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Sequences of Intuitionistic Fuzzy Soft *G*-Modules

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Abstract

The notion of intuitionistic fuzzy G-modules and the notion of exact sequence of intuitionistic fuzzy G-modules was introduced by P.K. Sharma [13, 15] and studies their properties. In this paper we develop the notion of exact sequence of intuitionistic fuzzy soft G-modules and studies their properties.

Keywords: Intuitionistic fuzzy set, intuitionistic fuzzy soft set, intuitionistic fuzzy G-submodule, intuitionistic fuzzy soft G-submodule, exact sequence

1. Introduction

Many practical problems in economics, engineering, environment, social science, medical science etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. The reason for these difficulties may be due to the inadequacy of the theories of parameterization tools. Molodtsov [12] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Maji et al. [11] presented the concept of fuzzy soft set. The theory of fuzzy sets, first developed by Zadeh in [18], is perhaps the most appropriate framework for dealing with uncertainties, a number of generalizations of this fundamental concept have come up. The notion of intuitionistic fuzzy sets introduced by Atanassov [3] is one among them. Algebraic structures play a vital role in Mathematics and numerous applications of these structure are seen in many disciplines such as computer sciences, information sciences, theoretical physics, control engineering and so on. This inspires researchers to study and carry out research in various concepts of abstract algebra in fuzzy setting. Biswas [4] applied the concept of intuitionistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroup of a group. Fuzzy submodules of a module M over a ring R were first introduced [10]. Aktaş and Çağman [2] defined soft groups and compared soft sets with fuzzy sets and rough sets. F. Feng et al. [5] gave soft semirings and U.Acar et al. [1] introduced initial concepts of soft rings.

After that the definition of fuzzy soft group was given by some authors [8]. Qiu-Mei Sun et al. [16] defined soft modules and investigated their basic properties. Gunduz (Aras) and Bayramov.S. in [6],[7] did some investigations on intuitionistic fuzzy soft modules. The notion of intuitionistic fuzzy G-modules was introduced by P.K. Sharma [13]. K.M. Veliyeva and S.A. Bayramov introduced fuzzy soft G-modules [17].
Many properties like representation, reducibility, complete reducibility and injectivity of intuitionistic fuzzy G-modules have been discussed in [13],[14],[15]. In this paper we develop the notion of exact sequence of intuitionistic fuzzy soft G-modules and studies their properties.

2. Preliminaries

Definition 2.1. ([12]) Let X be an initial universe set and E be a set of parameters. A pair (F,E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set X, i.e., $F:E\to P(X)$, where $P(X)$ is the power set of X.
Definition 2.2. ([16]) Let (F,A) be a soft set over M. (F,A) is said to be a soft module over M if and only if $F(x)<M$ for all $x\in A$.
Definition 2.3. ([7]) Let $IFS(X)$ denote the set of all intuitionistic fuzzy sets on X and $A\subset E$. A pair (F,A) is called an intuitionistic fuzzy soft set over X, where F is a mapping from A into $IFS(X)$. That is, for each $a\in A$, $F(a)=(F_a,F^a):X\to I$ is an intuitionistic fuzzy set on X, where $F_a,F^a:X\to I$ are fuzzy sets.
Definition 2.4. ([7) Let (F,A) be an intuitionistic fuzzy soft set over M. Then (F,A) is said to be an intuitionistic fuzzy soft module over M iff $\forall a\in A$, $F(a)=(F_a,F^a)$ is an intuitionistic fuzzy submodule of M.
Definition 2.5. ([7]) Let (F,A) and (H,B) be two intuitionistic fuzzy soft modules over M and N respectively, and let $f:M\to N$ be a homomorphism of modules, and let $g:A\to B$ be a mapping of sets. Then we say that $(f,g):(F,A)\to(H,B)$ is an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy soft modules, if the following conditions are satisfied: $f(F_a)=H_{g(a)}$, $f(F^a)=H^{g(a)}$.
We say that (F,A) is an intuitionistic fuzzy soft homomorphic to (H,B). Note that for $\forall a\in A$, $f:(M,F_a,F^a)\to(N,H_{g(a)},H^{g(a)})$ is an intuitionistic fuzzy homomorphism of intuitionistic fuzzy modules.

Definition 2.6. ([13]) Let G be a group and M be a G-module over K, which is a subfield of C. Then a intuitionistic fuzzy G-module on M is an intuitionistic fuzzy set $A=(\mu_A,\nu_A)$ of M such that following conditions are satisfied

(i) $\mu_A(ax+by)\geq\mu_A(x)\wedge\mu_A(y)$ and $\nu_A(ax+by)\leq\nu_A(x)\vee\nu_A(y), \forall a,b\in K$ and $x,y\in M$

(ii) $\mu_A(gm)\geq\mu_A(m)$ and $\nu_A(gm)\leq\nu_A(m), \forall g\in G; m\in M$

Definition 2.7. (15]) For any IFS $A=\{<x,\mu_A(x),\nu_A(x)>:x\in X\}$ of set X. We denote the support of the IFS set A by A^* and is defined as

$$A^*=\{x\in X:\mu_A(x)>0 \text{ and } \nu_A(x)<1\}.$$

Proposition 2.1. ([15]) Let $f:X\to Y$ be a mapping and A,B are IFS of X and Y respectively. Then the following result holds

(i) $f(A^*)\subseteq(f(A))^*$ and equality hold when the map f is bijective

(ii) $f^{-1}(B^*)=(f^{-1}(B))^*$

3. Sequences of intuitionistic fuzzy soft G-modules

Let (F,A) be intuitionistic fuzzy soft G-module on M, (G,B) be intuitionistic fuzzy soft G-module on N. $f:M\to N$ is a G-modules homomorphism, $\varphi:A\to B$ is a mapping of sets.

Definition 3.1. If for each $a\in A$ $f:(M,F_a,F^a)$ is a homomorphism of the intuitionistic fuzzy G-modules, then (f,φ) pair is called homomorphism of intuitionistic fuzzy soft G-modules $(f,\varphi):(F,A)\to(G,B)$.

Let $(f,\varphi):(F,A)\to(G,B)$ be homomorphism of IFSG(M) and $\ker f<M$ be the kernel of M, such as G submodule. Define structure of IFSG(M) on $\ker f$ following

$$\bar{F}(a)=(\bar{F}_a,\bar{F}^a),\ \bar{F}_a=F_a/_{Kerf}\ ,\bar{F}^a=F^a/_{Kerf}$$

for $\forall a\in A$.

In this way show IFSG(M) structure of $\operatorname{Im} f<N$,such as G submodule of N:

$$\bar{G}(b)=(\bar{G}_b,\bar{G}^b),\ \bar{G}_b=G_b/_{\operatorname{Im} f}\ ,\bar{G}^b=G^b/_{Kerf}\ .$$

Theorem 3.1. Let M and N be G-modules and let f be a G-module homomorphism. If (G,B) is a intuitionistic fuzzy G-module on N, then $(f^{-1}(G),B)$ is an intuitionistic fuzzy soft G-module over M.

Proof. If the mapping $f^{-1}(G):B\to IFSG(M)$ is defined by

$f^{-1}(G)_b(x)=G_b(f(x))$, $f^{-1}(G)^b(x)=G^b(f(x))$.

For $k_1,k_2\in K$, $x,y\in M$ and $g\in G$, we have

$f^{-1}(G)_b(k_1x+k_2y)=G_b(f(k_1x+k_2y))=G_b(k_1f(x)+k_2f(y))\geq$

$G_b(f(x))\wedge G_b(f(y))\geq f^{-1}(G)_b(x)\wedge f^{-1}(G)_b(y)$.

Thus, $f^{-1}(G)_b(k_1x+k_2y)\geq f^{-1}(G)_b(x)\wedge f^{-1}(G)_b(y)$.

Similarly, we can show that,

$f^{-1}(G)^b(k_1x+k_2y)=G^b(f(k_1x+k_2y))=G^b(k_1f(x)+k_2f(y))\leq$

$G^b(f(x))\vee G^b(f(y))\leq f^{-1}(G)^b(x)\vee f^{-1}(G)^b(y)$.

Thus, $f^{-1}(G)^b(k_1x+k_2y)\leq f^{-1}(G)^b(x)\vee f^{-1}(G)^b(y)$.

Also, $f^{-1}(G)_b(gm)=G_b(f(gm))=G_b(g\,f(m))\geq G_b(f(m))=f^{-1}(G)_b(m)$.

Thus, $f^{-1}(G)_b(gm)\geq f^{-1}(G)_b(m)$.

Similarly, we can show that, $f^{-1}(G)^b(gm)\leq f^{-1}(G)^b(m)$.

Hence $(f^{-1}(G),B)$ is intuitionistic fuzzy G-module on M.

Theorem 3.2. Let M and N be G-modules and $f:M\to N$ be a G-module homomorphism. If (F,A) is an intuitionistic fuzzy G-module on M, then $(f(F),A)$ is an intuitionistic fuzzy soft G-module on N.

Proof. If the mapping $f:(F):A-IFSG(N)$ is defined by

$(f(F))_a(y)=\sup\{F_a(x):f(x)=y\}$, $(f(F))^a(y)=\inf\{F^a(x):f(x)=y\}$.

Now we show that $(f(F),A)$ is an intuitionistic fuzzy soft G-module on N.

For $k_1,k_2\in K$, $x,y\in N$ and $g\in G$, we have

$(f(F))_a(k_1x+k_2y)=\vee\{F_a(z):z\in f^{-1}(k_1x+k_2y)\}=$

$\vee\{F_a(z):f(z)=k_1x+k_2y,\ x,y\in N,z\in M\}=$

$\vee\left\{F_a\left(k_1'z'+k_2'z''\right):f(z')=x,\ f(z'')=y\right\}=$

$\vee\left\{F_a\left(k_1'z'+k_2'z''\right):z'\in f^{-1}(x),z''\in f^{-1}(y)\right\}\geq$

$\vee\left[\{F_a(z')\wedge F_a(z'')\}:z'\in f^{-1}(x),z''\in f^{-1}(y)\right]\geq$

$\left[\vee\{F_a(z'):z'\in f^{-1}(x)\}\right]\wedge\left[\vee\{F_a(z''):z''\in f^{-1}(y)\}\right]\geq$

$(f(F))_a(x)\wedge(f(F))_a(y)$.

Thus, $(f(F))_a(k_1x+k_2y)\geq(f(F))_a(x)\wedge(f(F))_a(y)$.

Similarly, we can show that, $\left(f\left(F\right)\right)^{a}\left(k_1x+k_2y\right)\leq\left(f\left(F\right)\right)^{a}\left(x\right)\vee\left(f\left(F\right)\right)^{a}\left(y\right)$.
Also,
$\left(f\left(F\right)\right)_{a}\left(gn\right)=\vee\left\{F_a\left(x\right):x\in f^{-1}\left(gn\right)\right\}=\vee\left\{F_a\left(x\right):f\left(x\right)=gn,\,g\in G,\,n\in N\right\}=$
$\vee\left\{F_a\left(gm\right):gm\in M,\,f\left(gm\right)=gn,\,g\in G,\,n\in N,\,m\in M\right\}\geq$
$\vee\left\{F_a\left(m\right):m\in M,\,f\left(m\right)=n\in N\right\}\geq\vee\left\{F_a\left(m\right):m\in f^{-1}\left(n\right)\right\}=\left(f\left(F\right)\right)_{a}\left(n\right)$.
Thus, $\left(f\left(F\right)\right)_{a}\left(gn\right)\geq\left(f\left(F\right)\right)_{a}\left(n\right)$.
Similarly, we can show that $\left(f\left(F\right)\right)^{a}\left(gn\right)\leq\left(f\left(F\right)\right)^{a}\left(n\right)$.
Hence $\left(f\left(F\right),A\right)$ is an intuitionistic fuzzy soft G-module on N.
Let $\left\{F_i,A_i\right\}_{i\in I}$ be family of soft G-module over $\left\{M_i\right\}_{i\in I}$ and let the parameters set be a fixed point.
We denote the fixed point of A_i as a_{0_i} and let $F_i\left(a_{0_i}\right)=0$. For $A=\Pi_{i\in I}A_i$ and $M=\oplus_{i\in I}M_i$ we define the mapping $F:A\to M$ by $F\left(a\right)=\oplus_{i\in I}F\left(a_i\right)$, for all $a=\left\{a_i\right\}\in A$. Then $\left(F,A\right)$ is a soft G-module over M.

Definition 3.2. $\left(F,A\right)$ is said to be direct sum of $\left\{\left(F_i,A_i\right)\right\}_{i\in I}$ and denoted as $\oplus_{i\in I}\left(F_i,A_i\right)$.

Theorem 3.3. If $\left\{\left(F_i,A_i\right)\right\}_{i\in I}$ is a family of intuitionistic fuzzy soft G-modules over $\left\{M_i\right\}_{i\in I}$, then $\oplus_{i\in I}\left(F_i,A_i\right)$.is an intuitionistic fuzzy soft G-modules over $\left\{M_i\right\}_{i\in I}$.

Proof. Define $F:\Pi_{i\in I}A_i\to\oplus_{i\in I}M_i$ for all $\left\{a_i\right\}\in\Pi_{i\in I}A_i$ by
$F\left(\left\{a_i\right\}\right)=\left(\wedge_{i\in j_i}\left(F_i\right)_{a_i},\vee_{i\in j_i}\left(F_i\right)^{a_i}\right)$ where $j_i:M_i\to\oplus_{i\in I}M_i$ is an embedding mapping. Since $\left(j_i\left(F_i\right)_{a_i},j_i\left(F_i\right)^{a_i}\right)$-is an intuitionistic fuzzy G-module over $\oplus_{i\in I}M_i$ for all $i\in I$, $F\left(\left\{a_i\right\}\right)$ is an intuitionistic fuzzy G-submodule over $\oplus_{i\in I}M_i$.

Definition 3.3. For any IFSS $\left(F,A\right)$ of set X. We denote the support of the IFSS by $\left(F^*,A\right)$ and is defined as $F^*\left(a\right)=\left\{x\in X,\,F_a\left(x\right)>0,\,F^a\left(x\right)<1\right\}$. It is clear that $\left(F^*,A\right)$ is a soft set on X.

Proposition 3.1. Let $f:X\to Y$, $\varphi:A\to B$ be a two mappings and $\left(F,A\right),\left(G,B\right)$ are IFSS of X and Y respectively. Then the following result holds:
a) $\left(f,\varphi\right)\left(F^*,A\right)\subset\left(f\left(F\right)^*,\varphi\left(A\right)\right)$

b) $f^{-1}\left(G^{*},B\right)=\left(f^{-1}\left(G\right)^{*},A\right)$

Theorem 3.4.

a) Let $\left(F,A\right)$ is IFSG(M). Then $\left(F^{*},A\right)$ is a soft G-submodule of M.

b) For any $\left(F,A\right),\left(G,B\right)$ soft G-module of M, we have

$\left(\left(F,A\right)+\left(G,B\right)\right)^{*}=\left(F^{*},A\right)+\left(G^{*},B\right)$.

c) $\left(\left(F,A\right)\bigcap\left(G,B\right)\right)^{*}=\left(F^{*},A\right)\bigcap\left(G^{*},B\right)$

Definition 3.4. We define two IFSS $\left(\tilde{\Phi},A\right)$ and $\left(\tilde{M},A\right)$ of M as. For $\forall a\in A$ and $\forall x\in M$

$$\tilde{\Phi}\left(a\right)\left(x\right)=\begin{cases}\left(1,0\right), x=0\\ \left(0,1\right), x\neq 0\end{cases};\ \tilde{M}\left(a\right)\left(x\right)=\left(1,0\right).$$

Then the IFSS $\left(\tilde{\Phi},A\right)$, $\left(\tilde{M},A\right)$ are IFSSM, of M which are actually equivalent of $\left\{0\right\}$ and M in module theory.

Definition 3.5. If $\left(F,A\right),\left(G,B\right)\in IFSGM$ on M, then the sum $\left(F,A\right)+\left(G,B\right)$ is called the direct sum of $\left(F,A\right)$ and $\left(G,B\right)$ if $\left(F,A\right)\bigcap\left(G,B\right)=\left(\tilde{\Phi},A\bigcap B\right)$ and we write it is as $\left(F,A\right)\oplus\left(G,B\right)$.

Theorem 3.5. Let $\left(F,A\right),\left(G,B\right),\left(H,C\right)$ soft G-modules of M such that $\left(F,A\right)=\left(G,B\right)\oplus\left(H,C\right)$, then $\left(F^{*},A\right)=\left(G^{*},B\right)\oplus\left(H^{*},C\right)$.

Let

$$\cdots\xrightarrow{f_{i-1}}M_{i-1}\xrightarrow{f_{i}}M_{i}\xrightarrow{f_{i+1}}M_{i+1}\xrightarrow{f_{i+2}}\cdots \tag{1}$$

are a sequence of G-modules and G-module homomorphism.

Definition 3.6. Let $M_i, i\in Z$ be G-modules and let $\left(F_i,A\right)\in IFSG\left(M_i\right), i\in Z$. Suppose that (1) is exact sequence of G-modules. Then the sequence

$$\cdots\xrightarrow{(f_{i-1},1_A)}\left(F_{i-1},A\right)\xrightarrow{(f_{i},1_A)}\left(F_{i},A\right)\xrightarrow{(f_{i+1},1_A)}\left(F_{i+1},A\right)\xrightarrow{(f_{i+2},1_A)}\cdots \tag{2}$$

of intuitionistic fuzzy soft G-module is said to be exact it, for all $a\in A$ $\cdots\to\left(M_{i-1},F_{i-1}\left(a\right)\right)\to\left(M_{i},F_{i}\left(a\right)\right)\to\left(M_{i+1},F_{i+1}\left(a\right)\right)\to\cdots$ the sequence of intuitionistic fuzzy G-module is exact.

Theorem 3.6. Let $\left(F,A\right),\left(G,B\right)\in IFSG\left(M\right)$ be such that $\left(F,A\right)\oplus\left(G,B\right)$ is a direct sum of intuitionistic fuzzy soft submodules of G-module M so that $\left(F^{*},A\right)+\left(G^{*},A\right)$ is a direct sum of soft G-modules. Then the sequence $0\longrightarrow\left(F,A\right)\xrightarrow{(i,1_A)}\left(F,A\right)\oplus\left(G,A\right)\xrightarrow{(\pi,1_A)}\left(G,A\right)\longrightarrow 0$ is exact, considering $\left(F,A\right)\in IFSG\left(F^{*}\left(a\right)\right)$ and $\left(G,A\right)\in IFSG\left(F^{*}\left(a\right)\right)$.

Proof. Note that the sequence of soft G-modules is an exact sequence.

$0 \longrightarrow (F^*, A) \xrightarrow{(i,1_A)} (F^*, A) \oplus (G^*, A) \xrightarrow{(\pi,1_A)} (G^*, A) \longrightarrow 0$, where "$(i,1_A)$" and "$(\pi,1_A)$" are respectively the canonical injection and projection. We have to prove that the sequence
$0 \longrightarrow (F, A) \xrightarrow{(i,1_A)} (F, A) \oplus (G, A) \xrightarrow{(\pi,1_A)} (G, A) \longrightarrow 0$ is an exact sequence of intuitionistic fuzzy soft G-modules.

For each $a \in A$ let $F^*(a) + G^*(a)$. Then $iF(a)(x) = \left(i(F_a)(x), i(F^a)(x)\right)$, where

$$i(F_a)(x) = \begin{cases} \vee\{F_a(t) : t \in F^*(a), i(t) = x\}, \text{ if } i^{-1}(x) \neq \varnothing \\ 0, \qquad \text{otherwise} \end{cases} = \begin{cases} F_a(x), & \text{if } x \in F^*(a) \\ 0, & \text{if } x \notin F^*(a) \end{cases}$$

and

$$i(F^a)(x) = \begin{cases} \wedge\{F^a(t) : t \in F^*(a), i(t) = x\}, \text{ if } i^{-1}(x) \neq \varnothing \\ 1, \qquad \text{otherwise} \end{cases} = \begin{cases} F^a(x), & \text{if } x \in F^*(a) \\ 1, & \text{if } x \notin F^*(a) \end{cases}.$$

Thus, $i(F, A) = (F, A) \quad \forall x \in F^*(a)$..... (1)

Also, for $\forall a \in A$ $\left(F(a) + G(a)\right)(x) = \left((F+G)_a(x), (F+G)^a(x)\right)$, where

$$(F+G)_a(x) = \begin{cases} \vee\{F_a(y) \wedge G_a(z) : y, z \in M, y + z = x\}, \text{ if } x = y + z \\ 0, \qquad \text{if } x \neq y + z \end{cases} = \begin{cases} F_a(x), & \text{if } x \in F^*(a) \\ 0, & \text{if } x \notin F^*(a) \end{cases}$$

and

$$(F+G)^a(x) = \begin{cases} \wedge\{F^a(y) \vee G^a(z) : y, z \in M, y + z = x\}, \text{ if } x = y + z \\ 1, \qquad \text{if } x \neq y + z \end{cases} = \begin{cases} F^a(x), & \text{if } x \in F^*(a) \\ 1, & \text{if } x \notin F^*(a) \end{cases}.$$

[Note that $(F, A) \oplus (G, A)$ is a direct sum, so $(F, A) \cap (G, A) = \Omega$. If $x = y + z$ with $x \in F^*(a)$, then the only possibility is $x = x + 0$ or $x = y + z;\ y, z \in F^*(a)$. But in the second case $G_a(z) = 0, G^a(z) = 1$.]

Thus, $(F, A) + (G, A) = (F, A)$ if $x \in \left(F^*(a)\right)$ for $\forall a \in A$..... (2)

It follows from (1) and (2) that $i(F, A) \subseteq (F, A) + (G, A)$.

For $x \in \left(G^*(a)\right), \left(\pi\left(F(a) + G(a)\right)\right)(x) = \left\{\pi(F+G)_a(x), \pi(F+G)^a(x)\right\}$ where,

$\pi(F+G)_a(x) = \vee\{(F+G)_a(t) : t \in F^*(a)+G^*(a); \pi(t)=x\}$
$= \vee\{(F+G)_a(r+x) : r \in F^*(a)\}$ [Since $\pi : F^*(a)+G^*(a) \to G^*(a)$-is the projection] $= \vee\{F_a(r) \wedge G_a(x) : r \in F^*(a)\} = G_a(x)$. [Since $F_a(r)=1$ with $r=0$].
Similarly, we have $\pi(F+G)^a(x) = G^a(x)$. Hence $\pi(F(a)+G(a)) = G(a)$.
Now by (1), we have

$$(i(F,A))(x) = \begin{cases} (F_a(x), F^a(x)), & if\ x \in F^*(a) = \ker(\pi) \\ (0,1), & if\ x \notin F^*(a) = \ker(\pi) \end{cases} \text{ i.e.,}$$

$(i(F,A))^* = \ker(\pi)$.
Therefore, $0 \longrightarrow (F,A) \xrightarrow{i} (F,A)+(G,A) \xrightarrow{\pi} (G,A) \longrightarrow 0$ is an exact sequence of intuitionistic fuzzy soft G-modules.

Theorem 3.7. Let $M \xrightarrow{f} N \xrightarrow{g} P$ be a sequence of G-modules exact at N and let $(F,A) \in IFSG(M)$, $(G,A) \in IFSG(N)$, $(H,A) \in IFSG(P)$. Then the sequence $(F,A) \xrightarrow{(f,1_A)} (G,A) \xrightarrow{(g,1_A)} (H,A)$ of intuitionistic fuzzy soft G-modules is exact at (G,A) only if for each $a \in A$ the sequence $F^*(a) \xrightarrow{f'} G^*(a) \xrightarrow{g'} H^*(a)$ is a sequence of G-modules exact at $G^*(a)$, where f' and g' are restriction of f and g to $F^*(a)$ and $G^*(a)$ respectively.
Proof. Suppose that $(F,A) \xrightarrow{f} (G,A) \xrightarrow{g} (H,A)$ is exact at (G,A). Then by definition $f((F,A)) \subseteq (G,A)$, $g((G,A)) \subseteq (H,A)$ and $(f(F,A))^* = \ker(g)$.
Now, consider the sequence $F^*(a) \xrightarrow{f'} G^*(a) \xrightarrow{g'} H^*(a)$.
We claim that this sequence is exact at $G^*(a)$.
For $x \in (f(F,A))^*$, $\forall a \in A$:
$\Leftrightarrow (f(F))_a(x) > 0$ and $(f(F))^a(x) < 1$
$\Leftrightarrow \vee\{F_a(t) : f(t) = x, t \in M\} > 0$ and $\wedge\{F^a(t) : f(t) = x, t \in M\} < 1$
$\Leftrightarrow \exists' s\, t_1, t_2 \in M$ such that $x = f(t_1) = f(t_2)$, $F_a(t_1) > 0$, $F^a(t_2) < 1$
(As $F_a(t_1) + F^a(t_2) \le 1$ always, so if $F_a(t_1) > 0$ then $F^a(t_2) < 1$)
$\Leftrightarrow \exists' s\, t_1 \in M$ such that $x = f(t_1)$, $F_a(t_1) > 0$ and $F^a(t_1) < 1$ i.e., $t_1 \in F^*(a)$
$\Leftrightarrow x = f(t_1) \in f(F^*(a))$.
Thus, we get $(f(F,A))^* = f(F^*(a))$. Similarly, we get $(g(F,A))^* = g(F^*(a))$.
Therefore, $f'(F^*(a)) = f(F^*(a)) = (f(F,A))^* \subseteq (G,A)^*$ as $f(F,A) \subseteq (G,A)$.

Similarly, $g'\left(G^*(a)\right)=\left(g(G,A)\right)^* \subseteq (H,A)^*$.

Now, Since $\left(f(F,A)\right)^* = \ker(g)$ it follows that $f'\left(F^*(a)\right)=\ker(g')$.

Thus, the sequence $F^*(a) \xrightarrow{f'} G^*(a) \xrightarrow{g'} H^*(a)$ is exact at $G^*(a)$.

This completes the proof of the theorem.

4. Conclusions

The main focus of this article is to introduce the concept of exact sequence of G-modules by intuitionistic soft fuzzification the concept in crisp theory. We develop the notion of exact sequence of intuitionistic fuzzy soft G-modules and studies their properties.

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