The Building Block Game: A Didactic Approach
to Solve Complex Problem

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Abstract

This work presents a didactic method for the analysis and synthesis of problems, experimented by the authors in the teaching of mathematics and circuit theory in High School, but also applicable to the learning of other disciplines, not only in the scientific fields. It is shown how the resolution of a complex problem or the study of a project can be faced through the recognition of the elementary constituent objects and the examination of their connections. Examples drawn from circuit theory, mathematics and mineralogy are also provided. The approach has been validated and refined through years of use in the classroom, though it hasn’t been formally researched until now. In its current version, the method reflects a pedagogical reflection and development effort framed within the international EduHack project, focussed on developing innovative pedagogical approaches. The article therefore complements the presentation with interviews from former students, attesting its effectiveness and flexibility.
1 Introduction

In the study of a scientific discipline, it is possible to identify elementary components or elements which can allow the resolution of complex problems. Once the elementary components are recognized, these tasks can be tackled by examining their possible connections in order to obtain the final result. The elementary components can also be recombined in order to experiment with alternative solutions. This approach can certainly be applied not only to all technical and scientific disciplines but also to other disciplinary fields. For example, in teaching a foreign language, some words (verbs, nouns and adjectives) can be defined as elementary objects and used to create complex sentences with a logical meaning.

In addition to the traditional teaching methods, the method presented in this article, named "the building block game", can be applied to several other educational methods, such as, for example, TBL (Team-Based Learning; [1-2]), or EPBL (Example-Problem-Based Learning; [3]) which are based on the discussion and collaboration between students in solving a problem or carrying out a project.

2 The "building block game": four different approaches

For the teaching methods reported in the introduction, the "building block game" can be carried out according to one of the four different approaches described below.

1) The first approach consist in selecting a definitive number of elementary objects within the teaching discipline, to build a system, i.e. a complex object, which satisfies appropriate specifications, and establish any changes that can be obtained by varying the properties of one or more elementary objects (Fig. 1). In this case, it is necessary to identify, among the various possible connections of the assigned elementary objects, those that satisfy the required system. To verify alternative solutions, one or more modifiers can be inserted to make the complex object more efficient.

This approach requires the knowledge of the specific properties of elementary objects and allows the group of students to practice with the connection possibilities in accordance with these properties. This is an active learning method that is well suited to the criteria of TBL and Peer Learning [4-5] because it involves the exchange of knowledge, ideas and experiences between peers.
2) The second approach is aimed at recognizing the elementary objects that form a complex system and using them, even with the addition of new elementary objects, in order to create complex objects that satisfy appropriate properties. This approach is in contrast to the previous one: in fact a complex object is already assigned and it is required to recognize the elementary objects and their properties. Once known, the elementary objects have to be reconnected, possibly by adding new elementary objects, in order to obtain a new complex object with different properties (Fig. 2).

3) In the third approach (Fig.3) some elementary objects are assigned to identify the various complex objects that can be generated and it is also necessary to study how the choice of appropriate connections allows the creation of different complex systems. Moreover, it is also required to examine the possibility of suitable modifications that, through the use of regulatory parameters, lead to different complex systems with the same properties, among those already obtained.
Fig. 2. Recognition of the elementary objects (#1 through #7) of a complex object (#1) and their reuse to generate a new and more efficient complex object (#2). Two new elementary objects (#8, #9) can also be added to satisfy appropriate properties.

4) In the fourth approach (Fig. 4), complex objects composed of elementary objects can be considered as elementary blocks. These elementary blocks can be further connected to build even more complex composite systems. In fact, two or more complex objects can be connected together as the elementary objects, without it being necessary to know the properties and connections of the elementary objects that compose them. Consequently a new complex object with appropriate properties is obtained.

These four approaches for the application of the “building blocks game” show that this teaching strategy can be used to apply two apparently mutually exclusive strategies for analyzing a problem: top-down and bottom-up analysis. The top-down analysis allows any part of a problem to be examined in detail starting from its general vision, progressively breaking the complex objects into simpler parts until the identification of the elementary objects of the system has been achieved (as well as the related properties used for their connection). In the inverse bottom-up analysis, starting from elementary objects, a complete complex
object or system can be created reconnecting them, possibly in a more efficient alternative way, and the initial problem can be reformulated in a more general form.

The following sections show some examples of the application of the “the building block game” in some university courses in different disciplinary fields.

**Fig 3.** Study of the complex systems that can be obtained from a given number of elementary objects and of the necessary changes to be applied to a complex system to obtain other systems with the same properties. Eo: elementary object; R: regulatory parameter.
3 The building block game applied to circuit theory

A circuit can be considered a complex system, that is, a set of elementary objects connected to each other, in order to create a geometric figure with a certain number of sides (branches) and vertices (nodes). The elementary objects that can be inserted on the sides are classified into active components, i.e. voltage or current generators, and passive elements which in Direct Current (DC) are only resistors, whereas in Alternating Current (AC) they can also be inductors or capacitors. The problem to be solved usually consists of computing one of the following two electrical quantities in a circuit: the voltage between two nodes and/or the current in a branch. The circuit can be already assigned or alternatively the constituent elementary objects and connections are given to assemble it.

An effective calculation tool in solving these problems can be the use of a simulation software, such as PSpice [6]. This program is equipped with a very large number of libraries from which it is possible to take the components, i.e. the elementary objects, to assign the desired properties and to connect them in order to create more complex circuits.

The “building block game” can give a different and more complete approach to these problems, boosting the students' creativity and imagination. Below, we
report an exercise illustrating the different approach for the resolution of a problem in the "traditional" version and with the “building block game" method.

3.1. Circuit theory: two different approaches for the resolution of a problem

The exercise concerns the calculation of the quantities that characterize an electrical diagram within a pre-established circuit and requires the application of the laws and theorems that are studied in the theoretical part of teaching. It belongs to the typology of cases described in Fig. 1.

a) Traditional approach. A circuit consisting of a 60 V generator and three resistors (two 2 Ohms resistors and one 3 Ohms resistor) are assigned (Fig.5). Alternatively, it is described how to connect these components in order to obtain the desired scheme. In both cases it is requested to calculate: i) The electrical quantities (voltage and current) concerning the 3 Ohm resistor; ii) How these quantities are modified if an additional 5 Ohm resistor is inserted between two nodes indicated in the diagram of Figure 6.

b) "Building block game" approach. Four elementary objects are given: a generator of 60 V, two 2 Ohms resistors and one 3 Ohms resistor. It is asked to examine the connections that can be made between these four elements and to analyze the corresponding circuits obtained (complex objects). Then for each scheme it is also requested to compute the electrical quantities related to one of the three resistors (see Fig. 1 and 3). The three possible approaches that can be followed using this teaching methodology are the following.

1) Connect the four elementary objects by making the voltage and current of the 3 Ohms resistor in the corresponding circuit (i.e., the complex object), assume the assigned values (circuit #1, Fig. 5).

2) Draw a circuit, (i.e., the complex object), for which the current coming out from the generator assumes its minimum value (circuit #2, Fig.5). This type of exercise can be traced back to type 1 described before (Fig. 1). The M1 and M2 modifiers depend on the circuits that can be obtained by linking the assigned objects. These circuits, among all those possible, must be chosen to meet the specifications required for the generator and the 3 Ohms resistor.

Fig. 5 - Two composite systems having an equal number of components. The choice of different connections allows the properties of the resistor R3 to be modified (Circuit #1 on the left) or of the generator (Circuit #2 on the right).
3) After having chosen one of the various complex objects, that can be obtained by means of an appropriate connection of the four elementary objects, it is requested to decide between which pair of nodes of the circuit a new fifth elementary object, for example a 5 Ohms resistor (R4 in figure 6), has to be inserted so that, the current coming out from the generator, or the electrical quantities of the 3 Ohms resistor, assumes new given values (Fig. 6). This type of exercise is part of the type 2 described before (Fig. 2). If a specification on the current of the generator is required, the modifier M1 depends on M2 which, in turn, varies according to the new circuit obtained by adding the resistor R4 through a suitable connection.

![Diagram](image)

**Fig 6.** The addition of a new elementary object allows the properties of one of the components inside the system to be changed; in this example the generator or resistor R3, according to the requirements of the problem.

This examples show how the proposed method makes the students more free to express their creativity because they are free to search, among the possible solutions, that is the possible realizable compound objects, the one which satisfies the specific requests. On the contrary, the problem which requires traditional resolution, often “forces” the student to solve a scheme which is already assigned without having the freedom of choice to create and to solve new schemes that optimize some specifications over others.

### 4 The building block game applied to mathematics

One of the fundamental topics of the Mathematical Analysis courses is the study of functions, which can be considered "objects", characterized by an equation and
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A graph. The problem that most frequently arises consists in plotting the graph associated with an assigned equation. There are elementary functions whose graph is already known, and in this sense they could be defined as fundamental functions. These can be composed together to obtain new functions which have a more complex equation and graph. Therefore, referring to “building block game” reported in Figure 1, it is possible to propose the following three exercises

**Exercise 1):** available objects: an elementary function and two or more constants; problem: add and/or multiply the assigned objects together in order to obtain a more complex object, that means a new function, to which a given graph corresponds.

**Exercise 2):** available objects: two or more elementary functions; problem: “compose” the available functions together in the right order so that the resulting composite function has a given trend. The term "composition" in Mathematical Analysis means the replacement of the variable $x$ contained in the equation of a function with the complete expression of a second function, and the problem that usually arises is the study of the new function obtained. On the contrary, it is much more creative to choose the order in which it is possible to compose more assigned functions to obtain a graph.

**Exercise 3):** available objects: some periodic functions having a known period are given and they are grouped in pairs; problem: consider as new objects, the functions that are obtained by applying the four fundamental operations (addition, subtraction, multiplication, division) to the functions of each pair and evaluate their period by comparing it with that of the functions of the pair.

In the study of function graphs carried out by applying suitable transformations to the diagrams of simpler functions, the proposed method can also be used according to the scheme of Figure 2; an example is shown below.

Knowing a "complex system 1" consisting of a "function $f$ ", $n$ objects, like elementary translations, scaling or modulus operations, contained in the expression of $f$ (possibly with the addition of an external object), and the graph of a new "function $g$ " representing the "complex system 2 ", the problem is to understand which objects to use and in which order to transform them, so as to obtain the equation associated with the graph of the "function $g$ " from that of the "function $f$ ", that means the "complex system 1"

The example of a problem, assigned both in the traditional version and using the proposed method, is shown in the following section.

4.1 The study of functions: two different approaches for the resolution of a problem

How is possible to modify the graph of an elementary function as a consequence of some translations or alterations of the scales of its values?

1) **Traditional approach.** Once the equation or trend of an elementary function has been assigned, check how its graph changes by introducing appropriate translations or alterations of the scale into the equation.

2) **"Building block game" approach.** A "function $f$ " (considered as a composed object, whose properties, such as the equation or the trend, are already known)
and some elementary objects represented by simple constants are given. Identify which constants are to be multiplied and which are to be added to the equation of \( f \) in order to obtain the expression of a new function \( g \) that has an appropriate graph (Fig. 7).

Fig 7. The functions \( f \) and \( g \) are the objects given by the problem. \( h_1, h_2, k_1, k_2 \) represent the constants that must be added and/or multiplied with \( f \) to obtain \( g \).

For example, suppose that the graph of a function \( f(x) \) (Fig. 8) and three constants equal to \(-2, -3, +4\) are the given elementary objects. The required compound object can be the equation of a new function \( g(x) \) with the graph shown in Figure 8. To solve the problem it is necessary to modify the expression of the function \( f(x) \) by introducing these constants.

In practice, the new equation takes the form \( g(x) = af(xb) + c \) and the problem comes down to choosing the values among \(-2, -3\) and \(+4\) to replace to \( a, b, c \), so that the new function \( g(x) \), (i.e., the compound object) assumes the required graph. Solving the problem according to the traditional method requires to make changes to the assigned object, i.e., to the function \( f(x) \), which are already known to the student and which have to follow a certain order predetermined by the problem itself. Finally the graph of the new function \( g(x) \) as the result of these changes is obtained. On the contrary, the “building block game” approach allows the student to decide which modifications are necessary to apply and the way of applying them to a function \( f(x) \) to obtain the graph of a new function \( g(x) \), which is assigned together with that of \( f(x) \). Choosing the correct combination of possible changes stimulates the student's imagination and creativity, unlike the traditional approach which limits the student to carrying out only the required operations.

The Desmos Activity Builder platform (https://teacher.desmos.com/?lang=it) can be a valuable tool to solve examples of this type, based on the “building block game” method. This platform allows an immediate view of the changes made to
the graph of a function if any parameters, and therefore any elementary objects, contained in its expression, varies.

**Fig. 8.** Given the trend of $f(x)$, choose the values of $a$, $b$ and $c$ to be included in the expression of $g(x)$ to obtain the graph of $g(x)$ shown in the figure.

**5 The building block game applied to mineralogy**

According to the definition of the International Mineralogical Association "a mineral is a homogeneous and natural solid characterized by a defined chemical composition and with a periodic and stacking ordered structure according to precise geometric laws".

The “building block game” is particularly suitable to understand various aspects of mineralogy, from crystallography to systematic- and applied-mineralogy. Indeed, every mineral can be decomposed into elementary objects (atoms, molecules) which combine in specific physico-chemical conditions to form
unique complex structures governed by precise geometric laws (symmetry) and by precise constraints (electrostatic charges, ion size, lattice energy, bond strength, etc).

A specific example of the application of the proposed method to this discipline is given below.

5.1. Crystal Morphology: two different approaches for the resolution of a problem

In Crystal Morphology the term “Form” defines the set of one (pedion) or more identical flat faces of a crystal (not only geometrically but also for the type of constituting atoms or ions, for their geometric arrangement and physical properties) which is obtained applying symmetry operators, such as rotation axis, mirror planes, and inversion centers.

Every crystalline system is characterized by specific elements of symmetry through which all the forms of a crystal belonging to that system can be obtained. Forms can be open (such as a hexagonal prism) or closed (such as a cube). Furthermore, since in physical reality it is necessary that the volume delimited by the faces of a crystal be closed, the resulting morphology can consist of a single closed form (simple forms) or of two or more open and / or closed forms (composite forms) (Fig. 9).

The approach to crystal morphology using the “building block game” can be either top-down or bottom-up (see section 1).
In the first case (top-down) a complex morphology (for example the quartz crystal of Fig. 10) is assigned and it is required to identify all the elementary objects (components; i.e. faces and forms) and the symmetry operators (modifiers; e.g. rotation axis and mirror planes) that allow the assigned crystal morphology to be obtained.

In the second case (Fig. 11) the components (for example faces) and modifiers are provided and it is required to achieve the final morphology of a crystal that satisfies the symmetry operations of a given crystalline system. It may also be interesting to ask to verify how it is possible to obtain different crystal forms by acting on the modifiers (symmetry operators; Fig. 11), starting from identical components.

In addition, it can be also asked to modify or to add the starting components (faces) to verify the morphological variations that are obtained using the same symmetry operators (modifiers).
In both cases, the decomposition and recomposition of complex objects by applying the “building block game” method allows students to think about how the ideal morphology of the crystals is closely controlled by geometric operations which are necessary from the atomic up to the macroscopic scale to ensure the periodicity of the crystal lattice.

**Fig. 9.** Examples of simple (hexagonal bipyramid on the left) and composite (hexagonal prism and pinacoid* on the right) forms. The simple form is obtained with the repetition of a single triangular face by applying a 6-fold rotation axis (red dashed line; rotation of 60 °) and a mirror plane (m). The composite form is obtained by applying the same symmetry operators on a rectangular face and on a hexagonal face. * The pinacoid is a shape consisting of two identical parallel faces.

**Fig.10.** Starting from a real crystal (quartz crystal) it is required to identify the elementary components (faces) and to apply modifiers (rotation axis and mirror planes) to recompose the initial morphology. The intermediate shapes (pyramid and hexagonal prism) obtained by applying a 6-fold rotation axis are simple open forms that allow to obtain the final composite morphology corresponding to starting crystal, by means of the application of mirror plane.
Fig. 11. Starting from some elementary geometric components (triangular and rectangular faces) it is required to apply the appropriate modifiers to obtain assigned forms (i.e. hexagonal prism and bipyramid in the upper part of the figure). It is also requested to highlight the morphological variations which are possible to obtain by varying only the modifiers (i.e., 4-fold rotation axis and mirror plane in the lower part of the figure).

6 Basic evaluation of the proposed method

To provide a basic evaluation of how the method was experienced and of its impact, we have contacted a small sample of people who have been acquainted with the method, either directly (about 100 people as students of different levels – high school, freshmen, graduates) or indirectly (16 people as head teachers), and asked them to relay their perspective and experience. The people have been chosen considering different types of school (scientific, classical and artistic high
schools and technical institutes) as well as several graduating students of the degree course in mathematics from Torino University. While we acknowledge the limitations of the sample (in its size, as well as in terms of being selected a posteriori), we do maintain that, as a small pilot case study, this allowed us to provide some initial insights and generalisations, providing useful guidelines and directions for future research (see [7], on the value of small case studies). To document and evaluate the study participants’ experiences of the method while acknowledging this a posteriori element, we deemed most appropriate to deploy a scaffolded debriefing approach. We followed the “What? So what? Now what?” model [8-9], which was used to help participants reflect on the nature and implications of the approach, on the lived experience of engaging with it, and its implication for and impact on their broader learning trajectories. The resulting short interviews were then subject to thematic analysis and coding [10], allowing two key relevant themes to be extrapolated: the participants’ perceived effectiveness in terms of achieving complex learning and its flexibility of usage across different disciplinary domains. We will now briefly examine each of those and discuss possible implications for future implementations of the method.

6.1 Effectiveness
The participants’ response in terms of their subjective experience of the method is overwhelmingly positive. All respondents highlighted how the “building blocks” method heightened student participation and their investment in the discipline. In particular, participants discussed how the inherently creative aspects of the method support students in moving beyond mere repetition and use of both theoretical concepts and technologies, and towards being active producers of their knowledge – a strong example of constructionist learning approach [11] facilitated by the “building blocks” metaphor and mental model. Furthermore, the open discussion of these mental models in the classroom easily scaffolds a reflective approach to learning, enabling both students and teachers to reflect on and deconstruct established (and often ossified) patterns [12], and build divergent thinking and peer collaboration. Focussing on this social dimension of learning, most respondents cited how the method promoted student self-directedness and creative and autonomous problem solving, heightening their confidence.

The method’s advantages are not limited to facilitating the achievement of specific disciplinary learning outcomes, but rather, as consistently reported by the study participants, it has a psychological impact in terms of students’ confidence in their problem solving skills, within and beyond the classroom (as reported in studies pertaining other active learning methodologies; see for example [13]). Future implementations of the methodology could further empower students exactly by providing more emphasis on its social aspects and implications, with co-creation becoming more central – though it there might be some friction vis-à-vis the current individualised arrangement of educational assessment paradigms.
6.2 Flexibility
As discussed by the participants, the method’s successful applications can be found not only across a number of different “STEM” fields (Science, Technology, Engineering, and Mathematics), but also in History (e.g., by supporting students in building different categorisation and thematic links than the most widespread chronological approaches), Music (e.g., by breaking down different elements of the same compositions) and even in other expressive domains (among the examples discussed we found Style Slalom skating, and even the traditional Italian art of flag throwing).
Most respondents explicitly link this strong flexibility to the elements of creativity and imagination inherent to the method, a connection that is also being increasingly being made through the discussion of Higher Order Thinking Skills [14]. Linked to these considerations, respondents on the teacher side discuss of the method is in fact not only a pathway towards effective, disciplinary based instruction, but can be a way towards building an integrated, holistic approach to education.
Future implementations of the method might benefit from further exploring this creative aspect by explicitly addressing creative disciplines, and their possible links with STEM, providing a possibly fruitful scaffold for integrated and transdisciplinary STEAM (Science, Technology, Engineering, Arts and Mathematics) programmes [15].

6.3 Limitations
Participants highlighted a few limitations of the methodology, mainly in terms of onboarding students in utilising a new methodology requiring active engagement. Participants refer to having to “unlearn” previous ways of learning, to being “unsettled” by the method, and having to abandon more passive learning habits (all of which are often acknowledged challenges in active learning approaches; see [16]. However, the emphasis on self-directedness and simplification inherent to the method, together with an element of playfulness linked with its core metaphor [17], has successfully scaffolded students in overcoming the onboarding obstacle, and acknowledging the many advantages of the “building blocks”.
Future implementations of the methodology could partially sidestep this initial obstacle either by emphasising the play element, or by focussing on locally and culturally relevant [18] topics that immediately catch the students’ interest as a way of introducing the method.

7 Conclusions
The “building block game” can be used together with any teaching method and provide tools and ideas to teachers in order to propose new and original problems on their discipline that involve more actively the students. The possibility of composing and breaking down complex objects improve their creativity thanks to the possibility of exploring different paths and applying a possible original and free choice for the resolution of the assigned problems.
In fact, traditional exercises usually require the properties or characteristics of a complex system by presenting it already as a set of interconnected elementary objects or by describing the necessary connections between the objects to obtain it.

On the contrary, using the proposed method, students have only simple objects (and possible modifiers) available and are stimulated to identify the way to connect them together to obtain a complex object with the specific properties.

This approach can be used with school students of all levels: faced with a series of problems, firstly they will have to interpret them by identifying the constituent "objects" and which of their properties have to be applied in creating their interconnection, in order to gradually reach the solution. Then they will have to take a critical attitude towards the proposed theme, looking for alternative solutions based on other possible interconnections of the elementary "objects".

Following the small pilot study outlined in this article, we intend to more thoroughly embed in-process evaluation and debriefing in future implementations of the method. Having clearly highlighted the method’s principles, its effectiveness and its flexibility across a number of different disciplinary domains, we are open to collaborating and comparing outcomes and experiences with any reader who might be interested in experimenting with “building blocks”.

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