International Mathematical Forum, Vol. 13, 2018, no. 10, 447 - 454
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/imf.2018.8845

Strong 2-Jordan Product Preserving Maps on Operator Algebras

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Abstract

Let $\mathcal{R}$ be a ring having unit 1 and an idempotent element e_1. Assume that $f : \mathcal{R} \to \mathcal{R}$ is a surjective map. It is shown that, under some mild conditions, f satisfies $\{f(a), f(e)\}_2 = \{a, e\}_2$ for all $a \in \mathcal{R}$ and $e \in \{e_1, 1 - e_1, 1\}$ if and only if $f(1)$ is in the center of $\mathcal{R}$ with $f(1)^3 = 1$ and $f(a) = f(1)a$ holds for all $a \in \mathcal{R}$. As applications, such maps on prime rings, standard operator algebras and von Neumann algebras are characterized, respectively.

Mathematics Subject Classification: 16W10; 47B49

Keywords: Standard operator algebras, von Neumann algebras, Jordan products, k-Jordan products

1 Introduction

Let $\mathcal{R}$ be an associative ring and k a positive integer. Recall that the k-commutator of elements $a, b \in \mathcal{R}$ is defined by $[a, b]_k = [[a, b]_{k-1}, b]$ with $[a, b]_0 = a$ and $[a, b]_1 = [a, b] = ab - ba$ (see [4]). Recall that a map $f : \mathcal{R} \to \mathcal{R}$ is said to be strong k-commutativity preserving if $[a, b]_k = [f(a), f(b)]_k$ for all $a, b \in \mathcal{R}$ ([7]). Obviously, strong k-commutativity preserving maps are the usual strong commutativity preserving maps if $k = 1$. For the case $k = 1$, see [5, 8] and the references therein. For the case $k \geq 2$, in [7], it is shown that a surjective map on a unital prime ring with characteristic not 2 and containing a nontrivial idempotent is strong 2-commutativity preserving if and only if it has the form $a \mapsto \lambda a + h(a)$, where λ is an element in the extended centroid of the ring satisfying $\lambda^3 = 1$ and h is a map from the ring into its center. With k increasing, the problem of characterizing strong k-commutativity preserving maps becomes much more difficult. Let X be a complex Banach space with $\dim X \geq 2$ and $\mathcal{A}$ be a standard operator algebra on X. Hou and Qi in [1] proved that, if the range of a map $f : \mathcal{A} \to \mathcal{A}$ contains all operators of rank ≤ 1, then f is strong k-commutativity preserving if and only if there exist a functional h on $\mathcal{A}$ and a complex scalar λ with $\lambda^{k+1} = 1$ such that $f(A) = \lambda A + h(A)I$ for all $A \in \mathcal{A}$.

On the other hand, $\mathcal{R}$ is also a Jordan ring under Jordan product $\{a, b\} = ab + ba$. Jordan product is a kind of important products and had been studied intensively by many authors. Motivated by the k-commutator, we can define the k-Jordan product of $a, b \in \mathcal{R}$ by $\{a, b\}_k = \{\{a, b\}_{k-1}, b\}_1$, where $\{a, b\}_0 = a$ and $\{a, b\}_1 = \{a, b\} = ab + ba$. In addition, f is called to strong k-Jordan product preserving if $\{f(a), f(b)\}_k = \{a, b\}_k$ for each $a, b \in \mathcal{R}$ (see [10]).

Thus, a natural problem is how to characterize strong k-Jordan product preserving maps on rings or algebras. Assume that $\mathcal{R}$ is a unital ring containing a nontrivial idempotent and $f : \mathcal{R} \to \mathcal{R}$ is a surjective map. Wang and Qi [10] showed that, under some mild conditions, f is strong k-Jordan product preserving if and only if there exists $\lambda \in \mathcal{Z}(\mathcal{R})$ (the center of $\mathcal{R}$) with $\lambda^{k+1} = 1$ such that $f(x) = \lambda x$ holds for all $x \in \mathcal{R}$. Taghavi, Kolivand and Rohi in [11] proved that, if $\mathcal{A}$ is a unital algebra containing a nontrivial idempotent e_1, then a surjective map $f : \mathcal{A} \to \mathcal{A}$ satisfies $\{f(a), f(e)\}_1 = \{a, e\}_1$ for all $a \in \mathcal{A}$ and $e \in \{e_1, 1 - e_1\}$ if and only if $f(1)$ is in the center of $\mathcal{A}$, $f(1)^2 = 1$ and $f(a) = f(1)a$ for all $a \in \mathcal{A}$; in [12] gave a concrete form of strong 2-Jordan product preserving surjective maps on standard operator algebras, and particularly, showed that, if a surjective map $\Phi : \mathcal{M} \to \mathcal{M}$ (here, $\mathcal{M}$ is a properly infinite von Neumann algebra) satisfies $\{\Phi(A), \Phi(P)\}_2 = \{A, P\}_2$ for all $A \in \mathcal{M}$ and all idempotents $P \in \mathcal{M}$, then $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{M}$.

The purpose of this paper is to consider strong 2-Jordan product preserving surjective maps on general rings. Assume that $\mathcal{R}$ is a unital ring with an idempotent element e_1 and $f : \mathcal{R} \to \mathcal{R}$ is a surjective map. It is shown that, under some mild conditions, f satisfies $\{f(a), f(e)\}_2 = \{a, e\}_2$ for all $a \in \mathcal{R}$ and $e \in \{e_1, 1 - e_1, 1\}$ if and only if $f(1)$ is in the center of $\mathcal{R}$ with $f(1)^3 = 1$ and $f(a) = f(1)a$ holds for all $a \in \mathcal{R}$ (Theorem 2.1). As applications, such maps on prime rings, standard operator algebras and von Neumann algebras are characterized, respectively (Corollaries 2.2-2.4), which generalize the corresponding results in [10, 12].

2 Main result and its proof

In this section, we will give the main result in this paper and its proof.

Theorem 2.1 *Let $\mathcal{R}$ be a ring having unit 1 and an idempotent element e_1. Assume that the characteristic of $\mathcal{R}$ is not 2 and $f : \mathcal{R} \to \mathcal{R}$ is a surjective map. If $\mathcal{R}$ satisfies $a\mathcal{R}e_1 = \{0\} \Rightarrow a = 0$ and $a\mathcal{R}(1 - e_1) = \{0\} \Rightarrow a = 0$, then*

$$\{f(a), f(e)\}_2 = \{a, e\}_2 \tag{1}$$

holds for all $a \in \mathcal{R}$ and $e \in \{e_1, 1 - e_1, 1\}$ if and only if $f(1) \in \mathcal{Z}(\mathcal{R})$, the center of $\mathcal{R}$, $f(1)^3 = 1$ and $f(a) = f(1)a$ holds for all $a \in \mathcal{R}$.

For the convenience, write $e_1 = e$ and $e_2 = 1 - e_1$. Then $\mathcal{R}$ can be written as $\mathcal{R} = \mathcal{R}_{11} + \mathcal{R}_{12} + \mathcal{R}_{21} + \mathcal{R}_{22}$, where $\mathcal{R}_{ij} = e_i\mathcal{R}e_j$ $(i, j \in \{1, 2\})$.

Now, we give the proof of Theorem 2.1.

Proof of Theorem 2.1. The "if" part is obvious. In the following, we will prove the "only if" part by checking several claims.

Claim 1. $f(0) = 0$.

By the surjectivity of f, there exists some $s \in \mathcal{R}$ such that $f(s) = -f(0)$. Letting $a = s$ and $e = 1$ in Eq.(1), we have

$$\begin{aligned} 4s = & \{s, 1\}_2 = \{f(s), f(1)\}_2 \\ = & f(s)f(1)^2 + f(1)^2f(s) + 2f(1)f(s)f(1) \\ = & -f(0)f(1)^2 - f(1)^2f(0) - 2f(1)f(0)f(1) \\ = & -\{f(0), f(1)\}_2 = -\{0, 1\}_2 = 0, \end{aligned}$$

which means $\{s, 1\}_2 = 0$. Since the characteristic of $\mathcal{R}$ is not 2, one gets $s = 0$. That is, $f(0) = -f(0)$. It follows from char$\mathcal{R} \neq 2$ that $f(0) = 0$.

Claim 2. f is additive, i.e. $f(a + b) = f(a) + f(b)$ holds for all $a, b \in \mathcal{R}$.

For any $a, b \in \mathcal{R}$, by the surjectivity of f, there exists some element $c \in \mathcal{R}$ such that $f(c) = f(a + b) - f(a) - f(b)$. Let $i \in \{1, 2\}$. Note that

$$\begin{aligned} & ce_i + 2e_ice_i + e_ic = \{c, e_i\}_2 = \{f(c), f(e_i)\}_2 \\ = & \{f(a + b) - f(a) - f(b), f(e_i)\}_2 \\ = & \{f(a + b), f(e_i)\}_2 - \{f(a), f(e_i)\}_2 - \{f(b), f(e_i)\}_2 \\ = & \{(a + b), e_i\}_2 - \{a, e_i\}_2 - \{b, e_i\}_2 = 0. \end{aligned}$$

This implies $e_jce_i + e_ice_j + 4e_ice_i = 0$ for $i, j \in \{1, 2\}$. As the characteristic of $\mathcal{R}$ is not 2, we obtain $c = 0$. Thus $f(c) = 0$ by Claim 1, and so $f(a + b) = f(a) + f(b)$.

Claim 3. $f(e)^3 = e$, $f(e_i)e_j = e_jf(e_i)$ and $f(1)e_i = e_if(1)$, $i, j \in \{1, 2\}$.

For $e \in \{1, e_1, e_2\}$, by taking $a = e$ in Eq.(1), we have $\{f(e), f(e)\}_2 = \{e, e\}_2$, which implies $f(e)^3 = e$. Thus, for $i \neq j \in \{1, 2\}$, one gets

$$f(e_i)e_i = f(e_i)f(e_i)^3 = f(e_i)^4 = f(e_i)^3f(e_i) = e_if(e_i),$$

and so

$$f(e_i)e_j = f(e_i)(1 - e_i) = (1 - e_i)f(e_i) = e_j f(e_i).$$

Combining the above two equations and Claim 2 gives

$$f(1)e_i = (f(e_i) + f(e_j))e_i = e_i f(e_i) + e_i f(e_j) = e_i f(1), \quad 1 \leq i \neq j \leq 2.$$

Claim 4. $f(e_i) \in \mathcal{R}_{ii}$, $i \in \{1, 2\}$.

By letting $a = e_1$ and $e = e_2$ in Eq.(1), one can obtain

$$f(e_1)f(e_2)^2 + f(e_2)^2 f(e_1) + 2f(e_2)f(e_1)f(e_2) = 0.$$

Multiplying by $f(e_2)^2$ from the left and the right in the above equation, respectively, by Claim 3, one gets

$$f(e_2)^2 f(e_1)f(e_2)^2 + e_2 f(e_2)f(e_1) + 2e_2 f(e_1)f(e_2) = 0 \tag{2}$$

and

$$e_2 f(e_1)f(e_2) + f(e_2)^2 f(e_1)f(e_2)^2 + 2e_2 f(e_2)f(e_1) = 0. \tag{3}$$

Comparing Eq.(2) and Eq.(3) yields

$$e_2 f(e_1)f(e_2) - e_2 f(e_2)f(e_1) = 0. \tag{4}$$

Similarly, one can show

$$e_1 f(e_1)f(e_2) - e_1 f(e_2)f(e_1) = 0. \tag{5}$$

Combining Eqs.(4)-(5) gives $f(e_1)f(e_2) = f(e_2)f(e_1)$, and so

$$f(1)f(e_i) = (f(e_i)+f(e_j))f(e_i) = f(e_i)(f(e_i)+f(e_j)) = f(e_i)f(1), \;\; i \neq j \in \{1, 2\}.$$

On the other hand, taking $a = e_i$ $(i = 1, 2)$ and $e = 1$ in Eq.(1) gives

$$f(e_i)f(1)^2 + f(1)^2 f(e_i) + 2f(1)f(e_i)f(1) = 4e_i.$$

It follows from char$\mathcal{R} \neq 2$ that

$$f(1)^2 f(e_i) = e_i, \quad i = 1, 2.$$

Multiplying by $f(1)$ from the left in the above equation, and by Claim 3, one achieves $f(e_i) = f(1)e_i = e_i f(e_i)e_i \in \mathcal{R}_{ii}$, $i = 1, 2$.

Claim 5. For any $a \in \mathcal{R}$, we have $f(e_i)^2 f(a)f(e_j) = f(e_i)f(a)f(e_j)^2$, $1 \leq i \neq j \leq 2$.

For any $a \in \mathcal{R}$, by Eq.(1), one has

$$f(a)f(e_1)^2 + f(e_1)^2 f(a) + 2f(e_1)f(a)f(e_1) = ae_1 + e_1 a + 2e_1 a e_1, \tag{6}$$

$$f(a)f(e_2)^2 + f(e_2)^2 f(a) + 2f(e_2)f(a)f(e_2) = ae_2 + e_2 a + 2e_2 ae_2 \tag{7}$$

and

$$f(a)f(1)^2 + f(1)^2 f(a) + 2f(1)f(a)f(1) = 4a. \tag{8}$$

As $e_1 + e_2 = 1$, by Claims 2 and 4, Eq.(8) can be rewritten as

$$\begin{aligned} &f(a)f(e_1)^2 + f(a)f(e_2)^2 + f(e_1)^2 f(a) + f(e_2)^2 f(a) + 2f(e_1)f(a)f(e_1) \\ &+2f(e_1)f(a)f(e_2) + 2f(e_2)f(a)f(e_1) + 2f(e_2)f(a)f(e_2) = 4a. \end{aligned} \tag{9}$$

Combining Eq.(9) and Eqs.(2)-(3), and noting that char$\mathcal{R} \neq 2$, one obtains

$$f(e_1)f(a)f(e_2) + f(e_2)f(a)f(e_1) = e_1 ae_2 + e_2 ae_1.$$

This implies $f(e_1)f(a)f(e_2) = e_1 ae_2$ and $f(e_2)f(a)f(e_1) = e_2 ae_1$ by Claim 4.

On the other hand, multiplying by e_2 from the left and the right in Eq.(6), respectively, and by Claim 4, one gets $e_2 f(a)f(e_1)^2 = e_2 ae_1$ and $f(e_1)^2 f(a)e_2 = e_1 ae_2$. So

$$f(e_1)^2 f(a)e_2 = f(e_1)f(a)f(e_2) \ \text{ and } \ e_2 f(a)f(e_1)^2 = f(e_2)f(a)f(e_1).$$

It follows from Claim 4 that $f(e_1)^2 f(a)f(e_2) = f(e_1)f(a)f(e_2)^2$ and $f(e_2)f(a)f(e_1)^2 = f(e_2)^2 f(a)f(e_1)$.

Claim 6. $f(1) \in \mathcal{Z}(\mathcal{R})$.

For any $a \in \mathcal{R}$, note that Eq.(8) holds. Multiplying by $f(1)$ from the left and the right in Eq.(8), respectively, by Claim 3, one gets

$$f(1)f(a)f(1)^2 + f(a) + 2f(1)^2 f(a)f(1) = 4f(1)a$$

and

$$f(a) + f(1)^2 f(a)f(1) + 2f(1)f(a)f(1)^2 = 4af(1).$$

Comparing the above two equations gives

$$4[af(1) - f(1)a] = f(1)f(a)f(1)^2 - f(1)^2 f(a)f(1). \tag{10}$$

Note that, by Claims 2-5, we have

$$\begin{aligned} & e_1[f(1)f(a)f(1)^2 - f(1)^2 f(a)f(1)]e_2 \\ = \ & e_1[f(e_1)f(a)f(e_1)^2 + f(e_1)f(a)f(e_2)^2 + f(e_2)f(a)f(e_1)^2 + f(e_2)f(a)f(e_2)^2 \\ & -f(e_1)^2 f(a)f(e_1) - f(e_1)^2 f(a)f(e_2) \\ & -f(e_2)^2 f(a)f(e_1) - f(e_2)^2 f(a)f(e_2)]e_2 = 0; \end{aligned}$$

and similarly, $e_2[f(1)f(a)f(1)^2 - f(1)^2 f(a)f(1)]e_1 = 0$. These and Eq.(10) imply

$$e_1[af(1) - f(1)a]e_2 = e_2[af(1) - f(1)a]e_1 = 0.$$

Now, by Claim 3, it is clear that

$$f(1) = e_1 f(1) e_1 + e_2 f(1) e_2.$$

By a direct calculation, one can obtain

$$e_1[af(1) - f(1)a]e_2 = e_1 a e_2 f(1) e_2 - e_1 f(1) e_1 a e_2 = 0$$

and

$$e_2[af(1) - f(1)a]e_1 = e_2 a e_1 f(1) e_1 - e_2 f(1) e_2 a e_1 = 0$$

for all $a \in \mathcal{R}$. That is,

$e_1 a e_2 f(1) e_2 = e_1 f(1) e_1 a e_2$ and $e_2 a e_1 f(1) e_1 = e_2 f(1) e_2 a e_1$ hold for all $a \in \mathcal{R}$.

Note that, by [9, Lemma 3.1], the center of $\mathcal{R}$ is

$$\begin{aligned}\mathcal{Z}(\mathcal{R}) = \ & \{z_{11} + z_{22} : z_{11} \in \mathcal{R}_{11}, z_{22} \in \mathcal{R}_{22}, z_{11}a_{12} = a_{12}z_{22} \\ & \text{and } z_{22}a_{21} = a_{21}z_{11} \ \text{ for all } a_{12} \in \mathcal{R}_{12}, a_{21} \in \mathcal{R}_{21}\}.\end{aligned}$$

It follows that $f(1) = e_1 f(1) e_1 + e_2 f(1) e_2 \in \mathcal{Z}(\mathcal{R})$.

Claim 7. $f(a) = f(1)a$ holds for all $a \in \mathcal{R}$.

Letting $e = 1$ in Eq.(1), and by Claim 6, one gets

$$4a = \{a, 1\}_2 = \{f(a), f(1)\}_2 = 4f(1)^2 f(a) \ \text{ for all } \ a \in \mathcal{R},$$

which implies $f(1)^2 f(a) = a$ as char$\mathcal{R} \neq 2$. Multiplying $f(1)$ from the left in the equation and noting that $f(1)^3 = 1$, we achieve $f(a) = f(1)a$ for all $a \in \mathcal{R}$.

The proof of Theorem 2.1 is finished.

Recall that a ring $\mathcal{R}$ is called prime if, for any $a, b \in \mathcal{R}$, $a\mathcal{R}b = \{0\} \Rightarrow a = 0$ or $b = 0$. It is obvious that prime rings satisfy the assumption "$a\mathcal{R}e_1 = \{0\} \Rightarrow a = 0$ and $a\mathcal{R}(1 - e_1) = \{0\} \Rightarrow a = 0$" in Theorem 2.1.

Applying Theorem 2.1 to prime rings, we have

Corollary 2.2 *Let $\mathcal{R}$ be a unital prime ring with an idempotent element e_1. Assume that the characteristic of $\mathcal{R}$ is not 2 and $f : \mathcal{R} \to \mathcal{R}$ is a surjective map. Then f satisfies $\{f(a), f(e)\}_2 = \{a, e\}_2$ for all $a \in \mathcal{R}$ and $e \in \{e_1, 1 - e_1, 1\}$ if and only if $f(1)$ is in the center of $\mathcal{R}$ with $f(1)^3 = 1$ and $f(a) = f(1)a$ holds for all $a \in \mathcal{R}$.*

Let X be a Banach space with dimension greater than 1. Denote by $\mathcal{B}(X)$ the algebra of all bounded linear operators on X. Recall that a standard operator algebra on X is a subalgebra of $\mathcal{B}(X)$ which contains the identity operator and all finite-rank operators in $\mathcal{B}(X)$. It is well known that standard operator algebras are prime. Hence, by Corollary 2.2, the following result is obvious, which generalizes Theorem 2.1 in [12].

Corollary 2.3 *Let X be a Banach space with* $\dim X > 1$ *and $\mathcal{A}$ a standard operator algebra on X. Assume that $\Phi : \mathcal{A} \to \mathcal{A}$ is a surjective map. Then f satisfies $\{\Phi(A), \Phi(P)\}_2 = \{A, P\}_2$ for all $A \in \mathcal{A}$ and all idempotents $P \in \mathcal{A}$ if and only if $\Phi(A) = \lambda A$ holds for all $A \in \mathcal{A}$, where λ is a scalar with $\lambda^3 = 1$.*

Recall that a von Neumann algebra $\mathcal{M}$ is a subalgebra of some $\mathcal{B}(H)$, the algebra of all bounded linear operators acting on a complex Hilbert space H, which satisfies the double commutant property: $\mathcal{M}'' = \mathcal{M}$, where $\mathcal{M}' = \{T : T \in \mathcal{B}(H) \text{ and } TA = AT \ \forall A \in \mathcal{M}\}$ and $\mathcal{M}'' = \{\mathcal{M}'\}'$ ([2], [3]). For $A \in \mathcal{M}$, the central carrier of A, denoted by $\overline{A}$, is the intersection of all central projections P such that $PA = 0$. If A is self-adjoint, then the core of A, denoted by $\underline{A}$, is $\sup\{S \in \mathcal{Z}(\mathcal{M}) : S = S^*, S \leq A\}$. Particularly, if $A = P$ is a projection, it is clear that $\underline{P}$ is the largest central projection $\leq P$. A projection P is core-free if $\underline{P} = 0$. It is easy to see that $\underline{P} = 0$ if and only if $\overline{I - P} = I$. If $\mathcal{M}$ has no central summands of type I_1, then by [6], each nonzero central projection $C \in \mathcal{M}$ is the carrier of a core-free projection in $\mathcal{M}$; particularly, there exists a nonzero core-free projection $P \in \mathcal{M}$ with $\overline{P} = I$. For such P, note that $\overline{P} = \overline{I - P} = I$. It follows from the definition of the central carrier that both $\text{span}\{TP(x) : T \in \mathcal{M}, \ x \in H\}$ and $\text{span}\{T(I - P)(x) : T \in \mathcal{M}, \ x \in H\}$ are dense in H. So $A\mathcal{M}P = \{0\}$ implies $A = 0$ and $A\mathcal{M}(I - P) = \{0\}$ implies $A = 0$. Thus, if $\mathcal{M}$ has no central summands of type I_1, then $\mathcal{M}$ satisfies the corresponding assumption in Theorem 2.1. Therefore, By Theorem 2.1, the following result is true.

Corollary 2.4 *Let $\mathcal{M}$ be a von Neumann algebra without central summands of type I_1. Then a surjective map $\Phi : \mathcal{M} \to \mathcal{M}$ satisfies $\{\Phi(A), \Phi(P)\}_2 = \{A, P\}_2$ for all $A \in \mathcal{M}$ and all idempotent operators $P \in \mathcal{M}$ if and only if $\Phi(I) \in \mathcal{Z}(\mathcal{M})$ with $\Phi(I)^3 = I$ and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{M}$.*

Acknowledgements. This work is partially supported by National Natural Science Foundation of China (11671006) and Outstanding Youth Foundation of Shanxi Province (201701D211001).

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Received: September 3, 2018; Published: October 18, 2018