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Celebrating Equilateralism

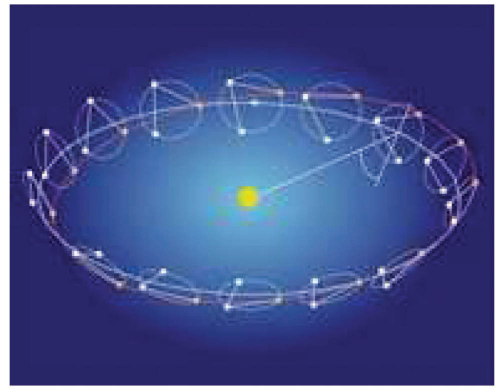
Mysteries of the Equilateral Triangle. *By Brian J. McCartin, Hikari Ltd., Ruse, Bulgaria, 2010, 246 pages; free for download, http://www.m-hikari.com/mccartin-2.pdf.*

BOOK REVIEW

By Philip J. Davis

From time to time mathematicians fall in love (a.k.a. become professionally and avidly engaged) with certain geometric figures. Apollonius fell in love with the conics. Bernoulli fell in love with the equiangular spiral. Lovers of the Gaussian normal curve are as numerous as the sands of the sea. In 1957 Daniel Pedoe came up with *Circles: A Mathematical View*.

I fell in love with what I called the Spiral of Theodorus and wrote about it. More generally, I began to accumulate reasons for thinking that spirals were the most important mathematical curve in the natural world. Frank Stenger fell in love with the sinc function. My Brown colleague Tom Banchoff fell in love with the 4D hypercube. He passed his ardor along to the artist Salvador Dali, and parlayed his association with this eccentric surrealist into a most engaging show for a general audience. Books dedicated to spline functions abound. Even now, Igor Najfeld is vigorously pursuing chains of circles that are tangent both to one another and to an enveloping equiangular spiral. And so it goes.



The LISA constellation's heliocentric orbit. From Mysteries of the Equilateral Triangle.

Contributor of a substantial addition to the literature of triangle geometry, Brian J. McCartin of Kettering University, a prize-winning author, has fallen in love with the equilateral triangle. Even as mathematician Lewis Carroll had his Snark pursued with "forks and hope," McCartin has pursued the equilateral triangle with a fine-tooth comb supported by an eagle eye. He has tracked down the multiple personality of the equilateral triangle as it appears in history, design; in theorems of plane geometry; in applications, games, recreational mathematics, competitions (e.g., the Olympiads); and in popular culture. McCartin's book lists 88 properties, 33 applications, 29 recreations, and 63 problems. Thus: Everything you might want to know or teach about equilateral triangles is here, and then some. Well, perhaps not quite, for my search engine came up with a million hits, although most of them, I suspect, would be of little interest.

The book contains mini-biographies and portraits, listed chronologically, of 45 mathematical eminences who have done something of interest with the equilateral triangle. As examples: We have the Euler line of a triangle. Lagrange gave us the equilateral solution of the three-body problem. Gauss (pushing the envelope a bit) proved that every positive number is the sum of at most three triangular numbers. James Maxwell gave us his color triangle. Sol Golomb dealt with replicating figures. Paul Erdös produced the Erdös–Mordell inequality. John Conway published a dissection proof of an area theorem.

The author has provided visual proofs of some theorems, in addition to full references (more than 300) for all the proofs. He also refers to the *Computer Generated Encyclopedia of Euclidean Geometry*, which was new to me and which, apparently, can spew out equilateral theorems with the ease of a Dunkin' Donuts machine punching out donuts.

Strangely not referenced is E.A. Abbott's 1884 chauvinistic social satire *Flatland*, recently puffed by Banchoff. Abbott's book describes a two-dimensional world populated by plane figures, such as circles, polygons, arranged in a social hierarchy. In the world of *Flatland*, the equilateral triangles emerge as members of the Middle Class of Craftsmen.

"Let no one ignorant of geometry" enter here is reputed to have been the message over the door of Plato's Academy. A subtitle of McCartin's book might have been "Let no non-equilateral triangle enter here." But occasionally, and luckily, a nondescript "lower-class" triangle does knock on the door and is let in. Fagnano's theorem welcomes such riffraff; and in the case of Napoleon's theorem, an arbitrary triangle is the progenitor of an emerging upper-class equilateral triangle.

What is my favorite equilateral triangle theorem? It is Napoleon's theorem, and interested readers can find it, as well as a number of its extensions, treated via circulant matrix theory in my book *Circulant Matrices*.

Judging from McCartin's title, and from a 2004 book produced in the same collecting or anthologizing mode (π: *A Biography of the World's Most Mysterious Number*, by Alfred Posamentier and Ingmar Lehmann), plane geometry is loaded with mysteries, many discovered and—who knows?—many yet to be discovered.

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