1$, under $m=0$; $m=1$; $m=3$

Table 1 tabulate the values of $\sigma_{\tau\tau}/|p|$ for various $E^{(2)}/E^{(1)}$, m and α. In this case the values of $\sigma_{\tau\tau}/|p|$ are calculated under $\kappa = 0.1$, $\chi_3/L = 1.0$ .It follows from the results given in Table 1 absolute maximal values of $\sigma_{\tau\tau}/|p|$ increase monotonically with increasing $E^{(2)}/E^{(1)}$ and m. This table indicates that the values of the normal stresses $\sigma_{\tau\tau}/|p|$ increase with α.

We assume that α is smaller than its critical values α_{cr} corresponding to micro buckling of the fiber in the matrix in Guz (1990).

Table 1: The values of $\sigma_{\tau\tau}/p$ obtained for various α and $E^{(2)}/E^{(1)}$

m	$\frac{E^{(2)}}{E^{(1)}}$	A.N.	$\alpha = \frac{p}{E^{(1)}}$							
			Tension				compression			
			0.0005	0.005	0.01	0.015	-0.0005	-0.005	-0.01	-0.015
0	20	1	1.0333	1.0331	1.0330	1.0329	-1.0333	-1.0334	-1.0336	-1.0337
		2	1.0250	1.0249	1.0248	1.0247	-1.0250	-1.0250	-1.0251	-1.0252
	50	1	1.0530	1.0523	1.0516	1.0510	-1.0530	-1.0537	-1.0544	-1.0552
		2	1.0538	1.0531	1.0525	1.0519	-1.0538	-1.0545	-1.05552	-1.0560
	100	1	1.0795	1.0774	1.0773	1.0737	-1.0795	-1.0818	-1.0842	-1.0911
		2	1.0837	1.0816	1.0797	1.0778	-1.0837	-1.0860	-1.0885	-1.0911

Table 1: (Continued): The values of $\sigma_{\tau\tau}/p$ obtained for various α and $E^{(2)}/E^{(1)}$

1	20	1	1.0642	1.0639	1.0637	1.0634	-1.0642	-1.0645	-1.0648	-1.0651
		2	1.0523	1.0521	1.0520	1.0518	-1.0523	-1.0525	-1.0528	-1.0530
	50	1	1.1005	1.0993	1.0981	1.0969	-1.1006	-1.1019	-1.1033	-1.1048
		2	1.0986	1.0974	1.0962	-1.0950	-1.0986	-1.0999	-1.1013	-1.1027
	100	1	1.1487	1.1449	1.1413	1.1379	-1.1488	-1.1530	-1.1576	-1.1626
		2	1.1506	1.1468	1.1432	1.1398	-1.1507	-1.1549	-1.1594	-1.1644
3	20	1	1.1485	1.1481	1.1477	1.1473	-1.1485	-1.1490	-1.1494	-1.1498
		2	1.1557	1.1552	1.1548	1.1543	-1.1557	-1.1561	-1.1566	-1.1571
	50	1	1.2367	1.2340	1.2314	1.2289	-1.2368	-1.2396	-1.2426	-1.2457
		2	1.2450	1.2422	1.2395	1.2369	-1.2451	-1.2481	-1.2512	-1.2545
	100	1	1.3619	1.3522	1.3432	1.3348	-1.3621	-1.3727	-1.3848	-1.3971
		2	1.3718	1.3619	1.3526	1.3441	-1.3920	-1.3829	-1.3948	-1.4079

6. Conclusion

In present paper, the investigations were made within scope of the piecewise homogeneous body model with the use of the three-dimensional geometrically nonlinear exact equations of the theory of elasticity. A single locally curved and hollow fiber has been investigated. The mathematical formulation of the corresponding boundary-value problem was given. The method is developed in such way as to obtain the normal stresses values up to the second approximation on the interface the hollow fiber and matrix material.

Consequently, the aim of the investigations was to study the influence of the normal stress distribution in an elastic body with a locally curved and hollow fiber for the second approximation

The numerical results were presented for a single locally curved and hollow fiber.

As a result of the numerical investigations it was established that:

(i) When the radius of hollow goes to 0 as a limit, it was obtained the same normal stresses values in a locally curved fiber in a composite material in same parameter values.

(ii) The Effect of geometrical nonlinearity increases monotonically with, $\frac{p}{E^{(1)}}$, as well as with $\frac{E^{(2)}}{E^{(1)}}$.

(iii) The maximum absolute value of the stresses considered increase oscillation frequency of the local curving.

The numerical results obtained agree with well-known mechanical consideration and, in some particular cases, coincide with known results.

References

[1] Yu.M. Tarnopolsky, I.G. Jigun and V.A. Polyakov, *Spatially-Reinforced Com-*

posite Materials, Handbook, Mashinostroyenia, Moscow, Russia, 1987.

[2] A.N. Guz, *Failure Mechanics of Composite Materials in Compression*, Naukova Dumka Russia, 1990.

[3] A. Kelly, Composite Materials, Impediments do wider use and some suggestions to overcome these, Chapter in *European Conference on Composite Materials (ECCM-8)*, Napoles-Italy, Vol. I, 1998, 15-18.

[4] S.D. Akbarov, A.N. Guz, *Mechanics of Curved Composites*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2000. http://dx.doi.org/10.1007/978-94-010-9504-4

[5] A.N. Guz, On one two-level model in the mesomechanics of compression fracture of Cracked Composites, *Int. Appl. Mech.*, **39** (2003), no. 3, 274-285. http://dx.doi.org/10.1023/a:1024410202053

[6] S.D. Akbarov, A.N. Guz, Method of Solving Problems in the Mechanics of Fiber Composites with Curved Structures, *Soviet Applied Mechanics*, **20** (1984), 777-785. http://dx.doi.org/10.1007/bf00888477

[7] A.N. Guz, *Fundamentals of the Three-Dimensional Theory of Stability of Deformable Bodies*, Springer New York, 1999. http://dx.doi.org/10.1007/978-3-540-69633-9

[8] R. Kosker, K. Simsek and S. D. Akbarov, Stress Distribution in an Infinite Elastic Body with a Locally Curved Fiber in a Geometrically Nonlinear Statement, *Mechanics of Composite Materials*, **41** (2005), no. 4, 291–302. http://dx.doi.org/10.1007/s11029-005-0055-3

[9] S.D. Akbarov, R. Kosker and K. Simsek, On the Normal Stresses in the Elastic Body with a Locally Curved Fiber under Geometrically Nonlinear Statement, *Sigma Journal of Engiineering and Natural Science*, **24** (2006), no. 3, 97–108.

[10] K.S. Alan, S.D. Akbarov, Analysis of stresses in an infinite elastic body with a locally curved covered nanofiber, *Mechanics of Composite Materials*, **47** (2011), no. 3, 343–358. http://dx.doi.org/10.1007/s11029-011-9213-y

[11] K.S. Alan, Normal Stresses in an Infinite Elastic Body with a Locally Curved and Hollow Nanofiber, *CMC-Computers Materials & Continua*, **44** (2014), no.1, 1-21.

Received: May 18, 2016; Published: June 21, 2016