

Pricing Quanto Forward and European Options

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Abstract

In this paper, using Black-Scholes assumptions, we derive an analytical closed form solution for the pricing of a quanto forward and option contract. We use techniques of stochastic calculus and continuous time in order to establish a closed form solution for a quanto forward and option.

Keywords: Quanto Option Pricing, Quanto Forward Price, Closed Form Solution, Black-Scholes

1 Introduction

The pricing of quanto options using the Black-Scholes model (Black and Scholes, 1973) are extremely inaccurate in an empirical framework, due to the fat tails observed in financial returns (see among others, Mittnik et al, 2000 and Opschoor et al, 2018). Park et al (2013) price quanto options in a stochastic volatility framework, within the Black and Scholes model. However, the skewness of financial data makes their findings imprecise. Kim et al (2015) improve

the Black and Scholes option pricing model for quanto options by introducing random time changes into normal tempered stable (NTS) distribution, which enables infinite jumps as well as capturing random varying time in stochastic volatility.

In this paper we present an analytical solution for the quanto forward and option prices, under Black-Scholes assumptions, which satisfies the general Feynman-Kac pricing partial differential equation (Janson and Tysk, 2006). In contrast with competing derivations assuming discrete time, the current research of developing a closed-form solution for quanto forwards and options allows the validation of the application of continuous time assumption and shows that errors can occur while adopting the former approach, especially when sampling frequencies are minuscule. Through this approach, extensions can be made to more complicated derivative securities, especially when they have dividend yield and intermittent income prior to the execution date. Also, numerical values can be very efficiently computed from the newly found analytical formula.

The next section introduces the notations and assumptions used in the pricing. The third section first prices the quanto forward contract. Based on the price of the quanto forward, we use the Black-Scholes pricing formula to price the European quanto option. Finally, we conclude by discussing possible shortcomings and how further research can improve the pricing, especially when parametric assumptions are changed.

2 Assumptions and notation

Consider a probability space (Ω, \mathcal{F}, P) where Ω is a set of outcomes, \mathcal{F} a σ -field, and P a probability measure and a time interval $\mathcal{T} = [0, T]$. For any $s, t \in \mathcal{T}$ with $s \leq t$ the family \mathcal{F}_t is called a filtration. We assume an underlying share price S follows a stochastic process. S is adapted to the filtration \mathcal{F}_t , if S_t can be determined by information available at time t . That is S_t is \mathcal{F}_t -predictable for any $t \in \mathcal{T}$. The return process X_t is a function of S_t , and $\langle X \rangle$ is the variation of X , which is known as the realized variance of the returns on S . B_t is a Brownian motion, which is the limit, as $\Delta_t \rightarrow 0$, of a Gaussian discrete-time random walk.

$$B_{t+\Delta_t} = B_t + \varepsilon_t \sqrt{\Delta_t}$$

Where ε_t are i.i.d standard normal distribution: $\varepsilon_t \sim N(0, 1)$. An Itô process X has the form:

$$X_t = X_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dB_s$$

Where X_0 a constant, μ_t and σ_t are adapted processes satisfying regularity conditions. The Itô process has the following properties:

$$\begin{aligned} dt \cdot dt &= 0, \\ dB_t \cdot dt &= 0, \\ dB_t \cdot dB_t &= dt. \end{aligned}$$

Consider a function $f(t, x)$ whose partial derivatives $\frac{\partial f}{\partial t}$, $\frac{\partial f}{\partial x}$ and $\frac{\partial^2 f}{\partial x^2}$ are well-defined and continuous, and an Itô process X_t

$$dX_t = \mu_t dt + \sigma_t dB_t.$$

This implies that $Y_t = f(t, X_t)$ is also an Itô process and

$$\begin{aligned} dY_t &= \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x} dX_t + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} (dX_t)^2 \\ &= \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \mu_t + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} \sigma_t^2 \right] dt + \frac{\partial f}{\partial x} \sigma_t dB_t. \end{aligned} \tag{1}$$

The second equality follows from the fact that $(dX_t)^2 = \sigma_t^2 dt$. The market is arbitrage-free if and only if there exists a risk-neutral measure Q .

According to Girsanov (1960), let B be a Brownian motion and P be the corresponding Wiener measure. Let η_t be an adapted process satisfying Novikov's conditions. Then a process

$$\xi_t = \exp \left(- \int_0^t \eta_\tau dB_\tau - \frac{1}{2} \int_0^t \eta_\tau^2 d\tau \right)$$

is a martingale and we can define a new measure Q by

$$Q(A) = \mathbb{E}^P(1_A \xi_T)$$

equivalent to P . Equivalent means that measures P and Q agree which events have zero probability. Moreover, the process B^Q defined by

$$B_t^Q = B_t + \int_0^t \eta_s ds,$$

is a Brownian motion under Q . Then

$$\mathbb{E}^Q[S_T] = \mathbb{E}^P[\xi_T S_T].$$

Consider the discounted price process $S_t/S_{0,t}$. Assume there exists an equivalent martingale measure Q such that for $T > t$,

$$\mathbb{E}_t^Q \left[\frac{S_T}{S_{0,T}} \right] = \frac{S_t}{S_{0,t}}.$$

At this point in the analysis, we fix an arbitrary time horizon $T > 0$ and introduce the following assumptions: (i) The real interest rate r is constant. (ii) Markets are arbitrage-free. (iii) The underlying share price S satisfies

$$\frac{dS_t}{S_t} = rdt + \sigma_t dB_t, \quad S_t > 0 \quad (2)$$

where B_t is a Brownian motion. (iv)

$$\int_t^T \sigma_\tau^2 d\tau \quad \text{is finite.} \quad (3)$$

(v) B_t and B_t^σ are independent

$$dB_t dB_t^\sigma = \rho dt. \quad (4)$$

From assumption (ii), we know that on a filtered probability space (Ω, \mathcal{F}, P) , there exists an equivalent probability measure Q such that $\forall \alpha, p \in \mathbb{R}$ and $\forall t \leq T$, a power contract paying at time T , αS_T^p has time- t price equal to $\alpha \mathbb{E}_t^Q S_T^p$, where \mathbb{E}_t^Q denotes the \mathcal{F}_t -conditional Q -expectation. We are now in the position to derive the pricing formulae for quanto forward contracts and quanto European options.

3 Pricing

Quanto Forward Contract: Assume that both the foreign asset price S_t and the exchange rate X_t follow a geometric Brownian motion:

$$\begin{aligned} \frac{dS_t}{S_t} &= \mu_s dt + \sigma_s dB_t^s, \\ \frac{dX_t}{X_t} &= \mu_x dt + \sigma_x dB_t^x, \\ dB_t^s dB_t^x &= \rho dt, \end{aligned}$$

where S_t is the foreign asset price, and X_t is the home price of one unit of the foreign currency. Let r and r_f be the domestic and the foreign interest rates

respectively,

$$\begin{aligned}\frac{dS_t^0}{S_t^0} &= r dt, \\ \frac{dS_t^{0,f}}{S_t^{0,f}} &= r_f dt.\end{aligned}$$

Consider a quanto forward contract which pays $\bar{X}S_T - F$ at time T . Here \bar{X} is a fixed exchange rate and F is the quanto forward price in units of domestic currency. We derive F that makes the market value of the quanto forward contract at time t equal to zero. Let V be the value of the derivative security described. Given no arbitrage and the fundamental theorem of asset pricing we obtain $V_T = \bar{X}S_T - F$. We derive the price of a European quanto option that pays $\max(\bar{X}S_T - K, 0)$ at time T , risk-neutral valuation, and asset pricing equation. Under the risk-neutral measure Q , we obtain

$$\begin{aligned}\frac{V_t}{S_t^0} &= \mathbb{E}_t^Q \left[\frac{V_T}{S_T^0} \right], \\ V_t &= \frac{S_t^0}{S_T^0} \mathbb{E}_t^Q [V_T], \\ &= e^{-r(T-t)} \mathbb{E}_t^Q [V_T].\end{aligned}$$

We require F such that $V_t = 0$, which means there is no money exchanged today. Hence

$$\begin{aligned}e^{-r(T-t)} \mathbb{E}_t^Q [V_T] &= 0, \\ \mathbb{E}_t^Q [V_T] &= 0, \\ \mathbb{E}_t^Q [\bar{X}S_T - F] &= 0, \\ F &= \mathbb{E}_t^Q [\bar{X}S_T].\end{aligned}$$

In order to find this value, we need to study the dynamics of S_t under risk-neutral measure Q . Under Q , the asset price is a martingale as the return under Q is equal to the risk free rate. Given that we have two Brownian motions B_t^s and B_t^x ; we assume that η_s and η_x are the prices of risks associated respectively. Now, by Girsanov's theorem we have

$$\begin{aligned}dB_t^{s,Q} &= dB_t^s + \eta_s dt, \\ dB_t^{x,Q} &= dB_t^x + \eta_x dt.\end{aligned}$$

We need to find η_x and η_s such that measure Q is an equivalent Martingale measure, i.e. the discounted prices should be martingales. Let domestic and foreign risk price assets have prices S_t^0 and $S_t^{0,f}$, and

$$\begin{aligned}\frac{dS_t^0}{S_t^0} &= r dt, \\ \frac{dS_t^{0,f}}{S_t^{0,f}} &= r_f dt.\end{aligned}$$

Due to the fact that these are risk-free assets, they do not have any diffusion (stochastic) terms. Steps (A)-(F) yield the requested results.

(A) From the domestic point of view, the foreign risky asset is an asset with risky price $Y_t = S_t X_t$. Applying Ito's lemma, we obtain a process for dY_t . If $Y_t = f(t, S_t, X_t)$,

$$\begin{aligned}dY_t &= f_t dt + f_{S_t} dS_t + f_{X_t} dX_t + \frac{1}{2} [f_{S_t S_t} (dS_t)^2 + f_{X_t X_t} (dX_t)^2 + 2f_{S_t X_t} (dS_t) (dX_t)] \\ &= 0 dt + X_t S_t (\mu_s dt + \sigma_s dB_t^s) + S_t X_t (\mu_x dt + \sigma_x dB_t^x) \\ &\quad + \frac{1}{2} [0 (dS_t)^2 + 0 (dX_t)^2 \\ &\quad + 2(X_t S_t (\mu_s \mu_x (dt)^2) + \mu_s \sigma_x dB_t^x dt + \sigma_s dB_t^s dt + \sigma_s dB_t^s \mu_x dt + \sigma_s \sigma_x dB_t^s dB_t^x)] \\ &= X_t S_t (\mu_s dt + \sigma_s dB_t^s) + S_t X_t (\mu_x dt + \sigma_x dB_t^x) + X_t S_t (\sigma_s \sigma_x \rho dt).\end{aligned}$$

Then we have

$$\frac{dY_t}{Y_t} = (\mu_s + \mu_x + \sigma_s \sigma_x \rho) dt + \sigma_s dB_t^s + \sigma_x dB_t^x. \quad (5)$$

(B) From the point of view of a domestic investor, the foreign risk free asset is a risky asset with price $Z_t = S_t^{0,f} X_t$. Applying Ito's lemma to get a process for dZ_t , where $Z_t = f(t, S_t^{0,f}, X_t)$. Thus

$$\begin{aligned}dZ_t &= f_t dt + f_S dS_t^{0,f} + f_X dX_t + \frac{1}{2} [f_{SS} (dS_t^{0,f})^2 + f_{XX} (dX_t)^2 + 2f_{SX} (dS_t^{0,f}) (dX_t)] \\ &= 0 dt + X_t S_t^{0,f} r_f dt + X_t S_t^{0,f} (\mu_x dt + \sigma_x dB_t^x) \\ &\quad + \frac{1}{2} [0 (dS_t^{0,f})^2 + 0 (dX_t)^2 + 2S_t^{0,f} X_t (r_f \mu_x (dt)^2 + r_f \sigma_x dB_t^x dt)] \\ &= X_t S_t^{0,f} r_f dt + X_t S_t^{0,f} (\mu_x dt + \sigma_x dB_t^x) + S_t^{0,f} x_t(0) \\ &= X_t S_t^{0,f} r_f dt + X_t S_t^{0,f} (\mu_x dt + \sigma_x dB_t^x),\end{aligned}$$

from which we obtain

$$\frac{dZ_t}{Z_t} = (r_f + \mu_x)dt + \sigma_x dB_t^x. \quad (6)$$

Now, under the risk-neutral measure, the discounted prices of non-dividend paying assets must follow a martingale. Thus, we apply Ito's lemma to $d\left(\frac{Y_t}{S_t^0}\right)$ and $d\left(\frac{Z_t}{S_t^{0,f}}\right)$.

$$\begin{aligned} d\left(\frac{Y_t}{S_t^0}\right) &= 0dt + \frac{dY_t}{S_t^0} + \left(\frac{-Y_t}{(S_t^0)^2}\right) dS_t^0 \\ &\quad + \frac{1}{2} \left[0(dY_t)^2 + \frac{2Y_t}{(S_t^0)^3} (dS_t^0)^2 + 2\frac{-1}{(S_t^0)^2} dS_t^0 dY_t \right] \\ &= \frac{Y_t}{S_t^0} [(\mu_s + \mu_x + \rho\sigma_x\sigma_s)dt + \sigma_s dB_t^s + \sigma_x dB_t^x] - \frac{Y_t}{S_t^0} rdt \\ &\quad + \frac{1}{2} \left[\frac{2Y_t}{(S_t^0)^3} (S_t^0)^2 r^2 (dt)^2 - \frac{2Y_t S_t^0}{(dS_t^0)^2} (r(\mu_s + \mu_x + \sigma_x\sigma_s\rho)(dt)^2 + r\sigma_x dt dB_t^x) \right], \end{aligned}$$

from which we obtain

$$d\left(\frac{Y_t}{S_t^0}\right) / \left(\frac{Y_t}{S_t^0}\right) = (\mu_s + \mu_x + \rho\sigma_x\sigma_s - r) dt + \sigma_s dB_t^s + \sigma_x dB_t^x. \quad (7)$$

(D)

$$\begin{aligned} d\left(\frac{Z_t}{S_t^0}\right) &= 0dt + \frac{dZ_t}{S_t^0} + \left(\frac{-Z_t}{(S_t^0)^2}\right) dS_t^0 \\ &\quad + \frac{1}{2} \left[0(dZ_t)^2 + \frac{2Z_t}{(S_t^0)^3} (dS_t^0)^2 + 2\frac{-1}{(S_t^0)^2} dS_t^0 dZ_t \right] \\ &= \frac{Z_t}{S_t^0} [(r_f + \mu_x)dt + \sigma_x dB_t^x] - \frac{Z_t}{S_t^0} rdt \\ &\quad + \frac{1}{2} \left[\frac{2Z_t}{(S_t^0)^3} (S_t^0)^2 r^2 (dt)^2 - \frac{2Z_t S_t^0}{(dS_t^0)^2} (r(r_f + \mu_x)(dt)^2 + r\sigma_x dt dB_t^x) \right]. \end{aligned}$$

From which we obtain

$$d\left(\frac{Z_t}{S_t^0}\right) / \left(\frac{Z_t}{S_t^0}\right) = (r_f + \mu_x - r) dt + \sigma_x dB_t^x. \quad (8)$$

We use Girsanov's theorem to express (3) and (4) under the risk-neutral measure. Rewriting $\left(\frac{Y_t}{S_t^0}\right)$ and $\left(\frac{Z_t}{S_t^0}\right)$ under Q .

(E)

$$d\left(\frac{Y_t}{S_t^0}\right) / \left(\frac{Y_t}{S_t^0}\right) = (\mu_s + \mu_x + \rho\sigma_x\sigma_s - r - \sigma_s\eta_s - \sigma_x\eta_x) dt + \sigma_s dB_t^{s,Q} + \sigma_x dB_t^{x,Q}. \quad (9)$$

(F)

$$d\left(\frac{Z_t}{S_t^0}\right) / \left(\frac{Z_t}{S_t^0}\right) = (r_f + \mu_x - r - \sigma_x\eta_x) dt + \sigma_x dB_t^{x,Q}. \quad (10)$$

Thus, both processes (9) and (10) have zero drift under Q . We use this property in order to solve for η_s and η_x . Therefore,

$$\begin{aligned} \sigma_s\eta_s &= \mu_s + \rho\sigma_s + \rho\sigma_s\sigma_x - r_f \\ \sigma_x\eta_x &= r_f + \mu_x - r \end{aligned}$$

yields

$$\begin{aligned} \frac{dS_t}{S_t} &= (\mu_s - \sigma_s\eta_s)dt + \sigma_s dB_t^{s,Q} \\ &= (r_f - \rho\sigma_x\sigma_s)dt + \sigma_s dB_t^{s,Q}. \end{aligned}$$

Applying Itô's lemma to $\ln(S_t)$ yields

$$\begin{aligned} d\ln(S_t) &= \frac{dS_t}{S_t} - \frac{(dS_t)^2}{2S_t^2} \\ &= (r_f - \rho\sigma_x\sigma_s)dt + \sigma_s dB_t^{s,Q}. \end{aligned}$$

Since $(dS_t)^2 = S_t^2\sigma_s^2 dt$,

$$d\ln(S_t) = (r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2)dt + \sigma_s dB_t^{s,Q}.$$

Integrating from t to T yields

$$\begin{aligned} \int_t^T d\ln(S_\tau) d\tau &= \int_t^T (r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2) d\tau + \int_t^T \sigma_s dB_\tau^{s,Q} d\tau. \\ \ln(S_T) - \ln(S_t) &= (r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2)(T - t) + \sigma_s(B_T^{s,Q} - B_t^{s,Q}), \\ S_T &= S_t \exp \left[(r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2)(T - t) + \sigma_s(B_T^{s,Q} - B_t^{s,Q}) \right]. \end{aligned}$$

We now conclude that

$$F = \mathbb{E}_t^Q(\bar{X}S_T) = \bar{X}\mathbb{E}_t^Q(S_T).$$

Using the property that $\mathbb{E} \exp x = \exp \left(\mathbb{E}(x) + \frac{1}{2} \text{var}(x) \right)$ we derive

$$\begin{aligned} \mathbb{E}_t^Q(S_T) &= \mathbb{E}_t^Q S_t \exp \left[(r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2)(T-t) + \sigma_s(B_T^{s,Q} - B_t^{s,Q}) \right] \\ &= S_t \mathbb{E}_t^Q \exp \left[(r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2)(T-t) + \sigma_s(B_T^{s,Q} - B_t^{s,Q}) \right] \\ &= S_t \exp \left[\mathbb{E}_t^Q \left\{ (r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2)(T-t) + \sigma_s(B_T^{s,Q} - B_t^{s,Q}) \right\} \right] \\ &\quad + \frac{1}{2} \text{var} \left\{ (r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2)(T-t) + \sigma_s(B_T^{s,Q} - B_t^{s,Q}) \right\} \\ &= S_t \exp \left\{ (r_f - \rho\sigma_x\sigma_s - \sigma_s^2/2)(T-t) + \frac{1}{2}\sigma_s^2(T-t) \right\} \\ &= S_t \exp \{ (r_f - \rho\sigma_x\sigma_s)(T-t) \}. \end{aligned}$$

It follows that

$$F = \bar{X}S_t \exp [(r_f - \rho\sigma_x\sigma_s)(T-t)] \quad (11)$$

as requested. Next, let's find the price of a European quanto option paying $\max(\bar{X}S_T - K, 0)$ at time T . Let this price be denoted by P . We apply the Black-Scholes pricing formula.

$$\max(\bar{X}S_T - K, 0) = \bar{X} \max(S_T - \frac{K}{\bar{X}}, 0). \quad (12)$$

Let P' be the price of an option that pays $\max(S_T - \frac{K}{\bar{X}}, 0)$ at maturity. Given no arbitrage and the fundamental theorem of asset pricing: $P = \bar{X}P'$. Thus, we price P' first then multiply it by \bar{X} . This yields

$$\begin{aligned} \frac{dS_t}{S_t} &= (r_f - \rho\sigma_x\sigma_s)dt + \sigma_s dB_t^{s,Q} \\ \mathbb{E}_t^Q \left(\frac{dS_t}{S_t} \right) &= (r_f - \rho\sigma_x\sigma_s)dt \neq rdt. \end{aligned}$$

This is not an arbitrage because the foreign asset S_t is denominated in foreign currency. At this point we cannot directly use the Black-Scholes formula. Let $q = r - r_f + \rho\sigma_x\sigma_s$, then

$$\frac{dS_t}{S_t} = (r - q)dt + \sigma_s dB_t^{s,Q}.$$

This is now analogous to pricing an asset using the Black-Scholes formula with constant dividend yield (q).

$$\begin{aligned}
 P' &= \exp(-q(T-t))S_t N(d_1) - \exp(-r(T-t))\frac{K}{\bar{X}}N(d_2) \\
 P &= \bar{X}P' = \exp((r_f - r - \rho\sigma_x\sigma_s)(T-t))\bar{X}S_t N(d_1) - \exp(-r(T-t))KN(d_2),
 \end{aligned} \tag{13}$$

where

$$\begin{aligned}
 d_1 &= \ln\left(\frac{\bar{X}S_t}{K} + (r_f - \rho\sigma_s\sigma_x + \frac{\sigma_s^2}{2})(T-t)\right) / \left(\sigma_s\sqrt{T-t}\right) \\
 d_2 &= d_1 - \sigma_s\sqrt{T-t}.
 \end{aligned}$$

4 Conclusions

Using Black-Scholes assumptions, this paper provides an analytical closed form solution for the pricing of a quanto forward and option contract. This is a desired contribution to the mathematical finance literature given that, most prior research obtains quanto option solutions using numerical techniques and Monte-Carlo simulations. Our solution provides very interesting avenues for further research. First, stochastic volatility could be incorporated into the pricing methodology (Heston, 1993), as both option and currency volatility is typically time-varying. More advanced derivatives can also be written on quanto, like quanto swaptions and quanto-forward floating-strike Asian options, for which higher parametric sophistication and processes for additional variables like interest rate would be required.

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