The Equivalent Form and the Evolution of Generalized Schrödinger Map Heat Flow

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Abstract

We present a study of the singular and smooth solutions of the generalized Schrödinger map heat flow equation (GSMF) on hyperbolic space. Considering the associated hyperbolic Landau-Lifshitz spin evolution equation with uniaxial anisotropy together with applied field, we study the dynamics in terms of the stereographic variable. Firstly, an equivalent equation of this system is obtained. Based on this equivalent equation, we construct some singular and smooth solution of GSMF in 2-dimensional $\mathcal{H}^2$ space. We study these norm $-1$ solutions that allow us depict the mechanism of the evolution.

Mathematics Subject Classification: 35K10, 35K65, 35Q40, 35Q55

Keywords: Schrödinger map heat flow, singular solution, smooth solution, hyperbolic

1 Introduction

In this paper, we investigate the nonlinear dynamics underlying the evolution of a 2D Schrödinger map heat flow with uniaxial anisotropy and applied field. This system which we call it GSMF can be regarded as a generalized Ferromagnetic material equation which we usually refer to Landau-Lifshitz equation (abbreviated LLE). The standard LLE which is the kernel equation of the ferromagnetic materials is proposed by L.D. Landau and E.M. Lifshitz.
LLE which exhibits vary nonlinear behavior exactly depict the evolution of magnetic domain wall is a dispersion equations.

There is a rich literature concerning the study of LLE (see, for instance, [11, 12]), the materials analyzing the singular solutions on hyperbolic space we are considering are very scarce. In fact, LLE exhibits the different properties in various target manifold. Different from the Euclid case, we will insist on the other possibility, choosing the hyperbolic plane $\mathcal{H}^2$ as the target space. In this case, known as the Schrödinger map equation on $\mathcal{H}^2$, is a particular case of the LLE for ferromagnetism. Exactly, in this paper, we study the hyperbolic LLE (or GSMF) without Gilbert damping term:

$$S_t = S \hat{\wedge} (\Delta S) + S \hat{\wedge} (SL) + S \hat{\wedge} H,$$

where $\hat{\wedge}$ denotes the pseudo cross product which means

$$\hat{a} \hat{\wedge} \hat{b} = (\hat{a} \wedge \hat{b}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$  

$S = (u, v, w)$ represents the spin vector. Furthermore, $|S|^2 = u^2 + v^2 - w^2 = -1, s_3 > 0$ Spin vector $S = (u, v, w) \in \mathcal{H}^2 \hookrightarrow R^{2+1}$. Magnetic anisotropy and applied field are $L = \text{diag}\{0, 0, \lambda\}$ and $H = (0, 0, \gamma)$ respectively. Here we set $\lambda$ and $\gamma$ are constants.

In this paper, we study the exact evolution of GSMF (1). In fact, the original Schrödinger map heat flow (SMF for short) which is an equation of differential geometry do not contain any magnetic anisotropy and applied field. Of course, if the Gilbert damping term disappear in the isotropic LLE, LLE will degenerate into the standard SMF. From this point of view , SMF is the most critical factor in LLE. Hence, it is useful to study the properties of the ferromagnetic materials by the SMF (or GSMF). Exactly, if we define the mapping $S : R^3 \times R \to (M, h, J)$ where $J$ and $h$ are the complex structure and the metric on Kähler manifold respectively. Furthermore, $D$ denotes the covariant derivative of $S^{-1}TM$. GSMF can be rewritten in a more geometric way as

$$\frac{\partial S}{\partial t} = J \sum_{i=1}^{n} D^i h S + J(SL) + JH.$$  

Under the different manifold and complex structure, (2) will represent different equation. For example, if the manifold and the complex structure are $\mathcal{H}^2$ and $\hat{\wedge}$ respectively, (2) will in the form of (1). However, if $M = S^2$ and $J(S) = S\Lambda$, (2) is magnetic anisotropy and applied field LLE which can be written in similar way as

$$\frac{\partial}{\partial t} S = S \wedge (\Delta S) + S \wedge (SL) + S \wedge H.$$  

(3)
It may also be noted that a number of investigations exist in the literature on the evolution of higher dimensional LLE corresponding to isotropic (see [2, 5, 6]). Due to these result, the existence of blowup solution of the critical dimension $n = 2$ and supercritical $n \geq 3$ are proved. Furthermore, the blowup rate of the solutions are also predicted in [5, 6]. In fact, some exact smooth and singular solutions of LLE are constructed recently [1, 3, 4, 8, 9, 10]. Despite these pioneering works, the investigations have been mostly limited to the isotropic case or the Euclid space. Although the authors construct the exact transformation matrix between isotropic LLE and LLE (or (3)) with magnetic anisotropy and applied field in [7]. The evolution dynamics of GSMF which admit a singularity in finite time on hyperbolic space is still not so clear. In fact, to the knowledge of the authors, concerning the systems coupled with other field as considered as in this paper, there exist very few studies.

The paper is organized as follows. In section 2, we deduce the equivalent equation of GSMF by stereographic projection. Then we study the exact solution of this equivalent system in section 3. In section 4, we analyse the evolution of the smooth and singular solution which deduced in section 3.

### 2 Equivalent equation of GSMF

There is not a generalization of Hasimoto transformation so that we can reduce (1) to the cubic Schrödinger equation in $n \geq 2$. In fact, the equivalent form of GSMF is an equation with a derivative term and a nonlocal term by Hasimoto transformation in this spatial dimension. Similarly, if the stereographic projection is used in $n \geq 2$, we can not deduce a cubic Schrödinger equation. However, the stereographic projection lead to a derivative Schrödinger equation without a nonlocal term. Let us compute which is the stereographic projection of (1). Firstly, we expand (1) as the following form

\[
v \Delta w - w \Delta v + v (w \lambda + \gamma) - u_t = 0, \quad (4)
\]

\[
-u \Delta w + w \Delta u - u (w \lambda + \gamma) - v_t = 0, \quad (5)
\]

\[
-u \Delta v + v \Delta u - w_t = 0. \quad (6)
\]

In order to find out the exact solution of (1), we firstly deduce the equivalent equation of it. If we employ the conversion

\[
(u, v, w) = \left( \frac{2 \Re(W)}{1 - \overline{W} W}, \frac{2 \Im(W)}{1 - \overline{W} W}, \frac{1 + \overline{W} W}{1 - \overline{W} W} \right), \quad (7)
\]

the various derivatives of space can be deduce as follows:

\[
\nabla u = \Psi \left[ W^2 \nabla W + \left( \overline{W} \right)^2 \nabla W + \nabla W + \nabla W \right], \quad (8)
\]
\[ \nabla v = i \Psi \left[ \left( \nabla W \right)^2 \nabla W - W^2 \nabla W - \nabla W + \nabla W \right], \]  
\[ \nabla w = 2 \Psi \left( \nabla W \nabla W + W \nabla W \right), \]

where \( \Psi = \left( -1 + W \nabla W \right)^{\frac{-1}{2}}. \)

Similar to the deducing of (8)-(10), we calculate the derivatives of the time as follows

\[ u_t = \Psi \left[ W^2 \nabla W_t + \left( \nabla W \right)^2 W_t + W_t + \nabla W_t \right], \]  
\[ v_t = i \Psi \left[ \left( \nabla W \right)^2 W_t - W^2 \nabla W_t - W_t + \nabla W_t \right], \]  
\[ w_t = 2 \Psi \left( \nabla W \nabla W + \nabla W \nabla W \right). \]

Based on (8)-(10), the Laplace derivatives of \( u, v \) and \( w \) are

\[ \Delta u = \Psi \left[ W^2 \Delta \nabla W + \left( \nabla W \right)^2 \Delta W + \Delta W + \Delta \nabla W \right] \\
- 2 \Psi^{3/2} \left[ \nabla W^2 \left( \nabla W \right)^3 + \left( \nabla W \right)^2 W^3 + \nabla W^2 \nabla W + 2 \nabla W \nabla W \nabla W \right] + 2 \nabla W \nabla W \nabla W + \left( \nabla W \right)^2 W, \]  
\[ \Delta v = -i \Psi \left[ W^2 \Delta \nabla W - \left( \nabla W \right)^2 \Delta W + \Delta W - \Delta \nabla W \right] \\
- 2 i \Psi^{3/2} \left[ \nabla W^2 \left( \nabla W \right)^3 - \left( \nabla W \right)^2 W^3 - \nabla W^2 \nabla W - 2 \nabla W \nabla W \nabla W \right] + 2 \nabla W \nabla W \nabla W + \left( \nabla W \right)^2 W, \]  
\[ \Delta w = -2 \Psi^{3/2} \left[ -W^2 \nabla W \Delta W - W \left( \nabla W \right)^2 \Delta W + W \Delta \nabla W + \Delta W \nabla W \right] \\
+ 2 \nabla W^2 \left( \nabla W \right)^2 \nabla W + 2 \nabla W \nabla W \nabla W + 2 \left( \nabla W \right)^2 W^2 + 2 \nabla W \nabla W \nabla W \right]. \]

Substituting (7)-(16) in the (1), a manipulations can deduce the equation of \( W \). Exactly, (6) adopt the following form

\[ W_t \nabla W + \nabla W_t = -i \left[ -\Delta W \nabla W + W \Delta \nabla W + 2 \frac{\nabla W^2 \left( \nabla W \right)^2}{W \nabla W - 1} - 2 \left( \frac{\nabla W}{W \nabla W - 1} \right) \right], \]

which can be expressed as

\[ \Re \left( W_t \nabla W \right) = -i \left[ -i \Im \left( \Delta W \nabla W \right) + i \Im \left( \frac{2 \nabla W^2 \left( \nabla W \right)^2}{W \nabla W - 1} \right) \right] \]
Similar to (17), the equivalent equation of (4) is

\[
\begin{align*}
\mathbf{W}' (W^2 + 1) + W_t \left( \overline{W}^2 + 1 \right) &= -i[\Delta \mathbf{W} (W^2 + 1) - \overline{\Delta W} (\overline{W}^2 + 1)] \\
-2 \frac{(\nabla W)^2 W (w^{2+1})}{-1 + WW} + 2 \frac{(\nabla W)^2 \overline{W} (\overline{w}^{2+1})}{-1 + WW} &- \frac{\lambda \overline{W} (w^{2+1}) W (w \overline{w} + 1)}{-1 + WW} \\
+ \frac{\lambda \overline{W} (w^{2+1}) (w \overline{w} + 1)}{-1 + WW} - \gamma \overline{W} (W^2 + 1) + \gamma W (\overline{W}^2 + 1),
\end{align*}
\]

which can be expressed as

\[
\begin{align*}
\mathfrak{R} \left( W_t \left( \overline{W}^2 + 1 \right) \right) &= -i \left[ -i \mathfrak{R} \left( \Delta W \left( \overline{W}^2 + 1 \right) \right) + i \mathfrak{I} \left( \frac{2(\nabla W)^2 \overline{W} (\overline{w}^{2+1})}{-1 + WW} \right) \right] \\
-\lambda i \mathfrak{I} \left( \frac{(\overline{w}^{2+1}) W (w \overline{w} + 1)}{-1 + WW} \right) + \gamma i \mathfrak{I} \left( W (\overline{W}^2 + 1) \right). \tag{18}
\end{align*}
\]

Similarly, the equivalent equation of (5) is

\[
\begin{align*}
-i \left( \mathbf{W}' (W^2 - 1) - W_t \left( \overline{W}^2 - 1 \right) \right) &= -\overline{\Delta W} (\overline{W}^2 - 1) - \overline{\Delta W} (W^2 - 1) \\
+2 \frac{(\nabla W)^2 W (w^{2-1})}{-1 + WW} + 2 \frac{(\nabla W)^2 \overline{W} (\overline{w}^{2-1})}{-1 + WW} &- \frac{\lambda \overline{W} (w^{2-1}) W (w \overline{w} + 1)}{-1 + WW} \\
- \frac{\lambda \overline{W} (w^{2-1}) (w \overline{w} + 1)}{-1 + WW} + \gamma \overline{W} (W^2 - 1) + \gamma W (\overline{W}^2 - 1),
\end{align*}
\]

which can be expressed as

\[
\begin{align*}
\mathfrak{I} \left( W_t \left( \overline{W}^2 - 1 \right) \right) &= -i \left[ -i \mathfrak{R} \left( \Delta W \left( \overline{W}^2 - 1 \right) \right) + i \mathfrak{I} \left( \frac{2(\nabla W)^2 \overline{W} (\overline{w}^{2-1})}{-1 + WW} \right) \right] \\
-\lambda i \mathfrak{R} \left( \frac{(\overline{w}^{2-1}) W (w \overline{w} + 1)}{-1 + WW} \right) + \gamma i \mathfrak{R} \left( W (\overline{W}^2 - 1) \right). \tag{19}
\end{align*}
\]

In fact, the evolution equation of $W$ can be deduced from (17) and (18) (or (17) and (19)) as follows

\[
i W_t = -\Delta W - \frac{2W}{1 - |W|^2} \sum_{j=1}^{n} (\delta_j W)^2 + \lambda \frac{1 + |W|^2}{1 - |W|^2} W + \gamma W, \tag{20}
\]

where $\overline{W}$ stands for the conjugate complex numbers of $W$. 

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In fact, the following conversion
\[ W = \frac{u + iv}{1 - w} \] (21)
can be regard as the inverse process of (7).

According to (7) and (21), we can realize the transformation between \( W \) and \( S \). By stereographic projection (21), (1) can be transformed into a derivative Schrödinger equation (20). In fact, this processing can be regarded as a projection from \( \mathcal{H}^2 \) to the extend complex plane \( C_\infty \).

Nonlinear terms of (20) appear on the right hand side of it. In fact, the first derivative term and the \( \lambda \) term lead to a strong nonlinearity which makes (20) hard to resolve. In particular, whether (20) is a completely integrable system is not so clear. In fact, if \( \lambda = \gamma = 0 \) and space dimension \( n = 1 \), (3) is essentially equivalent to a nonlinear Schrödinger equation which is a completely integrable system. If \( n \geq 2 \), (3) is not a completely integrable system under the same hypothesis of \( \lambda \) and \( \gamma \). Although the integrability on hyperbolic space is not clear, we expect the same situation occurs in this case.

3 Solution of GSMF

In this section, we construct the solution of (1) (or (20)) under the cylindrical coordinates. In this case, the deformation of (20) is as follows
\[ iW_t = -W_{rr} - \frac{n-1}{r} W_r - \frac{2W}{1 - |W|^2} W_r^2 + \frac{\lambda}{1 - |W|^2} W + \gamma W, \] (22)
where \( r = \left( \sum_{j=1}^{n} x_j^2 \right)^{1/2} \).

Under the cylindrical coordinates, some special form of the solutions are useful to construct the exact solution. We employ the following form of the solution
\[ W = e^{i(f(t)r^a + h(t))} g(t), \] (23)
where \( a \) is some special constant; \( f(t) \), \( h(t) \) and \( g(t) \) function is to be determined.

Substituting (23) into (22), a careful calculation show that the derivatives of time is
\[ iW_t = i e^{i(f(t)r^a + h(t))} \left( ir^a \left( \frac{d}{dt} f(t) \right) g(t) + i \left( \frac{d}{dt} h(t) \right) g(t) + \frac{d}{dt} g(t) \right). \] (24)

The Laplace section of \( W \) is in the following form
\[ -W_{rr} - \frac{n-1}{r} W_r = -f(t) ae^{i(f(t)r^a + h(t))} g(t) \left( -r^{2a-2} f(t) a + ir^{a-2} a \right) + ir^{a-2} n - 2 ir^{a-2}, \] (25)
One order derivatives of space term of (22) is

\[-\frac{2W}{1-|W|^2}W_r^2 = -2 \frac{e^{i(f(t)r^a + h(t))}}{(g(t))^2} \frac{(f(t))^2 r^2 a - a^2}{(g(t))^2 - 1}. \quad (26)\]

The magnetic anisotropy and applied field term of (22) is as follows

\[\lambda \frac{1 + |W|^2}{1 - |W|^2}W + \gamma W = -\frac{e^{i(f(t)r^a + h(t))}}{(g(t))^2} g(t) \left( -\gamma (g(t))^2 + (g(t))^2 \lambda + \gamma + \lambda \right). \quad (27)\]

(24)-(27) contain a same factor \(e^{i(f(t)r^a + h(t))}\). This is useful to us to simplify (22). Separating the real part and imaginary part of (24)-(27), (22) can be simplified to the equations as follows

\[-g(t) \left( \frac{d}{dt} f(t) \right) r^a - g(t) \frac{d}{dt} h(t) - \frac{(f(t))^2(r^a)^2 a^2 g(t)}{r^2} - 2 \frac{(g(t))^2(f(t))^2(r^a)^2 a^2}{(1-(g(t))^2)r^2} - \lambda g(t) \frac{(g(t))^2}{1-(g(t))^2} - \lambda g(t) \frac{(g(t))^3}{1-(g(t))^2} - \gamma g(t) = 0,\]

\[\frac{d}{dt} g(t) + \frac{f(t)r^a g(t)}{r^2} = 0. \quad (28)\]

(28) is a complicated equation. Hence, we just discuss a simple case of \(a = n = 2\). In this setting, (28) can be simplified as

\[-g(t) \left( \frac{d}{dt} f(t) \right) r^2 - g(t) \frac{d}{dt} h(t) - 4 (f(t))^2 r^2 g(t) - 8 \frac{(g(t))^2(f(t))^2 r^2}{1-(g(t))^2} - \lambda g(t) \frac{(g(t))^3}{1-(g(t))^2} - \gamma g(t) = 0,\]

\[\frac{d}{dt} g(t) + 4 f(t) g(t) = 0. \quad (29)\]

The first equation of (29) splits into

\[\left\{ \begin{array}{l}
\frac{d}{dt} h(t) + \lambda \frac{(g(t))^2 \gamma}{1-(g(t))^2} + \gamma = 0, \\
\left( \frac{d}{dt} f(t) \right) r^2 + 4 (f(t))^2 r^2 + 8 \frac{(f(t))^2(g(t))^2 r^2}{1-(g(t))^2} = 0.
\end{array} \right. \quad (30)\]

From the second equation of (29), we obtain

\[f(t) = -\frac{d}{dt} g(t) \left( \frac{4}{g(t)} \right). \quad (31)\]

Substituting (31) into (30), the equation of \(g(t)\) and \(h(t)\) is as follows

\[\left\{ \begin{array}{l}
\frac{d}{dt} h(t) + \lambda \frac{(g(t))^2 \gamma}{1-(g(t))^2} + \gamma = 0, \\
r^2 \frac{d^2}{dt^2} g(t) - \frac{r^2 (2 \frac{d}{dt} g(t))^2}{(g(t))^2} + r^2 (\frac{d}{dt} g(t))^2 = 0.
\end{array} \right. \quad (32)\]
Solving (32), the exact solution of it is as follows

\[
\begin{align*}
g(t) &= \frac{1}{2}C_2 t + \frac{1}{2}C_3 \mp \frac{1}{2}\sqrt{t^2C_2^2 + 2tC_2C_3 + C_3^2 - 4}, \\
h(t) &= \mp\lambda \sqrt{t^2C_2^2 + 2tC_2C_3 + C_3^2 - 4C_2 - \gamma t + C_1},
\end{align*}
\] (33)

where \(C_1, C_2\) and \(C_3\) are constants.

According to (31) and (33), we obtain

\[
f(t) = \pm \frac{C_2}{4\sqrt{t^2C_2^2 + 2tC_2C_3 + C_3^2 - 4}}
\] (34)

Based on (23) and (33)-(34), solution of (22) is as follows

\[
W = \frac{1}{2} e^{i\left(-4\Theta \gamma tC_2 \pm \Theta C_2^2 t^2 + 4\Theta^2\lambda + 4\Theta C_1 C_2\right)} (C_2 t \mp \Theta + C_3)
\] (35)

where \(\Theta = \sqrt{t^2C_2^2 + 2tC_2C_3 + C_3^2 - 4}\).

Combining (7) and (35), we obtain the following solution

\[
\begin{align*}
u &= \pm \frac{2}{\Theta} \cos \left(\frac{-4\Theta \gamma tC_2 \pm \Theta C_2^2 t^2 + 4\Theta^2\lambda + 4\Theta C_1 C_2}{4\Theta C_2}\right), \\
v &= \pm \frac{2}{\Theta} \sin \left(\frac{-4\Theta \gamma tC_2 \pm \Theta C_2^2 t^2 + 4\Theta^2\lambda + 4\Theta C_1 C_2}{4\Theta C_2}\right),
\end{align*}
\] (36)

\[4\] Energy of the solution

It is much easier to analyse the energy of the solution due to the exact form of solution. According to (36), the energy of the solution is as follows

\[
\int_0^K \left|S_r\right|^2 r dr = \frac{C_2^2 K^4}{4 \left(t^2C_2^2 + 2tC_2C_3 + C_3^2 - 4\right)^2}.
\] (37)

(37) is a function that has nothing to do with \(\lambda\) and \(\gamma\). Hence, the additional field plays an unimportant role in the energy of the solution. Furthermore, it is clearly that (36) can be a global solution(or a local solution) about time variables according to (37). In fact, if \(-\frac{C_3 \pm 2}{C_2} > 0\), (37) will be an infinity value at \(t = -\frac{C_3 \pm 2}{C_2}\). In this case, (36) is a finite time blowup solution. However, if \(-\frac{C_3 \pm 2}{C_2} < 0\), (37) will not be an infinity value at any \(t > 0\). Hence, (36) is a global solution in this case. In addition, if \(-\frac{C_3 \pm 2}{C_2} = 0\) (or \(-\frac{C_3 \pm 2}{C_2} = 0\), (36) is a singularity which blowup at \(t = 0\).
In fact, if the constants are selected appropriately, the same energy behavior will happen in (35): (35) covers two different types solutions (blowup solution and global solution). As we known, the difficulty of the previous PDE (22) is that the non-linear term involves derivatives. In fact, norm of this derivatives term is hard to be estimated. We rewrite (22) as follows

\begin{equation}
    iW_t = -W_{rr} - \frac{n-1}{r}W_r + F,
\end{equation}

where

\[ F = -\frac{2\bar{W}}{1 - |W|^2}W^2 + \lambda \frac{1+|W|^2}{1 - |W|^2} W + \gamma W. \]

Due to the fundamental and elemental chain of identities/inequalities, we obtain

\[ \| W \|_{L^2} \leq \| W_0 \|_{L^2} + \int_0^t \| F \|_{L^2} \, dt. \]  

(39)

If we replace \( F \) with \( F = |W|^2 W \), (38) is Schrödinger equation with

\[ \| F \|_{L^2} \leq \sup |W|^2 \| W \|_{L^2} . \]

Furthermore, \( \sup |W|^2 \) is bounded assuming regularity on \( W_0 \) and repeating the process for \( \nabla W, \Delta W, \ldots \). If for example \( F \) of (38), this method does not work. In fact, if we set \( \gamma = \lambda = 0 \) and

\[ \begin{align*}
    u &= \frac{2}{\Theta} \cos \left( \frac{C_2^2 r^2 + 4 \Theta C_1 C_2}{4 \Theta C_2} \right), \\
    v &= \frac{2}{\Theta} \sin \left( \frac{C_2^2 r^2 + 4 \Theta C_1 C_2}{4 \Theta C_2} \right), \\
    v &= t C_2 + C_3,
\end{align*} \]

(40)

a directly calculation (here we set \( r \in [0, K] \)) show

\[ \| W_0 \|_{L^2} + \int_0^t \| F \|_{L^2} \, dt = \left| -\frac{1}{2} \sqrt{\frac{C_3^2}{\Theta} - 4 + \frac{1}{2} C_3} \right| \sqrt{K} + \frac{1}{40} \int_0^t \sqrt{5(\Theta C_2 - \Theta + C_3)^2 C_2^2 K^{5/2}} \, dt. \]  

(41)

It is obviously that (41) will be an infinity value if \( t \) runs up to the blowup time. Hence, whether the left side of (39) is a finite value is not clear. Following the discussion above, the same situation will happen under the setting \( \gamma \) and \( \lambda \) are not equal to zero.
5 Conclusions

In this paper, we study the evolution of the GSMF. In order to understand the dynamics of this system, we firstly deduce the equivalent equation of it. In fact, it will be a little bit better by computing the stereographic projection form on the complex plane. The initial-value problem of GSMF has been studied extensively. It is known that sufficiently smooth solutions exist locally in time. As a conclusion, the SMF in 1d is solved (modulus solving Frenet equations for the curvature and the torsion). However, if the initial value of the GSMF is the large data and spatial dimension is more than 1, what will happen when $t$ tends to the infinity? Our results show that the solution can develop two different behaviors under the large data. Given the smooth data as in the initial time, solution can develop a finite time singularity or be a global one. In fact, if the spatial domain is finite, the energy of the solution of GSMF is also a finite one. The manifold $\mathcal{H}^2$ and the nonlinear term of GSMF which contains the spatial derivative play the important roles in the evolution of the solution.

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