Solving First Kind Abel Integral Equations Using the SBA Numerical Method

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Abstract

Our goal in this paper is to use the SBA numerical method (combination of Adomian method and Picard successive approximations), to solve first kind Abel integration equations.

In this work, we shall describe the SBA method and study the convergence of this method applied to first kind Volterra general integral.

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1 Introduction

These last years, a large number of numerical methods for resolution of integral equations and notably Laplace, Fourier and successive approximations, solving core and discretization, have been developed. In this paper, we propose to use the SBA numerical method to establish the convergence of the Adomian algorithm. We provide solutions to the following first kind Abel integral equations.

$$\int_0^x \frac{\varphi(t)}{\sqrt[n]{x-t}} dt = \left[\sum_0^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \frac{x^{m+1}}{\sqrt[n]{x}}; n \geq 2, m \in \mathbb{N} \quad (1)$$

$$\int_0^x \frac{\varphi(t)}{\sqrt[3]{x-t}} dt = \lambda x^p \sqrt[3]{x^2}, p \geq 1 \quad (2)$$

2 Description of the numerical SBA method

Let's consider the following functional equation

$$A\varphi = f \quad (3)$$

Where $A: H \rightarrow H$ is an operator not necessarily linear and H is a Hilbert space adequately chosen given the operator A .

Let:

$$A = L - R - N \quad (4)$$

Where L is an invertible operator in the Adomian sense, R the linear remainder and N a nonlinear operator.

Equation (3) therefore becomes:

$$L\varphi - R\varphi - N\varphi = f \Leftrightarrow \theta + L^{-1}(f) + L^{-1}(R\varphi) + L^{-1}(N\varphi) \quad (5)$$

Where θ is such that $L\theta = 0$.

Equation (5) is the Adomian canonical form [1]. Using the successive approximations [2], we get:

$$\varphi^k = \theta^k + L^{-1}(f^k) + L^{-1}(R(\varphi^k)) + L^{-1}(N(\varphi^{k-1})); k \geq 1 \quad (6)$$

This yields the following Adomian algorithm:

$$\begin{cases} \varphi_0^k = \theta^k + L^{-1}(f^k) + L^{-1}(N(\varphi^{k-1})); k \geq 1 \\ \varphi_n^k = L^{-1}(R(\varphi_{n-1}^k)); n \geq 1 \end{cases} \quad (7)$$

The Picard principle is then applied to equation (7): let φ^0 be such that $N(\varphi^0) = 0$, for $k = 1$, we get:

$$\begin{cases} \varphi_0^1 = \theta^1 + L^{-1}(f^1) + L^{-1}(N(\varphi^0)) \\ \varphi_n^1 = L^{-1}(R(\varphi_{n-1}^1)); n \geq 1 \end{cases}$$

If the series $\left(\sum_{n \geq 0} \varphi_n^1\right)$ converges, then $\varphi^1 = \sum_{n \geq 1} \varphi_n^1$.

For $k = 2$, we get:

$$\begin{cases} \varphi_0^2 = \theta^2 + L^{-1}(f^2) + L^{-1}(N(\varphi^1)) \\ \varphi_n^2 = L^{-1}(R(\varphi_{n-1}^2)); n \geq 1 \end{cases}$$

If the series $\left(\sum_{n \geq 0} \varphi_n^2\right)$ converges, then $\varphi^2 = \sum_{n \geq 1} \varphi_n^2$. This process is repeated to k .

If the series $\left(\sum_{n \geq 0} \varphi_n^k\right)$ converges, then $\varphi^k = \sum_{n \geq 1} \varphi_n^k$, therefore $\varphi = \lim_{k \rightarrow +\infty} \varphi^k$ is the solution of the problem.

Thus, given the problem $(p): A\varphi = f$, we combine ideas from the classical techniques to derive the following appropriate approximate scheme.

$$\begin{cases} \varphi_0^k = \theta^k + L^{-1}(f^k) + L^{-1}(N(\varphi^{k-1})); k \geq 1 \\ \varphi_n^k = L^{-1}(R(\varphi_{n-1}^k)); n \geq 1 \end{cases} \quad (8)$$

called SBA algorithm.

2.1 Study of the SBA algorithm convergence using the first class Volterra integral equation

Let us the following first kind Volterra general linear integral equation:

$$\int_a^x K(x, t) \varphi(t) dt = f(x), a \leq t \leq x \leq T < +\infty \quad (9)$$

Where φ is the unknown function, f is a continuous function, an $\sum_t = [a, T]$, K is continuous on $\Omega = [a, T] \times [a, T]$.

The search of the canonical form

Applying the substitution technique [4] to the equation (9), we get:

$$\int_a^x K(x, t) [\varphi(t) - \varphi(x)] dt + \varphi(x) K(x, t) dt = f(x) \quad (10)$$

Let put

$$h(x) = \int_a^x K(x, t) dt \quad (11)$$

Which is a known function continuous on $\sum_t = [a, T]$ and K is continuous on $\Omega = [a, T] \times [a, T]$. Equation (10) becomes:

$$h(x)\varphi(x) + \int_a^x K(x, t) [\varphi(t) - \varphi(x)] dt = f(x) \quad (12)$$

if $h(x) \neq 0$, we get:

$$\varphi(x) = \frac{f(x)}{h(x)} - \frac{1}{h(x)} \int_a^x K(x, t) [\varphi(t) - \varphi(x)] dt \quad (13)$$

which is the canonical form.

Applying the SBA algorithm to the equation (13), we get:

$$\begin{cases} \varphi_0^k(x) = \frac{f(x)}{h(x)} & , k \geq 1 \\ \varphi_n^k(x) = -\frac{1}{h(x)} \int_a^x K(x, t) [\varphi_{n-1}^k(t) - \varphi_{n-1}^k(x)] dt, n \geq 1 \end{cases} \quad (14)$$

Let us show that this algorithm converges

- f is assumed continuous on $\sum_t = [a; T]$ and given that h is continuous on $\sum_t = [a; T]$, there exist $m > 0, \lambda > 0$ and $m_1 > 0$ such that

$$\forall (x, t) \in \Omega = [a; T] \times [a; T], \left| \frac{f(x)}{h(x)} \right| \leq m, \left| \frac{1}{h(x)} \right| \leq \lambda \text{ and } \left| \frac{f(x)}{h(x)} - \frac{f(t)}{h(t)} \right| \leq m_1$$

- K being continuous on $\Omega = [a; T] \times [a; T]$, there exist $M > 0$ such $\forall (x, t) \in \Omega, |K(x, t)| \leq M$.

For $\varphi_n^k(x)$, we have successively, $\forall k \geq 1$

$$\left\{ \begin{array}{l} |\varphi_0^k(x)| \leq m \Rightarrow |\varphi_0^k(t) - \varphi_0^k(x)| \leq m_1 \\ |\varphi_1^k(x)| \leq m_1 \frac{|\lambda M(x-a)|}{1!} \Rightarrow |\varphi_1^k(t) - \varphi_1^k(x)| \leq m_2 \\ |\varphi_1^k(x)| \leq m_1 \frac{[\lambda M(x-a)]}{1!} \Rightarrow |\varphi_1^k(t) - \varphi_1^k(x)| \leq m_2 \\ |\varphi_2^k(x)| \leq m_2 \frac{[\lambda M(x-a)]^2}{2!} \Rightarrow |\varphi_2^k(t) - \varphi_2^k(x)| \leq m_3 \\ |\varphi_3^k(x)| \leq m_3 \frac{[\lambda M(x-a)]^3}{3!} \Rightarrow |\varphi_3^k(t) - \varphi_3^k(x)| \leq m_4 \\ \dots \\ |\varphi_n^k(x)| \leq m_n \frac{[\lambda M(x-a)]^n}{n!} \end{array} \right.$$

Let's put $m' = \text{Sup}(m_1, m_2, \dots, m_n)$, it follows that:

$$|\varphi_n^k(x)| \leq m' \frac{[\lambda M(x-a)]^n}{n!}$$

Let's put $\phi_n(x) = m' \frac{[\lambda M(x-a)]^n}{n!}$

The series:

$$\sum_{n \geq 0} \phi_n(x) = m' \sum_{n \geq 0} \frac{[\lambda M(x-a)]^n}{n!}$$

Converging geometrically toward the function:

$$g(x) = m' e^{[\lambda M(x-a)]} \text{ on } [a; T].$$

Therefore, the series $\sum_{n \geq 0} \varphi_n^k(x)$ converges normally and thus absolutely to

$\varphi^k(x)$ on $[a; T]$, $\forall k \geq 1$. Thus, $\varphi(x) = \lim_{k \rightarrow +\infty} \varphi^k(x)$ is the solution to problem (9).

For the function ζ , we have following SBA algorithm:

$$\left\{ \begin{array}{l} \zeta_0^k(x) = \frac{f(x)}{h(x)} \\ \zeta_n^k(x) = -\frac{1}{h(x)} \int_a^x K(x, t) [\zeta_{n-1}^k(t) - \zeta_{n-1}^k(x)] dt, n \geq 1 \end{array} \right. , k \geq 1 \quad (15)$$

Let's put $w(x) = \varphi(x) - \zeta(x)$, $w(x)$ checks the following SBA algorithm:

$$\begin{cases} w_0^k(x) = \varphi_0^k(x) - \zeta_0^k(x) & , k \geq 1 \\ w_n^k(x) = -\frac{1}{h(x)} \int_a^x K(x, t) [w_{n-1}^k(t) - w_{n-1}^k(x)] dt, n \geq 1 \end{cases} \quad (16)$$

By unfolding the algorithm (16) for $k \geq 1$, we get:

$$\begin{cases} w_0^k(x) = \frac{f(x)}{h(x)} - \frac{f(x)}{h(x)} = 0 \\ w_1^k(x) = -\frac{1}{h(x)} \int_a^x K(x, t) [w_0^k(t) - w_0^k(x)] dt = 0 \\ \vdots \\ w_n^k(x) = -\frac{1}{h(x)} \int_a^x K(x, t) [w_{n-1}^k(t) - w_{n-1}^k(x)] dt = 0, \forall n \geq 0 \end{cases}$$

we have $w^k(x) = \sum_{n=0}^{+\infty} w_n^k(x) = 0$ since $\forall n \geq 0, w_n^k(x) = 0$. And thus

$$w(x) = \varphi(x) - \zeta(x) = 0 \Leftrightarrow \varphi(x) = \zeta(x) \text{ and } \varphi = \zeta, \forall x \in \sum_t = [a; T].$$

This proves the uniqueness of the solution of the equation (9) and the convergence of the SBA algorithm.

2.2 Applications

Problem 1

Let's consider the following first kind Volterra linear integral equation:

$$\int_0^x \frac{\varphi(t)}{\sqrt[n]{x-t}} dt = \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \frac{x^{m+1}}{\sqrt[n]{x}}, n \geq 2 \quad (17)$$

The search for the Adomian canonical form

For this, we shall use the substitution technique as follows,

$$\varphi(x) \int_0^x \frac{1}{\sqrt[n]{x-t}} dt + \int_0^x \frac{\varphi(t) - \varphi(x)}{\sqrt[n]{x-t}} dt = \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \frac{x^{m+1}}{\sqrt[n]{x}} \quad (18)$$

by integration, we have,

$$\int_0^x \frac{1}{\sqrt[n]{x-t}} dt = \frac{nx}{(n-1)\sqrt[n]{x}}$$

and equation (17) becomes:

$$\varphi(x) = \frac{(n-1)x^m}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] - \frac{(n-1)\sqrt[n]{x}}{nx} \int_0^x \frac{\varphi(t) - \varphi(x)}{\sqrt[n]{x-t}} dt \quad (19)$$

The equation (19) is the canonical form.

Solving by the SBA method

Applying the SBA algorithm, it follows that (18):

$$\begin{cases} \varphi_0^k(x) = \frac{(n-1)x^m}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right], k \geq 1 \\ \varphi_p^k(x) = -\frac{(n-1)\sqrt[n]{x}}{nx} \int_0^x \frac{\varphi_{p-1}^k(t) - \varphi_{p-1}^k(x)}{\sqrt[n]{x-t}} dt, p \geq 1 \end{cases}$$

Computation of $\varphi_1^1(x)$

$$\varphi_1^1(x) = -\left(\frac{n-1}{n}\right)^2 \frac{\sqrt[n]{x}}{x} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \int_0^x \frac{t^m - x^m}{\sqrt[n]{x-t}} dt$$

we have:

$$\int_0^x \frac{t^m - x^m}{\sqrt[n]{x-t}} dt = \int_0^x \frac{t^m}{\sqrt[n]{x-t}} dt - \frac{nx^{m+1}}{(n-1)\sqrt[n]{x}}$$

Let's take $T = x - t$ and integrating, we get:

$$\begin{aligned} \int_0^x \frac{t^m}{\sqrt[n]{x-t}} dt &= \int_0^x (x-T)^m T^{-\frac{1}{n}} dt \\ &= \int_0^x \left[\sum_{k=0}^m C_m^k x^k (-1)^{m-k} T^{m-k} T^{-\frac{1}{n}} \right] dT \\ &= \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \frac{x^{m+1}}{\sqrt[n]{x}} \end{aligned}$$

$$\int_0^x \frac{t^m - x^m}{\sqrt[n]{x-t}} dt = \left[\sum_k^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \frac{x^{m+1}}{\sqrt[n]{x}} - \frac{nx^{m+1}}{(n-1)\sqrt[n]{x}}$$

Therefore, we get:

$$\varphi_1^1(x) = \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \left(1 - \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \right) x^m$$

Computation of $\varphi_2^1(x)$

$$\begin{aligned} \varphi_2^1(x) &= -\frac{(n-1)}{n} \frac{\sqrt[n]{x}}{x} \int_0^x \frac{\varphi_1^1(t) - \varphi_1^1(x)}{\sqrt[n]{x-t}} dt \\ &= \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \left(1 - \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \right)^2 x^m \end{aligned}$$

By induction on p , we get:

$$\left\{ \begin{array}{l} \varphi_0^k(x) = \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \\ \varphi_1^k(x) = \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \left(1 - \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \right) x^m \\ \varphi_2^k(x) = \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \left(1 - \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \right)^2 x^m \\ \dots \\ \varphi_p^k(x) = \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \left(1 - \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \right)^p x^m \end{array} \right.$$

And then :

$$\left\{ \begin{array}{l} \varphi_N^k(x) = \sum_{p=0}^{N-1} \varphi_p^k(x) \\ \\ = \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \frac{\left(1 - \left(1 - \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \right)^N \right)}{\left(1 - \left(1 - \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1+m-k-\frac{1}{n}\right)} \right] \right) \right)} x^m \end{array} \right.$$

Then the approached solution at the k^{th} step is:

$$\varphi^k(x) = \lim_{N \rightarrow +\infty} \varphi_N^k(x) = x^m$$

For $0 < \left(1 - \frac{n-1}{n} \left[\sum_{k=0}^m C_m^k \frac{(-1)^{m-k}}{\left(1 + m - k - \frac{1}{n} \right)} \right] \right) < 1$, the exact solution of the problem

is:

$$\varphi(x) = \lim_{k \rightarrow +\infty} \varphi^k(x) = x^m.$$

Problem 2

Let's consider the following Volterra linear equation of the first kind:

$$\int_0^x \frac{\varphi(t)}{\sqrt[3]{x-t}} dt = \lambda x^p \sqrt[3]{x^2}, p \geq 1 \quad (20)$$

The search for the canonical form

Let us search for the associated Adomian canonical form (20):

$$\varphi(x) \int_0^x \frac{1}{\sqrt[3]{x-t}} dt + \int_0^x \frac{\varphi(t) - \varphi(x)}{\sqrt[3]{x-t}} dt = \lambda x^p \sqrt[3]{x^2}$$

We get:

$$\frac{3}{2} \sqrt[3]{x^2} \varphi(x) = \lambda x^p \sqrt[3]{x^2} - \int_0^x \frac{\varphi(t) - \varphi(x)}{\sqrt[3]{x-t}} dt$$

Therefore, the canonical form is

$$\varphi(x) = \frac{2}{3} \lambda x^p - \frac{2}{3 \sqrt[3]{x^2}} \int_0^x \frac{\varphi(t) - \varphi(x)}{\sqrt[3]{x-t}} dt$$

Solving by the SBA method

Applying the SBA algorithm to the equation (20), it follows that

$$\begin{cases} \varphi_0^k(x) = \frac{2}{3} \lambda x^p, & k \geq 1 \\ \varphi_n^k(x) = -\frac{2}{3 \sqrt[3]{x^2}} \int_0^x \frac{\varphi_{n-1}^k(t) - \varphi_{n-1}^k(x)}{\sqrt[3]{x-t}} dt, n \geq 1 \end{cases}$$

Computation of $\varphi_1^k(x)$

$$\varphi_1^k(x) = -\frac{4\lambda}{9 \sqrt[3]{x^2}} \int_0^x \frac{t^p - x^p}{\sqrt[3]{x-t}} dt$$

We have

$$\int_0^x \frac{t^p - x^p}{\sqrt[3]{x-t}} dt = \int_0^x \frac{t^p}{\sqrt[3]{x-t}} dt - \frac{3x^{p+1}}{2 \sqrt[3]{x}}$$

Let's put $T = x - t$, and by integrating m times by parts, we get:

$$\begin{aligned}
\int_0^x \frac{t^p}{\sqrt[3]{x-t}} dt &= \int_0^x (x-T)^p T^{-\frac{1}{3}} dt \\
&= \frac{m(m-1)3^2}{2 \times 5} \int_0^x (x-T)^{p-2} T^{\frac{5}{3}} dt \\
&= \dots \\
&= \frac{p!3^{p+1}}{\prod_{k=1}^{p+1} (3k-1)} \sqrt[3]{x^{3(p+1)-1}}
\end{aligned}$$

Then

$$\int_0^x \frac{t^p - x^p}{\sqrt[3]{x-t}} dt = \frac{p!3^{p+1}}{\prod_{k=1}^{p+1} (3k+1)} \sqrt[3]{x^{3(p+1)-1}} - \frac{3x^{p+1}}{2\sqrt[3]{x}}$$

We have the following

$$\begin{aligned}
\varphi_1^k(x) &= -\frac{4\lambda}{9\sqrt[3]{x^2}} \left(\frac{p!3^{p+1}}{\prod_{k=1}^{p+1} (3k-1)} \sqrt[3]{x^{3(p+1)-1}} - \frac{3x^{p+1}}{2\sqrt[3]{x}} \right) \\
&= \frac{2}{3} \lambda \left(1 - \frac{p!3^p}{\prod_{k=2}^{p+1} (3k-1)} \right) x^p
\end{aligned}$$

By induction on n , we get:

$$\left\{ \begin{array}{l} \varphi_0^k(x) = \frac{2}{3} \lambda x^p \\ \varphi_1^k(x) = \frac{2}{3} \lambda \left(1 - \frac{p! 3^p}{\prod_{k=2}^{p+1} (3k-1)} \right) x^p \\ \varphi_2^k(x) = \frac{2}{3} \lambda \left(1 - \frac{p! 3^p}{\prod_{k=2}^{p+1} (3k-1)} \right)^2 x^p \\ \vdots \\ \varphi_n^k(x) = \frac{2}{3} \lambda \left(1 - \frac{p! 3^p}{\prod_{k=2}^{p+1} (3k-1)} \right)^n x^p \end{array} \right.$$

Let us put:

$$\begin{aligned} \varphi_N^k(x) &= \sum_{n=0}^{N-1} \varphi_n^k(x) \\ &= \frac{2}{3} \lambda x^p \left[\frac{1 - \left(1 - \frac{p! 3^p}{\prod_{k=2}^{p+1} (3k-1)} \right)^N}{1 - \left(1 - \frac{p! 3^p}{\prod_{k=2}^{p+1} (3k-1)} \right)} \right] \end{aligned}$$

Then the approached solution at the k^{th} step is:

$$\begin{aligned}\varphi^k(x) &= \lim_{N \rightarrow +\infty} \varphi_N^k(x) \\ &= \lambda \frac{\left[\prod_{k=1}^{p+1} (3k-1) \right]}{p! 3^{p+1}} x^p\end{aligned}$$

$$\text{For } 0 < \left(1 - \frac{p! 3^p}{\prod_{k=2}^{p+1} (3k-1)} \right) < 1$$

The exact solution of the problem is:

$$\varphi(x) = \lim_{k \rightarrow +\infty} \varphi^k(x) = \lambda \frac{\left[\prod_{k=1}^{p+1} (3k-1) \right]}{p! 3^{p+1}} x^p$$

3 Conclusion

In this paper, we first describe the SBA numerical method. Then we showed that the SBA method converges when applied to Volterra general integral equations of first kind.

Lastly, we used this method to solve the Abel integral equation of first kind.

The integral equations of this kind being not in the canonical form, applying the SBA method reveals difficult to counter this difficulty, we used the substitution technique to get the canonical form. The SBA method served us in solving two problems whose levels of difficulty pertain to the introduced parameters.

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