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Economic System Entanglement on Intra-Firm Trade Portfolios: The Impact of Counterparty Credit Ratings on Business-to-Business Credit Dynamics

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Abstract

In the last five years, Italy has seen a noticeable and steady increase in the supply of trade credit, granting of extensions, and general systemic business-to-business financial support. Focusing on system entanglement, this paper examines the impact in Italy of bank valuations of creditworthiness and credit intermediation on intra-firm trade portfolio dynamics. We further consider the impacts of exogenous shocks to the economy and other disruptive events on payment regularity and risks of insolvency in intra-firm transactions. Mapping portfolio dynamics to a quantum super-system with a Hamiltonian space of phases, we demonstrate that the performance of intra-firm portfolios depends concurrently on bank valuations and that system entanglement allows us to examine the extent to which economic disruptions shift portfolio dynamics from their state of equilibrium.

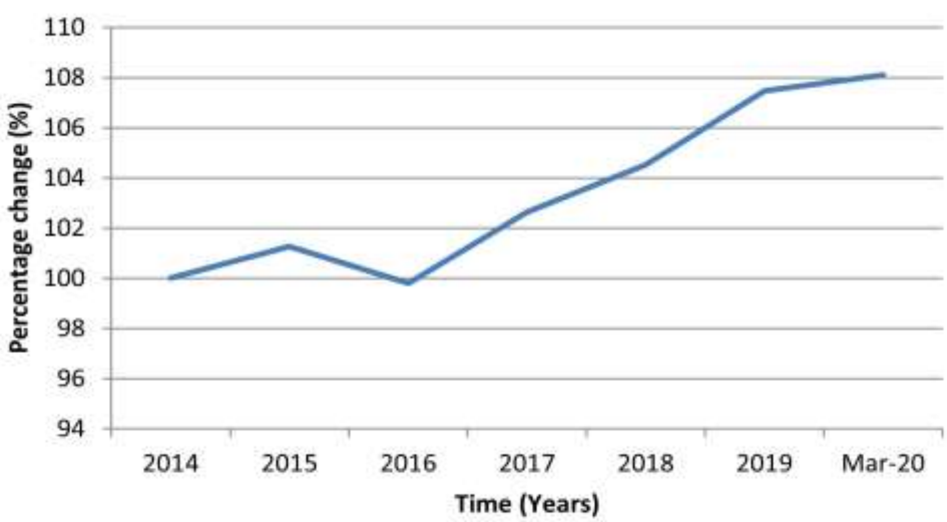
Keywords: System entanglement; intra-firm trade; portfolio dynamics; credit valuation

JEL Codes: C02; C61; G24; H12; M21

1. Introduction

The balance sheet liabilities of businesses consist of two macro components: equity and debt. Comprised predominantly of payables to banks, tax departments, and trade, debt accounts for 60 to 70 per cent of totals of the general balance of loans in business financial statements around the world [28]. This accumulation of debt follows a progressive upward trend in the trade credit supplied and extensions granted within the Italian intra-firm economy over the last five years [2], see Figure 1. Where this trend maps to increases in systemic business-to-business (B2B) financial support, the first quarter of 2020—the period immediately preceding responses in Europe to the COVID-19 pandemic—saw B2B forms of financing peak over the same five-year period [16].

Figure 1. Percentage change in micro and small enterprise trade receivables [9]



Applying concepts of system entanglement (most basically, that the quantum state of each entity occupying a given space can be measured and described only in relation to the state of other entity/ies in that space), this paper investigates the effects of exogenous shocks to the economy and those derived from discontinuous intervention by banks in credit intermediation. In particular, the bearing of internal bank valuation rationales—aiming for capital adequacy containment and performance maximization—on decisions to discontinue this intervention is considered [27]. Especial attention is in fact given to the impact of strictly financial shocks on the burgeoning system of B2B financial support. Further discussion analyzes the relationship between the accessibility of credit and the regularity of B2B payments alongside the impact of insolvencies on business financial statements under unfavorable economic conditions.

These considerations are of particular significance in Europe, where micro and small enterprises constitute a significant portion of the entrepreneurial fabric: in Italy, for instance, these businesses accounted for over 97% of all firms operating in 2018 [17].

2. Modelling credit systems

2.1 Micro and small enterprise bank ratings and access to credit

We begin by looking at the conventional credit subsystem and its processes for accessing bank leverage. Broadly speaking, access for businesses to this subsystem is highly contingent on assessments of the probability of a business defaulting, based on information provided in loan applications. Conducted by financial intermediaries, assessments follow a preliminary procedure involving rating algorithms, which aim to evaluate the expected performance of the applying business. From this probability of default, the creditworthiness of the business is also assessed.

Using F for the normal Gaussian cumulative distribution function, the probability of default determined for a generic enterprise i is verified when:

$$F^{-1}(PD_i) > \alpha K + \sqrt{1-\alpha^2}e_i \qquad (1)$$

Where:

- PD_i is the default probability of a generic enterprise i;
- α is a corrective known *a priori*;
- K is the random variable of the macroeconomic context;
- e_i represents the idiosyncratic factor of the enterprise i.

In the case of the micro and small enterprises dominating the European market, the e_i component means that the state of the enterprise i can be described by an emanation of a quantum system through wave function $\psi(r,t)$. This wave function indicates the probability amplitude of finding, in the time t, the state of the enterprise i at the generic point r of the interval $[\kappa,\lambda]$, where $\kappa = 0$ indicates the performing state, and $\lambda = 1$ represents the default state. Even if the state r is also “deterministic,” the result of measurement, in this case, becomes probabilistic.

$$P(r,t) \propto |\psi(r,t)|^2 \qquad (2)$$

Considering a generic state $|\psi\rangle$ expressed by the linear combination of the limit states $|\kappa\rangle$ and $|\lambda\rangle$ (eigenstates), we give the following linearity hypothesis:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|\kappa\rangle + |\lambda\rangle\} \qquad (3)$$

The measurement instrument for $|\psi\rangle$ therefore corresponds with the following algorithm used by a generic bank B to calculate the rating of the applicant enterprise i. Given its idiosyncratic nature, the factor e_i is specific to each micro and small enterprise. As well, due to the same micro size characteristics of the enterprises measured, the component can be said to exit simultaneously at any time and at all points in the range $[\kappa, \lambda]$.

The measurement of the rating, of a "macroscopic" nature, may result in three positions:

- (0), indicated by the state $|B_0\rangle$, if the instrument reads that the state of the microenterprise is $|\kappa\rangle$;
- (1), indicated by the state $|B_1\rangle$, if the instrument reads that the state of the microenterprise is $|\lambda\rangle$; and
- (0.5), which is the value registered by the instrument prior to a measurement.

$\hat{H}|B_r\rangle = B_r|B_r\rangle$ is the Hamiltonian operator, with $r = 0, 1$. Assuming that the corresponding eigenvectors are normalized, then $\langle B_r|B_{r'}\rangle = \delta_{rr'}$ and $\{|B_0\rangle, |B_1\rangle\}$ is an orthonormal basis of the Hilbert space $\mathscr{H}$ associated with the system [19].

The interaction between the phenomenon and the measurer then produces at time t:

$$|\psi(0)\rangle_1 = |\kappa\rangle|B_{0.5}\rangle \Rightarrow |\psi(t)\rangle_1 = |\kappa\rangle|B_0\rangle \qquad (4)$$

$$|\psi(0)\rangle_2 = |\lambda\rangle|B_{0.5}\rangle \Rightarrow |\psi(t)\rangle_2 = |\lambda\rangle|B_1\rangle \qquad (5)$$

Returning to the linearity proposed in (3), the superposition of $|\psi(0)\rangle_1$ and $|\psi(0)\rangle_2$ results in a generic initial state $|\psi(0)\rangle$ at time $t = 0$:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\{|\kappa\rangle|B_{0.5}\rangle + |\lambda\rangle|B_{0.5}\rangle\} \qquad (6)$$

which will evolve, following a measurement evidently carried out at a time $t > 0$, to the state:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\{|\kappa\rangle|B_0\rangle + |\lambda\rangle|B_1\rangle\} \quad (7)$$

Classical theory would deem the state expressed in (7) impossible. Practically speaking, a contextual superposition of the (macroscopic) measurement, which simultaneously marks $|B_0\rangle$ and $|B_1\rangle$, could never be verified. Taking a quantum theoretical approach, the linearity hypothesis applied to the measurer must first be contradicted such that, following the measurement of the *rating*, the state of the enterprise will no longer be a superposition of eigenstates of the measured quantity, but will *collapse* into κ or λ—or, more precisely, into any point r in the range $[\kappa, \lambda]$. The directionality of this collapse depends on whether the instrument measures 0, 1, or an intermediate quantity of the expected level of *performance* to which r will correspond [12].[1]

2.2 Commercial portfolios and debtors' creditworthiness: performance of scale

Returning to the B2B financial support system, let us now study the dynamics of the trade receivables portfolio of credit-granting business E for a given number of credit-receiving businesses ki , which were granted extended payment terms. Examining how the ability of each generic business i to access credit—as measured by bank rating algorithms—influences the regularity of its relationship with its creditor E, we determine how this ability affects the performance (and, negatively, the default) of scale within both inter- and intra- business systems.

Before granting deferrals, the business E may attempt to discern the reliability of its debtors by consulting public and private databases that report on prejudicial events, indicators of structural equilibrium, and credit payment history. However, while these reports may provide a useful snapshot of the financial standing of a generic business i, this is only a static view based on historical observations and characterized by an intrinsic time lag of as much as one to six months [11]. As well, financial statements providing information on bank and tax exposure are only publicly available for certain types of businesses; in Italy, joint stock companies are required to file publicly. Delays may also be built into such reporting processes,

[1] To empirically demonstrate this theory, an experiment was self-conducted on a sample of 250 Italian micro-enterprises operating during the 2018 and 2019 financial years, by setting up a reversal of the *rating* obtained and predicting its effects on their accounts Simulating a typical business management scenario with a temporal horizon of one year, tests were conducted on two categories: firms that managed to obtain credit through the B2B system (and therefore continued to be *performing*) and those that were unable to obtain credit. Findings revealed that, if denied credit, the former grouping would have shown signs of default, on average, at around the 220th day, while the latter, if granted credit, could have sustained regular operations for at least the first year.

ranging anywhere from five months to two years after the end of the financial year to which they refer.[2]

The supplier business E is not privy to the bank rating assigned to each business i, from which access to credit but also the ability of business i to cope with the elasticity of its own monetary cycle depends. The *performance* of the B2B transactions therefore depends also, and not secondarily, on the status—whether *performing*, in default, or at a certain point between these two extremes—of the business i, as determined by the measurement made by bank B.

It is worth noting, as well, that for the purposes of this modelling, the exclusive correlation for a generic business i between the existence of short-term bank supports—such as extensions of credit on current accounts, advances on invoices, and factoring—and its ability to properly meet its obligations is treated as an approximation. In other words, the argument discounts any endogenous solvency conditions resulting from the existence of regular cash flows. Reflecting a negative economic context caused by exogenous and/or strictly financial shocks, Figures 4 and 5—placed at the bottom of the document for greater consistency of exposure—provide significant empirical confirmation for this approximation.

From these conditions, the emergence of a quantum super-system, or second-order system, can be observed, wherein the observation by the bank B through the rating measurement operation—with its final event of assignment (*rectius*, collapse) of the state of i—becomes an isolated system for the second-order observer E. A structure akin to that of "Wigner's friend" is therefore achieved [25], [5].

For the first-order system in which the bank B makes its observation, we find:

- $|B_0\rangle \Rightarrow$ state of the microenterprise $|\kappa\rangle$ (performing) $\Rightarrow$ the enterprise i gets the loan $\mathcal{F}$
- $|B_1\rangle \Rightarrow$ state of the microenterprise $|\lambda\rangle$ (default) $\Rightarrow$ the enterprise i does not get the loan $\mathcal{F}$

There is therefore an association between funding from the macroscopic system $\mathcal{F}$ and two possible states: 'funding yes' $\mathcal{F}_0$ and 'funding no' $\mathcal{F}_1$.

The kets $\mathcal{F}_0$ and $\mathcal{F}_1$ are entangled with the states $|B_0\rangle$ and $|B_1\rangle$ such that, following the measurement operation by the bank B, the state of the whole first-order system will be:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\{|\kappa\rangle|B_0\rangle \otimes |\mathcal{F}_0\rangle + |\lambda\rangle|B_1\rangle \otimes |\mathcal{F}_1\rangle\} \qquad (8)$$

Until the moment a measurement is made, there is a superposition of states and the business i is simultaneously fundable and non-fundable.

[2] For instance, a commercial position for an LLC that you plan to open in February or March of the year *x* need only submit financial statements for the year $x - 2$.

As an external observer to the first system, the supplier, and likely creditor, business E functions within a quantum system of a second order. In this system, the results of the first-order system will cause a superposition of states between solvency or insolvency relative to the trade payables of i with E.
Put differently:

- $|\psi(t)\rangle \xrightarrow[\text{measurement}]{} |B_0\rangle \otimes |\mathcal{F}_0\rangle \Rightarrow$ state of *performing*, funding yes
- $|\psi(t)\rangle \xrightarrow[\text{measurement}]{} |B_1\rangle \otimes |\mathcal{F}_1\rangle \Rightarrow$ state of default, funding no

This first-order system is therefore part of a higher system, with a wave function:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\{|\kappa\rangle|B_0\rangle \otimes |\mathcal{F}_0\rangle \otimes |\mathfrak{S}_0\rangle + |\lambda\rangle|B_1\rangle \otimes |\mathcal{F}_1\rangle \otimes |\mathfrak{S}_1\rangle\} \qquad (9)$$

Where $\mathfrak{S}_j$ (with $j = 0, 1$) indicates the in/ability of the business i to make its payments to the supplying business E. Graphically:

- $|B_0\rangle \otimes |\mathcal{F}_0\rangle \otimes |\mathfrak{S}_0\rangle \Rightarrow$ state of *performing*, funding yes, regular payments to E
- $|B_1\rangle \otimes |\mathcal{F}_1\rangle \otimes |\mathfrak{S}_1\rangle \Rightarrow$ state of default, funding no, irregular payments to E

This last superposition collapses only when the business E receives full details of the loan disbursement by B to i. As happens in practice, however, this fundamental information asymmetry on the part of business E ensures that both states will exist for the entire duration of the issuance and full repayment, or not, of the trade receivable [22]. The qualitative tools for assessing solvency risk discussed at the beginning of section 3 are inadequate for resolving the "quantum" paradox of this model. A complete, or near complete, state of information is needed for the company E to successfully overcome these superposed states and improve the quality of its receivables and therefore the composition of its balance sheet.

Especially in periods of negative financial and economic growth—whether caused by endogenous shocks, as with the 2008 crisis, or exogenous ones, as with the present pandemic emergency—government measures to strengthen public guarantees through the reduction of the *LGD – Loss Given Default* factor of bank rating algorithms and reweighting of supervisory provisions would improve the creditworthiness of the company requesting leverage and orient it toward the state of $|\mathcal{F}_0\rangle$. Similar effects can be achieved through the implementation of tranched cover, with the junior tranche accounted for by the government [14], [1].

An extraordinary recovery measure that might only occur if the economic and productive fabric face a deep crisis, the provision of sinking fund contributions without restrictions on destination—a condition of which supplying companies are informed—would *ipso facto* sterilize the first (sub)system to directly affect the state of $\mathfrak{S}_j$ toward $\mathfrak{S}_0$.

2.3 Dynamics of the trade portfolio

The dynamics of the portfolio of trade receivables can be associated with a space of phases described by a Hamiltonian function [4]. When payments are regular, the Cartesian coordinates are measured according to the trend of pairs of monetary business cycles in the financial year. Though generally discrete, these periods are not necessarily sequential and are in fact more likely to show a superposition. We represent a given portfolio Z, where two flows of credit converge asynchronously, with each varying at any given time by oscillation—given by the joint contribution of the extensions granted—and by the strength of the relationship between the dimensional dynamics of credit and the relative period of manifestation.[3] This is shown in the Hamiltonian function as two components: one harmonic and another that either amplifies or dampens.

$$Z = \frac{x_m^2}{y_m^2} + \frac{x_n^2}{y_n^2} + \sin(y_m^2 + y_n^2) \qquad (10)$$

For pairs of motion coordinates, therefore—inverting, in formalism, for the elegance of the formula—the former component is determined by the sum of the ratio of variance in amounts granted with that of the variance in respective duration, while the latter is represented by the sine of the sum of the variance in periods of deferment [23]. More specifically, these coordinate pairs are indicated by:

$$x_{m/n}^2 = \sigma_{x_{m/n}}^2(M/N, \tau) = \frac{1}{2(M/N-1)} \sum_{m/n=1}^{M/N-1} [x_{m/n}(\tau) - x^*(\tau)]^2 \qquad (11)$$

and

$$y_{m/n}^2 = \sigma_{y_{m/n}}^2(M/N, \tau) = \frac{1}{2(M/N-1)} \sum_{m/n=1}^{M/N-1} [y_{m/n}(\tau) - y^*(\tau)]^2 \qquad (12)$$

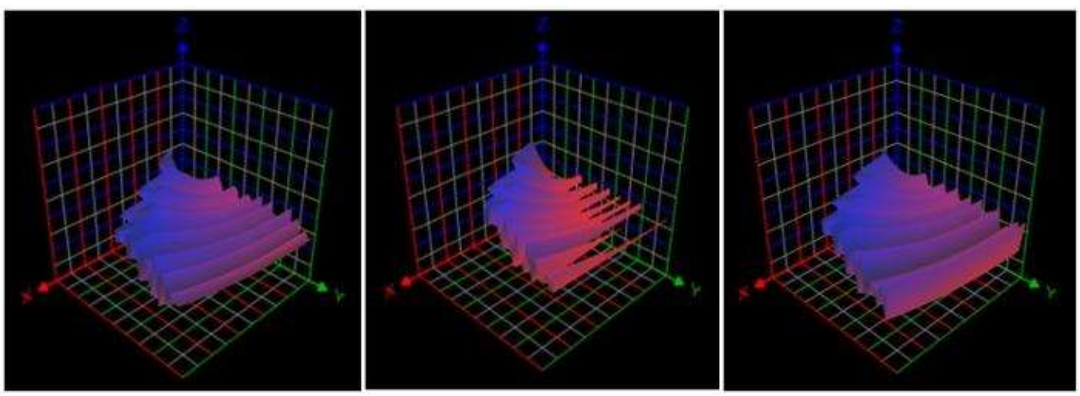
Where:

- M and N are the samplings of x_m, y_m and x_n, y_n;
- $x_{m/n}(\tau)$ and $y_{m/n}(\tau)$ represent the amount and timing of trade receivable lines at the times τ respectively;

[3] We have considered two flows to describe the dynamics of Z. In a given firm E, in fact, there are likely to exist several monetary flows (e.g. a clothing wholesaler that supplies the spring and autumn collections to its retail business customers) and these flows, through deferred payments, can overlap. This phenomenon may be more pronounced for some companies and less so for others.

– x^* and y^* are the expected or optimal values with which the acid-test ratio (current assets/current liabilities) calculated by business E, weighted with the reciprocal collection and payment times, equals 1. Put differently, they are those pairs of asset values and maturities by which the creditor business fully offsets its debts in the commercial treasury cycle, and therefore avoids drawing loans from its own resources or leveraging other debts (bank or tax). Therefore, the dynamic is designed on the variances from these points of perfect equilibrium noted in the working part of financial statements.

Figure 2. Portrait (qualitative representation) snapshots of three stages of Z in the motion of a B2B credit portfolio, as observed in the balance sheet assets of creditor company E in a time interval in which payments are substantially regular. The graphic representation of (10), the dynamics depicted, based on our investigation, are respectful of reality.



3. Analysis

Working from the above model, we discuss the impacts on portfolio dynamics from balance-sheet insolvency in B2B relationships that stem from exogenous shocks to the economy and/or negative economic conditions. Theoretical analyses are then applied to circumstances and responses in Italy arising from the COVID-19 pandemic.

3.1 Transformations in portfolio dynamics following debtor default

Let us assume a situation in which the total book value of the trade receivables of the supplier company E is sufficiently fractionated and therefore assumes a negligible concentration risk. In this hypothetical model, a factor that disturbs the *motion* is introduced to simulate the affected distributions of y compared to the idiosyncratic factors of e_i of the debtor enterprises, which contributes to the formation of a generating function *S* of transformation $(x, y) \longrightarrow (X, Y)$. In this way,

$X = [\bar{x} + \varepsilon\hat{x}]$ (with $\varepsilon\hat{x} > \rho$) also represents the output transformation of x. Here, $\bar{x}$ indicates an ergodic solution for $\dot{x}(t) = f(x(t), u(t))$, where $u(t)$ is the input function of the system in which x is located. This output transformation is brought about by the onset of losses on the commercial loans incurred by supplying firm E following delays or payment flow disruptions on the part of debtor businesses, which may be caused by the real devaluation of the receivable relative to its nominal value, the constitution of provisions for credit risk, opportunity costs, or other notional charges. $\varepsilon\hat{x}$, therefore, represents the disturbance to the dynamics described by Z.

The function $S(X, y)$ thus consists of the average of the variances of the marginal distribution of the durations in y_m and y_n multiplied by the respective coordinate *X(m)* of the transformation, and added to the product of the variance of y_m—approximated as a conditional mean—and the coordinate *X(n)*:

$$S = \frac{y_m^2 + y_n^2}{2} X_m + y_m^2 X_n \qquad (13)$$

The model also accounts for the permanence of the initial payment terms. For supplier company E in the B2B trade cycle, the lack of regular payment collection (even with a fixed duration for total repayment) could result in the dilution of its gross trade balance (in debit), which would compromise the regular dynamics of its receivables portfolio and disrupt the relationships in its balance sheet. As the variance of the deferrals increases, these effects are further aggravated and, after a certain point, may be irreversible and force an operating loss [15].

The outcome of the transformation will always exist within a particular region, as the standard deviations $\sigma_{y_m}(M, \tau)$ and $\sigma_{y_n}(N, \tau)$ are always positive or equal to zero. Limited to strictly positive cases, $y_m, y_n > 0$.

By virtue of the independence of the generating function *S* from the original coordinates—on which the perturbations also persist—and the new impulses, the transformation is implicitly defined by:

- $Y_m = \frac{y_m^2 + y_n^2}{2}$
- $Y_n = y_m^2$
- $x_m = y_m X_m + 2 y_m X_n$
- $Y_n = y_n X_m$

From which, we derive:

- $y_m = \sqrt{Y_n}$
- $y_n = \sqrt{2Y_m - Y_n}$
- $x_m = \sqrt{Y_n}(X_m + 2X_n)$

– $x_n = \sqrt{2Y_m - Y_n X_m}$

Substituting (10), we obtain the Hamiltonian W with the new variables:

$$W = (X_m + 2X_n)^2 + X_m^2 + \sin(2Y_m) \qquad (14)$$

Where the reasoning leading to (14) demonstrates the cyclical nature of the coordinate Y_n, the variable X_n is therefore preserved. Positing X_n as a fixed parameter, the one-dimensional Lagrangian for the variable Y_m is then:

$$L = X_m \dot{Y}_m - W \qquad (15)$$

With:

$$\dot{Y}_m = \frac{\partial W}{\partial X_m} = 4(X_m + X_n) \qquad (16)$$

Therefore:

$$L = \frac{1}{8}\dot{Y}_m^2 - \dot{Y}_m X_n - 2X_n^2 - \sin(2Y_m) \qquad (17)$$

Assuming that the variable X_n is constant, and therefore $-2X_n^2$ is constant, and considering that $\dot{Y}_m X_n = \frac{d}{dt}(Y_m X_n)$ and therefore does not affect the equations of the dynamic,

$$L = \frac{1}{8}\dot{Y}_m^2 - \sin(2Y_m) \qquad (18)$$

Carrying these assumptions forward, the Hamiltonian W can now be rewritten as:

$$W = \frac{1}{8}\dot{Y}_m^2 + \sin(2Y_m) \qquad (19)$$

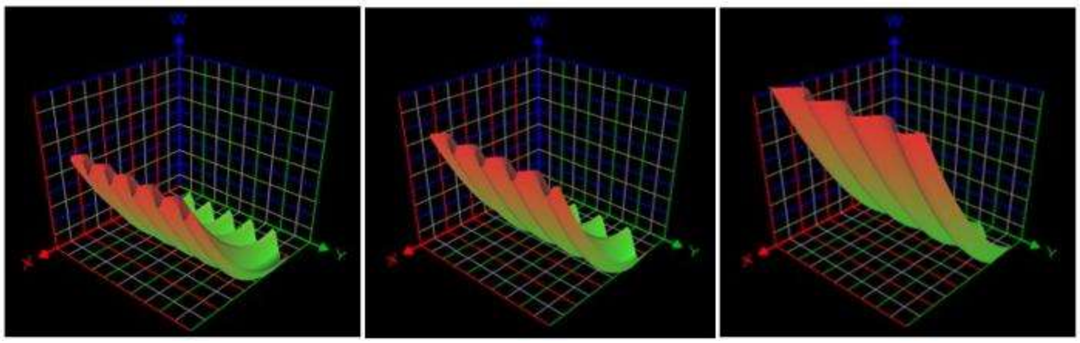
From (19), we can derive the dynamic of the variable Y_m:

$$t = \int \frac{1}{\sqrt{8[W - \sin(2Y_m)]}} dY_m \qquad (20)$$

The *motion* of the other variables can then be found through the integration of Hamilton’s equations:

– $X_n(t) = X_n(0)$
– $X_m(t) = \frac{1}{8}\dot{Y}_m(t) - X_n(0)$
– $Y_n(t) = Y_n(0) + Y_m(t) - Y_m(0) + 4tX_n(0)$

Figure 3. Portrait (qualitative representation) snapshots of three stages of W, which reflect the dynamics of a B2B portfolio that has become non-performing. There is a progressive flattening of the fluctuation and a marked incremental upward trend of the dependent variable (W, which replace Z after the transformation with perturbation), which highlight the "out of control" growth of the total portfolio and the worsening probability of recovery.



The conditions just discussed are most relevant when a (more or less) large portion of the portfolio is characterized by disruptive events (i.e. the irregularity of payments, and even the declaration of insolvency) from a certain number $h > \omega$ of debtors. When ω remains small, the disruptions (unless of significant granularity) will be of the type $\varepsilon\hat{x} \leq \rho$; the business E will manage to compensate setbacks through its own contributions or other types of leverage. However, as ω increases, the business E will likewise incur increasing losses on its balance sheet. This, in turn, will undermine its creditworthiness and, therefore, its ability to access bank credit, bringing the business E closer to systemic insolvency in its B2B relationships until a point of irreversibility is reached.

Indeed, business E will experience further repercussions due to these disruptions, as the unexpected losses negatively impact its ability to meet the terms of supply for other enterprises ki, who will then strategically concentrate payments toward other supplying companies and paradoxically create an *ipso facto* exacerbation of their own insolvency status relative to E. At the same time, regular debtor companies will be similarly affected by the inconstancy of the supply relationship and begin to delay their payments [18].

3.2 The Italian case study

The system expressed by (10) shows a point of equilibrium, generically denoted by z_{eq}, which is stable for each $\varepsilon\hat{x} > \rho$, according to Lyapunov [7]. For every perturbation $\varepsilon\hat{x} > \rho$, the vicinity with radius r that exists around the point of equilibrium z_{eq} delimits the bounds of the system trajectory: for these values, an α-

limit cycle is formed. For $\varepsilon\hat{x} > \rho$, a subcritical Hopf bifurcation can be observed as the portfolio dynamic distorts through progressive stretching and flattening of oscillations and a tendency toward exponential increase [24], [21]. For details on the procedure leading to this conclusion, see *ex multis* [20], [3].

Where the degree of disruption (*rectius*, the increase in the insolvency of debtor businesses) is linked to the ability of the same businesses ki to access bank credit in support of their current obligations, the equilibrium in the trade receivables system has been generally maintained in recent years by regular payments. The COVID-19 emergency, however, has placed pressure on corporate liquidity, which has inevitable repercussions for companies' payment practices.

Measures introduced by the Italian government in response to the COVID-19 emergency, especially the 2020 Decree-Laws of March (No. 18) and April (No. 23), increased the coverage of the public guarantee to 90%—and, in some cases, 100%—on credit lines granted by banks. These exogenous interventions sought to improve the (even though external) creditworthiness of businesses [13], affecting the ability of businesses ki to obtain financing, as expressed in Figure 4, and moving them toward the measurement of ratings $|B_0\rangle$. As shown in Figure 5, this has contributed to the reduction of impaired loans on commercial portfolios: among the phenomena described in the figures just mentioned, there is a cause-effect delay of about 30 to 45 days.

Figure 4. Percentage change in the volume of loans disbursed to micro and small enterprises in Italy, Sep 2019 to Jul 2020. Data from the Bank of Italy [8], Confindustria Studies Office [6], and ISTAT [10] processed internally.

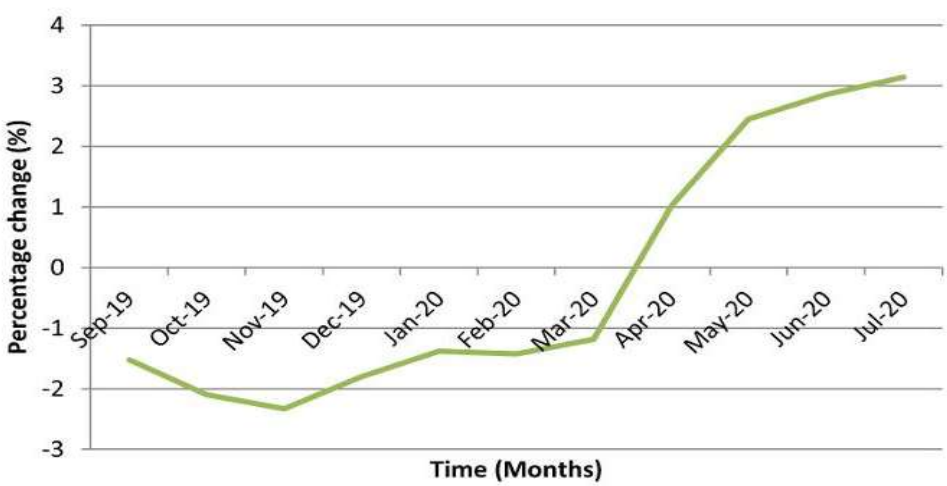
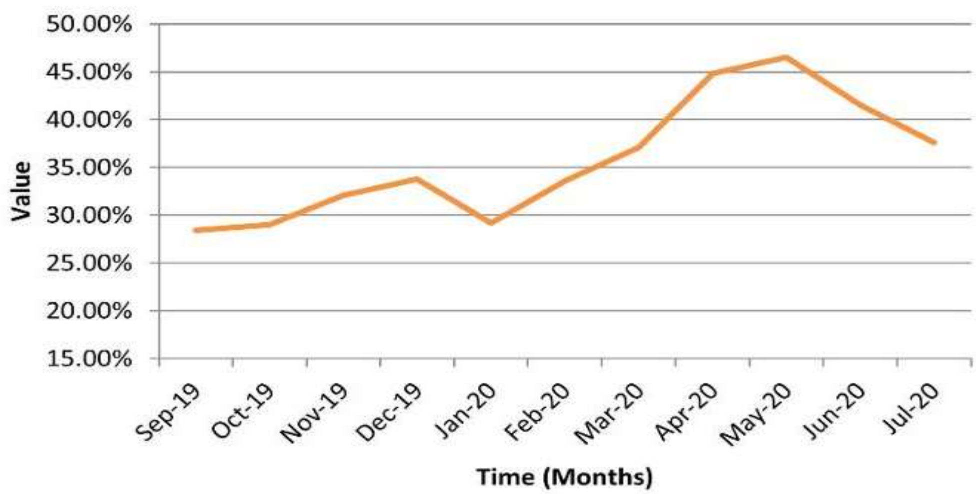


Figure 5. Value of unsettled business-to-business items maturing and outstanding in Italy, Sep 2019 to Jul 2020. Data from Cerved Group [9] and ISTAT [10] processed internally.



In the absence of government mechanisms to sustain the credit system, businesses would see balance sheet deficits and more extreme outcomes [26]. This would have severe repercussions on the resilience of the system.

4. Conclusion

This work demonstrates the correlation between the creditworthiness—and therefore the ability to access credit—of debtor companies in trade credit relationships and their solvency in B2B payments. Taking the position of the supplier and creditor company, we find that the supplier side suffers from the outcome of bank ratings of its counterparties, as its sudden manifestation at the time of measurement inhibits the ability of the supplier to react.

The proposed model and mathematical processing describe the dynamic phenomenon of the trade receivables portfolio Z reflected in the financial statements of a generic supplier business E by comparing the state of system equilibrium, see (10) and Figure 2, with the situation generated by a perturbation $\varepsilon\hat{x} > \rho$ caused by the default of a group of debtors in Figure 3. The effect of the perturbation leads to a critical condition of the portfolio, however, (governmental) support to stabilize bank credit disbursements can prevent—or significantly reduce—compromised performance of the balance sheet assets of the creditor company and of the overall system to which it belongs.

The limits of this research are typical of modeling: in fact, a confluence of two flows of variables—therefore, two monetary cycles—has been assumed to approximate the real heterogeneity of the cases. Furthermore, the contributions offered by other resources were neglected and no additional compensating effect has been included.

The model, thus far, offers a stress-testing tool for the portfolio. Further investigations may consider more complex integrated subsets of companies or industrial sectors to evaluate the combined effects and isolate any evidence of self-normalizing synergies.

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CRediT author statement:
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Elisa Casu: Data Collection, Curation, Systematization, and Investigation; Software and Graphics.

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