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Compactification of a Soft Topological Space

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Abstract

In this paper we introduce a compactification of a soft topological space via soft ultrafilters.

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1 Introduction

Soft sets was introduced by D. Molodtsov 1999 [4] as ageneral mathematical tool for dealing with uncertain objects. Operations on soft sets was introduced by P.K. Maji , R . Biswas and A. R. Roy 2003 [3]. Sabir and Nas 2011 [7] introduced and studied the concept of soft topological spaces over soft sets and some related concepts. In 2011 [1] Aygunogla , Aygun introduced the soft product topology, E. Peygh and B. Samadi , A.Tayebi 2013 [5] introduced soft locally connected of a soft point and soft connected spaces depending on soft disjoint non-null soft open sets.
Let $SS(X, A)$ be the collection of all soft sets over the set X where A is the set of parameters. Let (X, τ, A) be a soft topological space, We show that $\mathcal{B}(X, \tau, A)$ is a compactification of (X, τ, A) which is Hausdorff.

2 Preliminary Notes

Definition 2.1. [2] Let X be an initial universe set and A a set of parameters. A pair (F, A), where F is a map from A to $\mathcal{P}(X)$, is called a soft set over X. In what follows, by $SS(X, A)$ we denote the family of all soft sets (F, A) over X.

0_A will denote the soft set (F, A) where $F(a) = \phi$ for all $a \in A$ and 1_A will denote the soft set (F, A) where $F(a) = X$ for all $a \in A$.
0_A is called $A-$null soft set while 1_A is called $A-$absolute soft set.

Definition 2.2. [2] Let $(F, A), (G, A) \in SS(X, A)$. We say that the pair (F, A) is a soft subset of (G, A) if $F(a) \subseteq G(a)$ for every $a \in A$. Symbolically, we write $(F, A) \sqsubseteq (G, A)$. Also we say that the pairs $(F, A), (G, A)$ are soft equal if $(F, A) \sqsubseteq (G, A)$ and $(G, A) \sqsubseteq (F, A)$. Symbolically, we write $(F, A) = (G, A)$.

Definition 2.3. [2] Let I be an arbitrary index set and $\{(F_i, A) : i \in I\} \subseteq SS(X, A)$.

1. The soft union of these sets is the soft set $(F, A) = \sqcup\{(F_i, A) : i \in I\}$ where $F(a) = \bigcup\{(F_i(a)) : i \in I\}$, for every $a \in A$.
2. The soft intersection of these sets is the soft set $(F, A) = \sqcap\{(F_i, A) : i \in I\}$ where $F(a) = \bigcap\{(F_i(a)) : i \in I\}$, for every $a \in A$.

Definition 2.4. [2] Let (F, A) be a soft set over X and $x \in X$. We say that $x \in (F, A)$ whenever $x \in F(a)$ for all $a \in A$. If $U \subseteq X$, $U \subseteq F(a)$ for all $a \in A$, then we write $U \widetilde{\subseteq} (F, A)$.

Definition 2.5. [6] Let $x \in X$. Then the soft set (F, A) over X, where $F(a) = \{x\}$ $\forall a \in A$, is called the singleton soft set and denoted by x_A or (x, A).

Definition 2.6. [2] Let X be an initial universe set and A be a set of parameters, and $\tau \subseteq SS(X, A)$. We say that the family τ defines a soft topology on X if the following axioms are true:

1. $0_A, 1_A \in \tau$.
2. If $(G, A), (H, A) \in \tau$, then $(G, A) \sqcap (H, A) \in \tau$.
3. If $(G_i, A) \in \tau$ for every $i \in I$, then $\sqcup\{(G_i, A) : i \in I\} \in \tau$.

The triple (X, τ, A) is called a soft topological space or soft space. The members of τ are called soft open sets on X. Also, a soft set (F, A) is called soft closed if the complement $(F, A)^c \in \tau$. The family of soft closed sets is denoted by τ^c.
If $\tau = SS(X, A)$, then τ is called the soft discrete topology on X and (X, τ, A) is said to be the soft discrete space. Also for any $(F, A) \in SS(X, A)$, by $\overline{(F, A)}$ we mean the closure of (F, A) in (X, τ, A).

Definition 2.7. let (X, τ_X) be a topological space and (Y, τ_Y, B) be a soft topological space. A function $f : X \to Y$ is continuous at the point $x \in X$ if for every soft open nhood (G, B) of $f(x)$ in (Y, τ_Y, B), there exists an open nhood V of x in X such that $f(V)\widetilde{\subset}(G, B)$. If f is continuous at every point of X, then we say that f is continuous.

Theorem 2.8. *Let $f : (X, \tau) \to (Y, \tau_Y, B)$. Then the function f is continuous if and only if for each soft open set $(G, B) \in SS(Y, B)$, $f^{-1}(G, B)$ is open in X.*

Proof. Let $(G, B) \in SS(Y, B)$ be a soft open set and let $x \in f^{-1}(G, B)$. Then $f(x) \in (G, B)$. Since f is continuous at x, there exists an open set $V \subseteq X$, $x \in V$ such that $f(V)\widetilde{\subset}(G, B)$. So $x \in V \subseteq f^{-1}(G, B)$.
Conversely, let $x \in X$, $(G, B) \in SS(Y, B)$ be a soft open set containing $f(x)$. Then $x \in f^{-1}(G, B)$ which is open by assumption. So there exists an open set $V \subseteq X$ such that $x \in V \subseteq f^{-1}(G, B)$. This implies that $f(x) \in f(V)\widetilde{\subset}(G, B)$. Hence f is continuous at x. Since x is arbitrary, f is continuous. □

Definition 2.9. [8] Let (X, τ, A) be a soft topological space over X, (G, A) be a soft closed set and $x \in X$ such that $x \notin (G, A)$. If there exist soft open sets (F_1, A) and (F_2, A) such that $x \in (F_1, A)$, $(G, A) \sqsubseteq (F_2, A)$ and $(F_1, A) \sqcap (F_2, A) = 0_A$, then (X, τ, A) is called a soft regular space.

Theorem 2.10. [8] A soft topological space (X, τ, A) is soft regular if and only if for every $x \in X$ and every soft open set (F,A) of x, there is a soft open set (G,A) of x such that $x \in (G, A) \sqsubseteq \overline{(G, A)} \sqsubseteq (F, A)$.

Definition 2.11. [9] let (X, τ, A) be a soft topological space. A soft filter on(X, τ, A) is a non empty set $\mathcal{U} \subseteq SS(X, A)$ such that :

1. If $(G, A), (H, A) \in \mathcal{U}$, then $(G, A) \sqcap (H, A) \in \mathcal{U}$.
2. If $(G, A) \in \mathcal{U}$ and $(G, A) \sqsubseteq (H, A) \in SS(X, A)$, then $(H, A) \in \mathcal{U}$.
3. $0_A \notin \mathcal{U}$.

A soft filter on (X, τ, A) is called a soft ultrafilter if it is not properly contained in any other soft filter.
Note that if $\mathcal{U}$ and $\mathcal{V}$ are two soft ultrafilters on (X, τ, A), then $\mathcal{U} = \mathcal{V}$ iff $\mathcal{U} \subseteq \mathcal{V}$.

Theorem 2.12. [6] *Let $SS(X, A)$, $SS(Y, B)$ be the families of all soft sets on X and Y, respectively and φ_{fs} be a soft mapping from $SS(X, A)$ to $SS(Y, B)$.*

1. *If $\mathcal{U}$ is a soft filter on X, then $\varphi_{fs}(\mathcal{U}) = \{(G, B) : \varphi_{fs}^{-1}(G, B) \in \mathcal{U}\}$ is a soft filter on Y.*
2. *If $\mathcal{U}$ is a soft ultrafilter on X, then $\varphi_{fs}(\mathcal{U}) = \{(G, B) : \varphi_{fs}^{-1}(G, B) \in \mathcal{U}\}$ is a soft ultrafilter on Y.*

Definition 2.13. [6] Let (X,τ,A) be a soft topological space and $\mathcal{U}$ be a soft ultrafilter on X. $\mathcal{U}$ is said to be a soft compact if it contains some (F,A) such that $\overline{(F,A)}$ is a soft compact.

Theorem 2.14. [6] *Let (X,τ,A) be a soft Hausdorff space and $\mathcal{U}$ be a soft compact ultrafilter on X. Then $\sqcap\{\overline{(F,A)} : (F,A)\in\mathcal{U}\}$ is a singeltion soft set.*

3 Basic Results

Definition 3.1. Let (X,τ,A) be a soft topological space, then

(a) $\mathcal{B}(X,\tau,A)=\{\mathcal{U} : \mathcal{U}\ is\ a\ soft\ ultrafilter\ on\ (\text{X},\tau,A)\}$.

(b) Given $(G,A)\in SS(X,A)$, $\widehat{(G,A)}=\{\mathcal{U}\in\mathcal{B}(X,\tau,A) : (G,A)\in\mathcal{U}\}$.

Lemma 3.2. *let $\mathcal{U}$ be a soft filter on (X,τ,A) and let $(F,A)\in SS(X,A)$. Either*

1. *there is some $(G,A)\in\mathcal{U}$ such that $(G,A)\sqcap(F,A)=0_A$ or*
2. *$\{(C,A)\in SS(X,A) :$ there is some $(H,A)\in\mathcal{U}$ with $(H,A)\sqcap(F,A)\sqsubseteq (C,A)\}$ is a soft filter on (X,τ,A).*

Proof. Let $(F,A)\in SS(X,A)$ and suppose for any $(G,A)\in\mathcal{U}$, $(G,A)\sqcap (F,A)\neq 0_A$. We want to show that

$$\mathcal{V}=\{(C,A)\in SS(X,A) :\ for\ some\ (H,A)\in\mathcal{U},\ (F,A)\sqcap(H,A)\sqsubseteq(C,A)\}$$

is a soft filter on (X,τ,A). To show this we first note that $1_A\in\mathcal{V}$, since $1_A\in\mathcal{U}$ and $1_A\sqcap(F,A)\sqsubseteq 1_A$.
Hence $\mathcal{V}$ is a non empty subset of $SS(X,A)$. Now let $(C_1,A),(C_2,A)\in\mathcal{V}$, and pick $(H_1,A),(H_2,A)\in\mathcal{U}$ with $(H_1,A)\sqcap(F,A)\sqsubseteq(C_1,A)$ and $(F,A)\sqcap(H_2,A)\sqsubseteq(C_2,A)$. So
$[(F,A)\sqcap(H_1,A)]\sqcap[(F,A)\sqcap(H_2,A)]\sqsubseteq(C_1,A)\sqcap(C_2,A)$. Hence,
$\Rightarrow (F,A)\sqcap[(H_1,A)\sqcap(H_2,A)]\sqsubseteq(C_1,A)\sqcap(C_2,A)$. Therefore,
$(C_1,A)\sqcap(C_2,A)\in\mathcal{V}$. Let $(C_1,A)\in\mathcal{V}$ and$(C,A)\sqsubseteq(M,A)\in SS(X,A)$. Then there exists $(H,A)\in\mathcal{U}$ with $[(F,A)\sqcap(H,A)]\sqsubseteq(C,A)\sqsubseteq(M,A)$. Therefore $(M,A)\in\mathcal{V}$.
Assume on the contrary that $0_A\in\mathcal{V}$. So there is some $(H,A)\in\mathcal{U}$, with $(F,A)\sqcap(H,A)\sqsubseteq 0_A$. Therefore, $(F,A)\sqcap(H,A)=0_A$ which is a contradiction. □

In the following we let $\mathcal{P}_f(H))=\{\phi\neq\mathcal{F} : \mathcal{F}\subseteq H,\ and\ \mathcal{F}\ is\ finite\}$ where H is any set.

Theorem 3.3. *Let (X,τ,A) be a soft topological space and let $\mathcal{U}\subseteq SS(X,A)$. Then the following statements are equivalent:*

(a) *$\mathcal{U}$ is a soft ultrafilter on (X, τ, A).*

(b) *$\mathcal{U}$ has the finite intersection property and for each $(G, A) \in SS(X, A) \setminus \mathcal{U}$, there is some $(H, A) \in \mathcal{U}$ such that $(G, A) \sqcap (H, A) = 0_A$.*

(c) *$\mathcal{U}$ is maximal w.r.t finite intersection property, that is ; $\mathcal{U}$ is maximal member of $\{\mathcal{V} \subseteq SS(X, A) : \mathcal{V}$ has the finite intersection property$\}$.*

(d) *$\mathcal{U}$ is a soft filter on (X, τ, A) and for all $\mathcal{F} \in \mathcal{P}_f(SS(X, A))$, if $\sqcup \mathcal{F} \in \mathcal{U}$, then $\mathcal{F} \cap \mathcal{U} \neq \phi$.*

(e) *$\mathcal{U}$ is a soft filter on (X, τ, A) and for all $(G, A) \in SS(X, A)$ either $(G, A) \in \mathcal{U}$ or $(G, A)^c \in \mathcal{U}$.*

Proof.

$(a \Rightarrow b)$ By condition (1) and (3) of definition(2.11), $\mathcal{U}$ has the finite intersection property. Let $(G, A) \in SS(X, A) \setminus \mathcal{U}$ and

$$\mathcal{V} = \{(C, A) \in SS(X, A) : \text{ for some } (H, A) \in \mathcal{U},\ (G, A) \sqcap (H, A) \sqsubseteq (C, A)\}$$

Then $(G, A) \in \mathcal{V}$ so $\mathcal{U} \subsetneq \mathcal{V}$ so $\mathcal{V}$ is not a soft filter on (X, τ, A). Thus by lemma(3.2), there is some $(H, A) \in \mathcal{U}$ such that $(G, A) \sqcap (H, A) = 0_A$.

$(b \Rightarrow c)$ Let $\mathcal{U}$ has the finite intersection proprty, let $\mathcal{U} \subsetneq \mathcal{V} \subseteq SS(X, A)$. Pick $(G, A) \in \mathcal{V} \setminus \mathcal{U}$ and $(H, A) \in \mathcal{U}$ such that $(G, A) \sqcap (H, A) = 0_A$. Then $(G, A), (H, A) \in \mathcal{V}$. So $\mathcal{V}$ does not have the finite intersection property .

$(c \Rightarrow d)$ Assume $\mathcal{U}$ is maximal with respect to the finite intersection property among subsets of $SS(X, A)$. Then one has immediately that $\mathcal{U}$ is a nonempty subset of $SS(X, A)$. Since $\mathcal{U} \cup \{1_A\}$ has finite intersection property and $\mathcal{U} \subseteq \mathcal{U} \cup \{1_A\}$, one has $\mathcal{U} = \mathcal{U} \cup \{1_A\}$.
That is; $1_A \in \mathcal{U}$. Given $(G, A), (F, A) \in \mathcal{U}$, $\mathcal{U} \cup \{(G, A) \sqcap (F, A)\}$ has the finite intersection property. So $(G, A) \sqcap (F, A) \in \mathcal{U}$. Given $(G, A), (F, A)$ with $(G, A) \in \mathcal{U}$ and $(G, A) \sqsubseteq (F, A) \in SS(X, A)$, $\mathcal{U} \cup \{(F, A)\}$ has finite intersection property, since if $(T, A) \in \mathcal{U}$ and $(T, A) \sqcap (F, A) = 0_A$, then $(G, A) \sqcap (T, A) = 0_A$ which is a contradiction. Hence$(F, A) \in \mathcal{U}$. Now let $\mathcal{F} \in \mathcal{P}_f(SS(X, A))$ with $\sqcup \mathcal{F} \in \mathcal{U}$ and suppose that for each $(G, A) \in \mathcal{F}$, $(G, A) \notin \mathcal{U}$. Then given $(G, A) \in \mathcal{F}$, $\mathcal{U} \subsetneq \mathcal{U} \cup \{(G, A)\}$.
So $\mathcal{U} \cup \{G, A\}$ does not have the finite intersection property. So there exist $g_{(G,A)} \in \mathcal{P}_f(\mathcal{U})$ such that $(G, A) \sqcap (\sqcap g_{(G,A)}) = 0_A$. Let $\mathcal{H} = \cup_{(G,A) \in \mathcal{F}}(g_{(G,A)})$. Then $\mathcal{H} \cup \{\sqcup \mathcal{F}\} \subseteq \mathcal{U}$. While $(\sqcup \mathcal{F}) \sqcap (\sqcap \mathcal{H}) = 0_A$ which is a contradiction.

$(d \Rightarrow e)$ let $\mathcal{F} = \{(G, A), 1_A \setminus (G, A)\}$. Then $\sqcup \mathcal{F} = 1_A \in \mathcal{U}$.
Then $\mathcal{F} \cap \mathcal{U} \neq \phi$ (by d). This implies that $(G, A) \in \mathcal{U}$ or $(G, A)^c = 1_A \setminus (G, A) \in \mathcal{U}$.

$(e \Rightarrow a)$ Assume $\mathcal{U}$ is a soft filter on (X,τ,A) and for all $(G,A) \in SS(X,A)$,$(G,A) \in \mathcal{U}$ or $1_A \diagdown (G,A) \in \mathcal{U}$.
Let $\mathcal{V}$ be a soft filter with $\mathcal{U} \subseteq \mathcal{V}$ and suppose that $\mathcal{U} \neq \mathcal{V}$. Pick$(G,A) \in \mathcal{V} \diagdown \mathcal{U}$. Then $1_A \diagdown (G,A) \in \mathcal{U} \subseteq \mathcal{V}$ while $1_A \diagdown (G,A) \sqcap (G,A) = 0_A$ (a contradiction).

□

Proposition 3.4. *Let $x \in X$, let $a_x \in A$ be fixed. Let*

$$e(x) = \{(G,A) : x \in G(a_x)\}$$

Then $e(x)$ is a soft ultrafilter on (X,τ,A) which is called the soft Principal ultrafilter on (X,τ,A).

Theorem 3.5. *[9] Let $\mathcal{A} \subseteq SS(X,A)$ has the soft finite intersection property. Then there is a soft ultrafilter $\mathcal{U}$ on (X,τ,A) such that $\mathcal{A} \subseteq \mathcal{U}$.*

Lemma 3.6. *Let (X,τ,A) be a soft topological space, $(G,A),(F,A) \in SS(X,A)$, then*

(a) $[\widehat{(G,A) \sqcap (F,A)}] = \widehat{(G,A)} \cap \widehat{(F,A)}$.

(b) $\widehat{(G,A) \sqcup (F,A)} = \widehat{(G,A)} \cup \widehat{(F,A)}$.

(c) $\widehat{(G,A)}^c = \mathcal{B}(X,\tau,A) \diagdown \widehat{(G,A)}$.

(d) $\widehat{(G,A)} = \phi$ *iff* $(G,A) = 0_A$.

(e) $\widehat{(G,A)} = \mathcal{B}(X,\tau,A)$ *if and only if* $(G,A) = 1_A$.

Proof. *(a)* Let $\mathcal{U} \in \widehat{(G,A) \sqcap (F,A)}$. Since $(G,A) \sqcap (F,A) \sqsubseteq (F,A)$ and$(G,A) \sqcap (F,A) \sqsubseteq (G,A)$, we get $(G,A),(F,A) \in \mathcal{U}$. Hence $\mathcal{U} \in \widehat{(F,A)}$ and $\mathcal{U} \in \widehat{(G,A)}$ and therefore $\mathcal{U} \in \widehat{(F,A)} \cap \widehat{(G,A)}$.
On the other hand, suppose $\mathcal{U} \in \widehat{(F,A)} \cap \widehat{(G,A)}$. Then $(F,A) \in \mathcal{U}$ and $(G,A) \in \mathcal{U}$. Thus $(F,A) \sqcap (G,A) \in \mathcal{U}$ and so $\mathcal{U} \in \widehat{(F,A) \sqcap (G,A)}$.

(b) $\mathcal{U} \in \widehat{(F,A) \sqcup (G,A)}$
$\Leftrightarrow (F,A) \sqcup (G,A) \in \mathcal{U}$
$\Leftrightarrow I_A \diagdown [(G,A) \sqcup (F,A)] \notin \mathcal{U}$
$\Leftrightarrow [I_A \diagdown (G,A)] \sqcap [I_A \diagdown (G,A)] \notin \mathcal{U}$
$\Leftrightarrow I_A \diagdown (G,A) \notin \mathcal{U}$ or $I_A \diagdown (F,A) \notin \mathcal{U}$
$\Leftrightarrow (G,A) \in \mathcal{U}$ or $(F,A) \in \mathcal{U}$
$\Leftrightarrow \mathcal{U} \in \widehat{(G,A)}$ or $\mathcal{U} \in \widehat{(F,A)}$
$\Leftrightarrow \mathcal{U} \in \widehat{(G,A)} \cup \widehat{(F,A)}$.

(c) $\mathcal{U} \in \widehat{(G,A)}^c$
$\Leftrightarrow (G,A)^c \in \mathcal{U}$
$\Leftrightarrow (G,A) \notin \mathcal{U}$
$\Leftrightarrow \mathcal{U} \notin \widehat{(G,A)} \Leftrightarrow \mathcal{U} \in \mathcal{B}(X,\tau,A)\diagdown\widehat{(G,A)}$

(d) $\widehat{(G,A)} = \phi$
$\Leftrightarrow (G,A) \notin \mathcal{U}$ where $\mathcal{U}$ is any soft ultrafilter in $\mathcal{B}(X,\tau,A)$
$\Leftrightarrow (G,A) = 0_A$.

(e) $\widehat{(G,A)} = \mathcal{B}(X,\tau,A)$
$\Leftrightarrow (G,A) \in \mathcal{U}$ where $\mathcal{U}$ is any soft ultrafilter in $\mathcal{B}(X,\tau,A)$
$\Leftrightarrow (G,A) \in 1_A$.

□

Proposition 3.7. $\mathfrak{B} = \{\widehat{(G,A)} : (G,A) \in SS(X,A)\}$ *is a basis for a topology on* $\mathcal{B}(X,\tau,A)$.

Proof. Let $\mathcal{U} \in \mathcal{B}(X,\tau,A)$, then $\mathcal{U} \neq \phi$. Pick $(G,A) \in \mathcal{U}$, then $\mathcal{U} \in \widehat{(G,A)}$. let $\widehat{(G,A)}, \widehat{(F,A)} \in \mathfrak{B}$ and $\mathcal{U} \in \widehat{(G,A)} \sqcap \widehat{(F,A)}$. By Lemma (3.6), $\widehat{(G,A)} \sqcap \widehat{(F,A)} = \widehat{(G,A) \sqcap (F,A)} \in \mathfrak{B}$. Hence $\mathcal{U} \in \widehat{(G,A) \sqcap (F,A)} \in \mathfrak{B}$. □

The following Theorem describes some of the basic topological properties of $\mathcal{B}(X,\tau,A)$.

Theorem 3.8. *Let* (X,τ,A) *be a soft topological space*

(a) $\mathcal{B}(X,\tau,A)$ *is a compact Hausdorff space .*

(b) *the mapping* $e : X \to \mathcal{B}(X,\tau,A)$ *is injective and* $e[X]$ *is a dense subset of* $\mathcal{B}(X,\tau,A)$.

Proof. (a) Suppose $\mathcal{U}$ and $\mathcal{V}$ are distinct elements of $\mathcal{B}(X,\tau,A)$. If $(G,A) \in \mathcal{U}\diagdown\mathcal{V}$, then $1_A\diagdown(G,A) \in \mathcal{V}$ so by Proposition (3.7), $\widehat{(G,A)}$ and $\widehat{(G,A)}^c$ are disjoint open subsets of $\mathcal{B}(X,\tau,A)$ containing $\mathcal{U}$ and $\mathcal{V}$ respectively. Thus $\mathcal{B}(X,\tau,A)$ is T_2 space.

By lemma (3.6).c, we observe that the sets of the form $\widehat{(G,A)}$ are also a base for the closed sets.

Next, we show $\mathcal{B}(X,\tau,A)$ is compact. So consider a family $\mathcal{A}$ of sets of the form $\widehat{(G,A)}$ with the finite intersection property and show that $\mathcal{A}$ has a nonempty intersection .

Let $\mathcal{B} = \{(G,A) \in SS(X,A) : \widehat{(G,A)} \in \mathcal{A}\}$. If $\mathcal{F} \in \mathcal{P}_f(\mathcal{B})$, then there is some $\mathcal{U} \in \cap_{(G,A)\in\mathcal{F}}\widehat{(G,A)}$.
and so $\sqcap\mathcal{F} \in \mathcal{U}$ and thus $\sqcap\mathcal{F} \neq 0_A$.
That is $\mathcal{B}$ has the soft finite intersection property. So by Theorem (3.5) pick $\mathcal{V} \in \mathcal{B}(X,\tau,A)$ with $\mathcal{B} \subseteq \mathcal{V}$. So for each $\widehat{(G,A)} \in \mathcal{A}, (G,A) \in \mathcal{V}$.
Hence $\mathcal{V} \in \widehat{(G,A)}$ for each $\widehat{(G,A)} \in \mathcal{A}$. Thus $\mathcal{V} \in \cap\mathcal{A}$.

(b) If $x, y \in X$ are distinct, $1_A \diagdown (x, A) \in e(y) \diagdown e(x)$. So $e(y) \neq e(x)$. Hence e is injective.
If $\widehat{(G, A)}$ is a non empty basic open subset of $\mathcal{B}(X, \tau, A)$, then $(G, A) \neq 0_A$. So there exists $a_t \in A$ such that $t \in G(a_t)$. Therefore $(G, A) \in e(t)$ and consequently, $e(t) \in \widehat{(G, A)}$. Thus $e(t) \in \widehat{(G, A)} \cap e[X]$.

4 Main Results

In this section we show that $\mathcal{B}(X, \tau, A)$ is the soft Stone-Čech Compactification of the soft discrete topological space (X, τ, A). We remind the reader that we are assuming that all hypothesized topological spaces and soft topological spaces are Hausdorff.

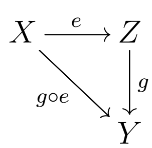
Definition 4.1. Let X be a soft discrete topological space. A soft Stone-Čech Compactification of (X, τ, A) is a pair (e, Z) such that:

(a) Z is a compact space.

(b) e is an embedding of (X, τ, A) into Z.

(c) $e[X]$ is dense in Z and

(d) giving any soft compact space Y and any continuous soft mapping $\varphi_{fS} : (X, \tau, A) \to (Y, \tau_Y, B)$, there exists a continuous $g : Z \to Y$ such that $g \circ e = f$.
(That is the following diagram commutes).



Proposition 4.2. *Let (Y, τ, B) be a soft compact Hausdorff space. Then (Y, τ, B) is soft regular space.*

Proof. Let $x \in (V, B)$ where (V, B) is open set. We want to show that there exists a soft open set (U, B) in (Y, τ, B) such that $x \in (U, B) \sqsubseteq \overline{(U, B)} \sqsubseteq (V, B)$
For any $y \in 1_B \diagdown (V, B)$, we have $x \neq y$, so there exists (W_y, B) a nhood of y and (U_y, B) a nhood of x such that $(W_y, B) \sqcap (U_y, B) = 0_B \to (*)$
Now $\{(W_y, B) : y \in 1_B \diagdown (V, B)\}$ is a cover of $1_B \diagdown (V, B)$ which is a soft compact (soft closed in soft Hausdorff space is soft compact). So let $(W_{y_1}, B), ..., (W_{y_n}, B)$ be a finite cover of $1_B \diagdown (V, B)$. Let $(U, B) = \sqcap_{i=1}^{n} (Uy_i, B)$ which is a nhood of x. Now from $(*)$ we get $(U_{y_i}, B) \sqsubseteq 1_B \diagdown (W_{y_i}, B)$.

Also $1_B \diagdown (V,B) \sqsubseteq \sqcup_{i=1}^{n} \overline{(W_{y_i},B)} \Rightarrow \sqcap_{i=1}^{n} I_B \diagdown (W_{y_i},B) \sqsubseteq (V,B)$.
So $\overline{(U,B)} = \overline{\sqcap_{i=1}^{n}(U_{y_i},B)} \sqsubseteq \sqcap_{i=1}^{n} \overline{(U_{y_i},B)} \sqsubseteq \sqcap_{i=1}^{n}[1_B \diagdown (W_{y_i},B)] \sqsubseteq (V,B)$.
□

Proposition 4.3. *Let $\varphi_{fs} : SS(X,A) \to SS(Y,B)$. Then $\varphi_{fS}(x,A) = (F,B)$ is a soft closed set in (Y,τ,B)*

Proof. Note that
F(b) =

$$\begin{cases} \{f(x)\} & , s^{-1}(\{b\}) \neq \emptyset\,, \\ \emptyset & , s^{-1}(\{b\}) = \emptyset\,. \end{cases}$$

for all $b \in B$. We want to show $\varphi_{fS}(x,A) = (F,B)$ is closed in (Y,τ,B). Let $y \in 1_B \diagdown (F,B)$. So $y \notin F(b)$ for all $b \in B$. This implies that $y \notin \{f(x)\}$. So $y \neq f(x)$. Since Y is T_2 space , we can pick two soft open sets $(G,B),(T,B)$ such that $y \in (G,B)$ and $f(x) \in (T,B)$, $(G,B) \sqcap (T,B) = 0_B$. This implies that $f(x) \notin (G,B)$. Now $f(x) \in (T,B)$ and so $f(x) \in T(b)$ for all $b \in B$. Hence, $F(b) \subseteq T(b)$ for all $b \in B$, and so, $(F,B) \sqsubseteq (T,B)$. Therefore, $1_B \setminus (T,B) \sqsubseteq 1_B \setminus (F,B)$. So $y \in (G,B) \sqsubseteq I_B \setminus (T,B) \sqsubseteq 1_B \setminus (F,B)$. Hence $1_B \setminus (F,B)$ is soft open set. Thus (F,B) is a soft closed set . □

Proposition 4.4. *Let (X,τ,A) be a soft topological space and (Y,τ_Y,B) be a soft compact topological space, $\varphi_{fS} : SS(X,A) \to SS(Y,B)$. For each $\mathbf{P} \in \mathcal{B}(X,\tau,A)$, let $\mathcal{A}_{\mathbf{P}} = \{\overline{(\varphi_{fS}(G,A))} : (G,A) \in \mathbf{P}\}$. Then, there exists $y \in Y$ such that $(y,B) \sqsubseteq \sqcap \mathcal{A}_{\mathbf{P}}$.*

Proof. For each $\mathbf{P} \in \mathcal{B}(X,\tau,A)$, let $\mathcal{S} = \{\varphi_{fS}(G,A)) : (G,A) \in \mathbf{P}\}$. Now for each $(G,A) \in \mathbf{P}$, we have $(G,A) \sqsubseteq \varphi_{fS}^{-1}(\varphi_{fS}(G,A))$. Since $\mathbf{P}$ is a soft ultrafilter, $\varphi_{fS}^{-1}(\varphi_{fS}(G,A)) \in \mathbf{P}$. Hence $\mathcal{S} \subseteq \varphi_{fS}(\mathbf{P}) = \{(G,B) \in SS(Y,B) : \varphi_{fS}^{-1}(G,B) \in \mathbf{P}\}$. But by Theorem (2.12), $\varphi_{fS}(\mathbf{P})$ is a soft ultrafilter on Y. Since (Y,τ_Y,B) is a soft compact space, we have for each $(G,A) \in \mathbf{P}$, $\overline{(\varphi_{fS}(G,A))}$ is a soft compact set. So $\varphi_{fS}(\mathbf{P})$ is a soft compact ultrafilter on Y. Hence by Theorem (2.14), $\sqcap\{\overline{(F,B)} : (F,B) \in \varphi_{fS}(\mathbf{P})\}$ is a singleton soft set, say,
$\sqcap\{\overline{(F,B)} : (F,B) \in \varphi_{fS}(\mathbf{P})\} = (y,B)$ for some $y \in Y$. Thus
$(y,B) = \sqcap\{\overline{(F,B)} : (F,B) \in \varphi_{fS}(\mathbf{P}\} \sqsubseteq \sqcap \mathcal{A}_{\mathbf{P}}$. □

Theorem 4.5. *Let (X,τ,A) be a soft discrete topological space. Then $(e,\mathcal{B}(X,\tau,A))$ is the soft Stone-Čech Compactification of (X,τ,A).*

Proof. Condition (a), (b) and (c) of definition (4.1) hold by Theorem (3.8). It remains for us to verify condition(d). Let Y be a soft compact space and $\varphi_{fS} : SS(X,A) \to SS(Y,B)$. For each $\mathcal{U} \in \mathcal{B}(X,\tau,A)$, let $\mathcal{A}_{\mathcal{U}} = \{\overline{(\varphi_{fS}(G,A))} : (G,A) \in \mathcal{U}\}$. By Proposition(4.4), choose $g(\mathcal{U}) \in \sqcap \mathcal{A}_{\mathcal{U}}$. Then we have the following diagram:

$$\begin{array}{ccc} (X,\tau,A) & \xrightarrow{e} & \mathcal{B}(X,\tau,A) \\ & \searrow^{g\circ e} & \downarrow g \\ & & (Y,\tau_Y,B) \end{array}$$

We need to show that the diagram commutes and that g is continuous. For the first assertion, let $x \in X$, $(x,A) \in e(x)$. So by Proposition (4.3), $g(e(x)) \in \overline{(\varphi_{fS}(x,A))} = \varphi_{fS}(x,A)$. Let $\varphi_{fS}(x,A) = (F,B) \in SS(Y,B)$. So $g(e(x)) \in F(b)$ *for all* $b \in B$. So $g(e(x)) \in \cup\{f(\{x\})\} = \cup\{f(x)\} = \{f(x)\}$. So $g \circ e = f$ as required.

To see g is continuous, let (G,B) be a soft nhood of $g(\mathcal{U})$ in Y. By Proposition (4.2), Y is regular. So by Proposition (2.10), pick a soft nhood (H,B) of $g(\mathcal{U})$ with $\overline{(H,B)} \sqsubseteq (G,B)$ and let $(F,A) = \varphi_{fS}^{-1}(H,B)$. We claim that $(F,A) \in \mathcal{U}$. Suppose instead that $I_A \setminus (F,A) \in \mathcal{U}$. Then $g(\mathcal{U}) \in \overline{\varphi_{fS}(1_A \setminus (F,A))}$ and (H,B) is a nhood of $g(\mathcal{U})$. So
$(H,B) \sqcap \varphi_{fS}(1_A \setminus (F,A)) \neq 0_B$. But $(H,B) \sqcap \varphi_{fS}(1_A \setminus (F,A))$
$= (H,B) \sqcap \varphi_{fS}[(\varphi_{fS}^{-1}(H,B))^c]$
$= (H,B) \sqcap \varphi_{fS}[\varphi_{fS}^{-1}((H,B)^c)] \sqsubseteq (H,B) \sqcap (H,B)^c = 0_B$. We have a contradiction. Thus $\widehat{(F,A)}$ is a nhood of $\mathcal{U}$. We claim $g(\widehat{(F,A)}) \widetilde{\subset} (G,B)$. So let $q \in \widehat{(F,A)}$, so $(F,A) \in q$. Hence $q \in \overline{(\varphi_{fS}(F,A))} \sqsubseteq \overline{(H,B)} \sqsubseteq (G,B)$. Hence $g(\widehat{(F,A)}) \widetilde{\subset} (G,B)$.

□

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