

## Comparing US and Russian Grading Scales

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### Abstract

In the US, grades are usually based on comprehensive written exams: the larger the proportion of topic in which the student shows knowledge, the higher the student's grade. In contrast, in Russia, the grades are based on oral exams, and the bulk of the grade comes from a student answering questions of a few (usually, three) randomly selected topics. A natural question is: what is the relation between the two grading scales? It turns out that "excellent" and "good" grades means the same in both scales, while the US "satisfactory" level is higher than a similar Russian level.

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## 1 US and Russian Grading Systems: A Brief Description

**US grading system.** In the US, typically, exams are written and comprehensive. In general, the number of points is proportional to the number of correctly answered questions – or, in case of partially correct answers, to the number of correctly answered parts of the questions. Based on this number of

points, a letter grade is assigned. The best is A “excellent”, the next best is B “good”, followed by C “satisfactory”, by D, and finally, by F “fail”.

Usually, at least 90 points out of 100 mean an A. This means that if a student knows at least 90% of the topics, this student gets an A. Similarly:

- if a student knows between 80% and 90% of the topics, he or she usually gets a B,
- between 70% and 80% of the topics correspond to a C,
- between 60% and 70% correspond to a D, and
- below 60% means an F.

**Russian grading system.** In Russia, grades are 5 “excellent”, 4 “good”, 3 “satisfactory”, 2 “bad”, and 1 “very bad”. Usually, all exams are oral. A student randomly selects a “ticket” that contains three randomly selected topics.

If a student showed knowledge of all three topics, at least 4 (“good”) is guaranteed. If this is the case, the examiners start asking additional questions. If the student successfully answers the vast majority of these questions, this student gets a 5 (“excellent”), otherwise, his grade is 4.

If a student knows only two out of the three randomly selected topics, this student gets a 3 (“satisfactory”). If the student knows only one of the three topics – or none – he or she get a 2 (“fail”).

**Formulation of the problem.** What is the relation between the grades from different grading scales? For example, if a student gets a 4 in a Russian system, what grade does that correspond to in the US grading system?

The answer is reasonably straightforward for the Russian grade of 5 (“excellent”): this means that the student has shown mastery of practically all the topics, so this seems to correspond to the US A (“excellent”) grade. For 4 or 3, there is no such immediate connection; so let us find a one.

## 2 What Is the US Analog of the Russian 4 (“Good”) Grade?

**Analysis of the problem.** In the US system, the numerical grade corresponds to the proportion  $p$  of topics that the student knows.

In the Russian system, as we have mentioned, the grade of 4 means that the student has successfully answered three randomly selected questions.

Of course, in contrast to a comprehensive exam that checks the knowledge of all the topics, an oral exam with a few randomly selected questions is a lottery. A student with the same level of knowledge sometimes gets a higher grade

and sometimes gets a lower grade, depending on luck. If this student happens to know all three questions, the student gets a 4, otherwise this student may get a 3 or even a 2.

Once a level of knowledge  $p$  is given, the student may get a 4 or this student may get a lower grade. The resulting grade of a student is not determined only by his/her level of knowledge, it is also influenced by luck. So, for each level of knowledge  $p$ , we can only talk about the *probability* of getting a 4.

**A natural way to establish equivalence between the two grading systems.** For large values of  $p$ , the probability of getting a 4 is high, so the vast majority of students with this high level of knowledge will get at least a 4. For low level of knowledge  $p \ll 1$ , this probability of getting a 4 is low, so very few students with this level of knowledge will get a 4, most of them will get 3 or even 2. As the level  $p$  of knowledge increases from 0 to 1, the probability of getting a 4 increases. At some threshold value, there will be a transition from the cases in which the majority of students *do not* get a 4 to the cases in which, vice versa, majority of students *do* get a 4. This threshold value corresponds to the case when the probability of getting a 4 is exactly  $1/2$ .

It is therefore reasonable to associate the lower end of the interval corresponding to the Russian grade of 4 with the level of knowledge  $p$  for which the probability of getting a 4 is exactly  $1/2$ .

**So what level of knowledge corresponds to the Russian grade of 4?** To find the desired threshold value, let us compute the probability of getting a 4.

Suppose that a student knows part  $p$  of the material. Then, the probability that this student answers one of the three questions is equal to the probability that this student knows the corresponding topic – i.e., is equal to  $p$ . For three independently selected questions, the probability that the student successfully answers all three questions is thus equal to the product of the three values  $p$  – i.e., is equal to  $p^3$ .

So, the desired threshold probability  $p_0$  can be determined from the condition that  $p^3 = 0.5$ . This corresponds to  $p \approx 0.8$ . So, we arrive at the following conclusion.

**Conclusion of this section.** The Russian grade of 4 corresponds to the student knowing at least 80% of the material – i.e., to the US B grade.

In other words, similarly to how the Russian “excellent” grade (5) corresponds to the US “excellent” grade (A), the Russian “good” grade (4) also corresponds to the US “good” grade (B).

### 3 What Is the US Analog of the Russian 3 (“Satisfactory”) Grade?

**Natural question.** What is a natural US-grade analogue of the Russian “satisfactory” (3) grade?

**What is 3 grade: a brief reminder.** As we have mentioned earlier, the Russian grade of 3 corresponds to the case when a student shows knowledge in two out of three randomly selected topics.

**Idea.** To find a US grade equivalent to the Russian grade of 3, we will – similarly to the previous section – find a threshold value  $p_0$  at which the probability to get at least a 3 is exactly  $1/2$ .

**Analysis of the problem.** To get a 3 means to correctly answer at least two of the randomly selected questions. For each level of knowledge  $p$ , the probability of getting a 3 is thus equal to the sum of the following four probabilities:

- the probability  $p^3$  of correctly answering all three questions,
- the probability  $p^2 \cdot (1 - p)$  that the first two questions were answered correctly and the third answer was wrong,
- the probability  $p \cdot (1 - p) \cdot p$  that the first and the third question were answered correctly, but the second answer was wrong, and
- the probability  $(1 - p) \cdot p^2$  that the second and the third question were answered correctly, but the first answer was wrong.

The resulting sum is equal to  $p^3 + 3 \cdot p^2 \cdot (1 - p)$ .

One can easily see that the condition  $p^3 + 3 \cdot p^2 \cdot (1 - p) = 1/2$  is satisfied for  $p = 0.5$ . Thus, we arrive at the following conclusion.

**Conclusion of this section.** The Russian grade of 3 may correspond to C, may correspond to D, or may even correspond to F – all the grade of 3 says is that the student knows at least half of the material.

In other words, contrary to what we might expect based on the US-grade equivalences of 5 (“excellent”) and 4 (“good”), the Russian grade of 3 (“satisfactory”) is not necessarily equivalent to the US “satisfactory” grade C, it may also correspond to the US “unsatisfactory” (“fail”) grade F.

### 4 This Was About 3 for a Single Class, What About Several Classes?

**Additional information about the Russian grading system.** In top Russian universities, to stay in school, each semester a student must pass (i.e.,

get at least satisfactory grades) in at least 3 of the 4 courses. If there is an unsatisfactory grade for one of the courses, the student usually gets a chance to study again and re-take the previously failed exam after such study.

**Natural question.** In the previous section, we learned that a Russian grade of 3 means that a student knows at least half of the material. However, if the student knows half of the material, this student will pass only half of the classes – and thus, will be expelled from the university. To stay in school, the student needs to know more than that.

A natural question is: what is the smallest level of knowledge that enables a student to stay in school?

**Idea.** Similarly to the previous sections, we will be looking for the threshold level of knowledge  $p_0$  for which the probability to stay in school is exactly 0.5.

**Analysis of the problem.** Let  $p_0$  denote the desired threshold level of knowledge. For this level of knowledge, and for each of the four classes, the probability  $c$  to pass this class is – as we have shown in the previous section – equal to  $c = p_0^3 + 3 \cdot p_0^2 \cdot (1 - p_0)$ . The probability to pass at least three out of four classes is similarly equal to  $c^4 + 4 \cdot c^3 \cdot (1 - c)$ . We want this probability to be equal to 1/2:  $c^4 + 4 \cdot c^3 \cdot (1 - c) = 0.5$ .

One can see that this equality is satisfied for  $c \approx 0.64$ . So, to find the desired threshold value  $p_0$ , we need to find  $p_0$  from the condition that  $p_0^3 + 3 \cdot p_0^2 \cdot (1 - p_0) = 0.64$ .

One can check that this equality is satisfied for  $p_0 = 0.6$ . This level corresponds to US grades D and higher. Thus, we arrive at the following conclusion.

**Conclusion of this section.** Staying in school in Russia is equivalent to having an average grade at least D.

## 5 General Conclusion

The Russian “excellent” grade (5) corresponds to the US “excellent” grade (A). The Russian “good” grade (4) corresponds to the US “good” grade (B).

For an individual class, the Russian “satisfactory” grade may mean C, D, or even F. For the whole semester, passing the semester means having a level of knowledge corresponding to the US grade D – intermediate between “satisfactory” (C) and “unsatisfactory” (F).

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## References

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