On Orthosymmetric Multilinear Mappings

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Abstract

In this paper, it is shown that n-th adjoint of an orthosymmetric multilinear mapping on vector lattices is again an orthosymmetric multilinear mapping.

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1 Introduction

Orthosymmetric bilinear mappings have been worked by many authors. We note that extensions of orthosymmetric bilinear mappings have been studied by M.A. Toumi [11], [12], K. Boulabair [6] R, Yilmaz [13]. M.A. Toumi studied the extensions of orthosymmetric bilinear mapping to Dedekind completions of vector lattice. And also he studied the extension of orthosymmetric bilinear mapping to the order continuous order bidual of a vector lattice. R. Yilmaz extent the orthosymmetric bilinear mapping to the order bidual and order continuous order bidual of a vector lattice by a difference way. They used the second adjoints of bilinear mappings called Arens extensions, [2, 9]. In this work, our aim is to extend the orthosymmetric multilinear mappings on vector lattices to the order biduals and the order continuous order bidual of vector lattices by using the n-th adjoint of multilinear mappings called Arens extension, [3].

For unexplained notion and terminology, we refer to the book [1].
2 Preliminary Notes

An ordered real vector space $E$ with the property that for every $x, y \in E$ the supremum and infimum of $\{x, y\}$ exist in $E$ is called a Riesz space or a vector lattice. We denote the following notations for supremum and infimum: $x \vee y = \sup \{x, y\}$ and $x \wedge y = \inf \{x, y\}$. The absolute value or modulus of $x \in E$ is given by the formula: $|x| = x \vee -x$. The set of all positive elements in a vector lattice $E$ is said to be the positive cone and denoted by $E^+$. So, $E^+ = \{x \in E : x \geq 0\}$. A vector lattice $E$ is called Archimedean if $\frac{x}{n} \downarrow 0$ holds in $E$ for every $x \in E^+$. In this paper, we will assume that all vector lattices are Archimedean. We say that a vector lattice $E$ is Dedekind complete if every subset of $E$ which is bounded from above has a supremum. Let $E$ be a vector lattice and let $x, y \in E$. The set $[x, y] = \{z \in E : x \leq z \leq y\}$ is said to be an order interval. A subset $A$ of $E$ is called an order bounded set if there exist $x, y \in E$ such that $A \subseteq [x, y]$. Let $E$ be a vector lattice. A positive linear functional $f$ on $E$ is called order continuous if $x_n \downarrow 0$ in $E$, then $f(x_n) \downarrow 0$. The vector space of all order continuous linear functionals on $E$ is denoted by $E_\sim$. A linear functional $f$ on $E$ is said to be order bounded if $f(A)$ is order bounded for every order bounded subset $A$ in $E$. The vector space of all order bounded linear functionals on $E$ is denoted by $E_\sim$. By $E_\sim\sim$ we denote the second order dual or order bidual of $E$, $[4, 5]$.

Definition 2.1 [8,10] Let $E_1, E_2, ..., E_n$ and $F$ be vector lattices. A multilinear mapping

$$T : E_1 \times E_2 \times ... \times E_n \to F$$

is called positive if $T(x_1, x_2, ..., x_n) \geq 0$ for all $x_i \in E_i$ for $i = 1, ..., n$.

Positive multilinear mapping $T$ is denoted by $T \geq 0$.

We say that a multilinear mapping $T$ is regular if $T$ can be written as a difference of two positive multilinear mappings.

Definition 2.2 [8,10] Let $E_1, E_2, ..., E_n$ and $F$ be vector lattices. A multilinear mapping

$$T : E_1 \times E_2 \times ... \times E_n \to F$$

is called order bounded if $T(B_1 \times B_2 \times ... \times B_n)$ is an order bounded set in $F$ for all order bounded sets $B_i$ in $E_i$ for $i = 1, ..., n$.

Definition 2.3 [8,10] Let $E_1, E_2, ..., E_n$ and $F$ be vector lattices. A multilinear mapping

$$T : E_1 \times E_2 \times ... \times E_n \to F$$

is called lattice multimorphism or lattice $n$-morphism if the linear operator $T^{(j)} : x \to T(x_1, ..., x_{j-1}, x, x_{j+1}, ..., x_n)$ is a lattice homomorphism for any choice of $1 \leq j \leq n$ and $x_k \in E_k^+$, $j \neq k \leq n$. 

Definition 2.4 [8,10] Let $E_1, E_2, \ldots, E_n$ and $F$ be vector lattices. A multilinear mapping
\[ T : E_1 \times E_2 \times \ldots \times E_n \to F \]
is positive if
\[ |T(x_1, x_2, \ldots, x_n)| \leq T(|x_1|, \ldots, |x_n|) \]
for all $x_i \in E_i$ for $i = 1, 2, \ldots, n$.

Definition 2.5 [8,10] Let $E_1, E_2, \ldots, E_n$ and $F$ be vector lattices. A multilinear mapping
\[ T : E_1 \times E_2 \times \ldots \times E_n \to F \]
is said to be a lattice multimorphism if
\[ |T(x_1, x_2, \ldots, x_n)| = T(|x_1|, \ldots, |x_n|) \]
for all $x_i \in E_i$ for $i = 1, 2, \ldots, n$.

Definition 2.6 [8,10] Let $E, F$ be vector lattices. An $n$-linear operator
\[ T : E^n = E \times E \times \ldots \times E \to F \]
is called an orthosymmetric multilinear mapping if $T(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in E$ such that $|x_i| \land |x_j| = 0$ for some pair of indices $1 \leq i, j \leq n$.

Let $E_1, E_2, \ldots, E_n$ and $F$ be vector lattices. Suppose that
\[ T : E_1 \times E_2 \times \ldots \times E_n \to F \]
is a positive multilinear mapping. Then, the Arens adjoint of $T$ is the positive multilinear mapping:
\[ T^\sim : F^\sim \times E_1 \times \ldots \times E_{n-1} \to E_n^\sim \]
defined by
\[ T^\sim(\zeta', x_1, \ldots, x_{n-1})(x_n) = \zeta'(T(x_1, \ldots, x_n)). \]
We do this process $(n+1)$ times.

\[ T^{\sim\sim} : E_n^{\sim\sim} \times F^\sim \times \ldots \times E_{n-2} \to E_{n-1}^{\sim\sim} \]
defined by $T^{\sim\sim}(x_n'', \zeta', \ldots, x_{n-2})(x_{n-1}) = x_n''(T^\sim(\zeta', x_1, \ldots, x_{n-1}), \ldots)$.

\[ T^{\sim(n+1)} : E_1^{\sim\sim} \times E_2^{\sim\sim} \times \ldots \times E_{n-1}^{\sim\sim} \to F^{\sim\sim} \]
defined by $T^{\sim(n+1)}(x_1'', x_2'', \ldots, x_n'')(\zeta') = x_1'(T^{\sim(n)}(x_2'', \ldots, x_n'', \zeta'))$, [3], [7].
3 Main Results

Theorem 3.1 Assume that $E$ is a vector lattice and $F$ is a Dedekind complete vector lattice. Let

$$T : E \times E \times \ldots \times E \to F$$

be a positive orthosymmetric multilinear mapping. Then, $n$-th order adjoint of orthosymmetric multilinear mapping $T$ on the order biduals $E^{\sim \sim}$ is an orthosymmetric multilinear mapping.

Proof. It is proved by using Arens multiplication and similar techniques in the paper R. Yilmaz [13].

Theorem 3.2 Assume that $E$ is a vector lattice and $F$ is a Dedekind complete vector lattice. Let

$$T : E \times E \times \ldots \times E \to F$$

be a positive orthosymmetric multilinear mapping. Then, $n$-th extension of $T$ to order continuous order bidual $(E^{\sim})_{n}^{\sim}$ is also an orthosymmetric multilinear mapping.

Proof: It is proved by similar way in M.A. Toumi [12] and Arens product [3,2].

References


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