**Abstract**

In this paper, we introduce the notion of \( \phi \)-complement of intuitionistic fuzzy graph structure \( \bar{G} = (A, B_1, B_2, ..., B_k) \) where \( \phi \) is a permutation on \( \{B_1, B_2, ..., B_k\} \) and obtain some results. We also define some elementary definitions like self complementary, totally self complementary, strong self complementary intuitionistic fuzzy graph structure and study their properties.

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**Keywords:** Intuitionistic fuzzy graph structure, \( \phi \)-complement, self complementary, totally self complementary, strong self complementary

**I. Introduction**

The concept of intuitionistic fuzzy graph structure \( \bar{G} = (A, B_1, B_2, ..., B_k) \) is introduced and studied by the authors in [6]. Zadeh [7] in 1965 introduced the notion of fuzzy sets. Then Rosenfeld [8] gave the idea of fuzzy relations and fuzzy graph in 1975. Atanassov [5] proposed the first definition of intuitionistic fuzzy graph. E. Sampatkumar in [1] has generalized the notion of graph \( G = (V, E) \) to graph structure \( G = (V, R_1, R_2, ..., R_k) \) where \( R_1, R_2, ..., R_k \) are relations on \( V \) which are mutually disjoint and each \( R_i \), \( i = 1, 2, 3, ..., k \) is symmetric and irreflexive. T. Dinesh and T. V. Ramakrishnan [2] introduced the notion of fuzzy...
graph structure and studied their properties. In this paper, we will introduce and study \( \bar{\phi} \) - complement of intuitionistic fuzzy graph structure \( \bar{G} \).

2. Preliminaries

In this section, we review some definitions that are necessary to understand the content of this paper. These are mainly taken from [2], [7], [8], [9], [10] and [11].

**Definition (2.1) [11]:** \( G = (V, R_1, R_2,...,R_k) \) is a graph structure if \( V \) is a non empty set and \( R_1, R_2,...,R_k \) are relations on \( V \) which are mutually disjoint such that each \( R_i, i=1, 2, 3, \ldots, k \), is symmetric and irreflexive.

**Definition (2.2) [9, 10]:** An intuitionistic fuzzy graph is of the form \( G = (V, E) \) where

(i) \( V = \{v_1, v_2, \ldots, v_n\} \) such that \( \mu_1 : V \to [0,1] \) and \( \gamma_1 : V \to [0,1] \) denote the degree of membership and non membership of the element \( v_i \in V \), respectively and \( 0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1 \), for every \( v_i \in V \), \( i = 1, 2, \ldots, n \),

(ii) \( E \subseteq V \times V \) where \( \mu_2 : V \times V \to [0,1] \) and \( \gamma_2 : V \times V \to [0,1] \) are such that

\[
\mu_2(v_i, v_j) \leq \min\{\mu_1(v_i), \mu_1(v_j)\} \quad \text{and} \quad \gamma_2(v_i, v_j) \leq \max\{\gamma_1(v_i), \gamma_1(v_j)\}
\]

and \( 0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1 \), for every \( (v_i, v_j) \in E \), \( i, j = 1, 2, \ldots, n \).

**Definition (2.3) [8]:** Let \( G = (V, R_1, R_2, \ldots, R_k) \) be a graph structure and \( A, B_1, B_2, \ldots, B_k \) be intuitionistic fuzzy subsets (IFSs) of \( V \), \( R_1, R_2, \ldots, R_k \) respectively such that

\[
\mu_{A_i}(u,v) \leq \mu_A(u) \wedge \mu_A(v) \quad \text{and} \quad \nu_{A_i}(u,v) \leq \nu_A(u) \vee \nu_A(v) \quad \forall u,v \in V \text{ and } i = 1, 2, \ldots, k.
\]

Then \( \bar{G} = (A, B_1, B_2, \ldots, B_k) \) is an IFGS of \( G \).

**Example (2.4) [8]:** Consider the graph structure \( G = (V, R_1, R_2, R_3) \), where \( V = \{u_0, u_1, u_2, u_3, u_4\} \) and \( R_1 = \{(u_0, u_1), (u_0, u_2), (u_3, u_4)\}, R_2 = \{(u_1, u_2), (u_2, u_4)\}, \text{ and } R_3 = \{(u_2, u_3), (u_0, u_4)\} \) are the relations on \( V \). Let \( A = \{< u_0, 0.5,0.4 >, < u_1, 0.6,0.3 >, < u_2, 0.2,0.6 >, < u_3,0.1,0.8 >, < u_4, 0.4,0.3 >\} \) be an IFS on \( V \) and \( B_1 = \{(u_0, u_1), 0.5,0.3 >, < (u_0, u_2), 0.1,0.3 >, < (u_3, u_4), 0.1,0.2 >\}, B_2 = \{< (u_1, u_2), 0.2,0.1 >, < (u_2, u_4), 0.1,0.2 >\}, B_3 = \{< (u_2, u_3), 0.1,0.5 >, < (u_0, u_4), 0.3,0.2 >\} \) are intuitionistic fuzzy relations on \( V \).

Here \( \mu_{B_i}(u,v) \leq \mu_A(u) \wedge \mu_A(v) \) and \( \nu_{B_i}(u,v) \leq \nu_A(u) \vee \nu_A(v) \quad \forall u, v \in V \) and \( i = 1, 2, 3 \).

\( \therefore \) \( \bar{G} \) is an intuitionistic fuzzy graph structure.

**Definition (2.5) [7]:** The complement of a fuzzy graph \( G = (\sigma, \mu) \) is a fuzzy graph \( \bar{G} = (\bar{\sigma}, \bar{\mu}) \) where \( \bar{\sigma} = \sigma \) and \( \bar{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v), \forall u, v \in V \).
\textbf{Definition (2.6) [7]:} Consider the fuzzy graphs \( G_1 = (\sigma_1, \mu_1) \) and \( G_2 = (\sigma_2, \mu_2) \) with \( \sigma_1 = V_1 \) and \( \sigma_2 = V_2 \). An isomorphism between \( G_1 = (\sigma_1, \mu_1) \) and \( G_2 \) is a one to one function \( h \) from \( V_1 \) onto \( V_2 \) that satisfies \( \sigma_1(u) = \sigma_2(h(u)) \) and \( \mu_1(u, v) = \mu_2(h(u), h(v)) \), \( \forall \, u, v \in V \).

\section{3. \( \phi \)-Complement of Intuitionistic Fuzzy Graph Structure}

\textbf{Definition (3.1):} Let \( \tilde{G} = (A, B_1, B_2,...,B_k) \) be an intuitionistic fuzzy graph structure of graph structure \( G = (V, R_1, R_2,...,R_k) \). Let \( \phi \) denotes the permutation on the set \{\( R_1, R_2,...,R_k \)\} and also the corresponding permutation on \{\( B_1, B_2,...,B_k \)\} i.e., \( \phi(B_i) = B_i^{\phi} = B_j \) (i.e., \( \phi \mu_{R_i} = \mu_{B_{j_i}} \) and \( \phi \nu_{R_i} = \nu_{B_{j_i}} \)) if and only if \( \phi(R_i) = R_j \), then the \( \phi \) - complement of \( \tilde{G} \) is denoted \( \tilde{G}^{\phi} \) and is given by \( \tilde{G}^{\phi} = (A, B_1^{\phi}, B_2^{\phi},...,B_k^{\phi}) \) where for each \( i = 1,2,3,...,k \), we have

\[ \mu_{B_i}^{\phi}(uv) = \mu_A(u) \land \mu_A(v) - \sum_{j \neq i} (\phi \mu_{B_j})(uv) \]  

\[ \nu_{B_i}^{\phi}(uv) = \nu_A(u) \lor \nu_A(v) - \sum_{j \neq i} (\phi \nu_{B_j})(uv). \]

\textbf{Example (3.2):} Consider an intuitionistic fuzzy graph structure \( \tilde{G} = (A, B_1, B_2) \) such that \( V = \{u_0, u_1, u_2, u_3, u_4, u_5\} \). Let \( R_1 = \{(u_0, u_1), (u_0, u_2), (u_3, u_4)\}, R_2 = \{(u_1, u_2), (u_4, u_5)\}, A = \{< u_0, 0.8,0.2>, < u_1, 0.9,0.1>, < u_2, 0.6,0.3>, < u_3, 0.5,0.4>, < u_4, 0.6,0.1>, < u_5, 0.7,0.2>\}, B_1 = \{(u_0, u_1), 0.8,0.1>, <(u_0, u_2), 0.5,0.3>, <(u_3, u_4), 0.4,0.2>\}, B_2 = \{(u_1, u_2), 0.6,0.2>, <(u_4, u_5),0.5,0.1>\}.

Let \( \phi \) be a permutation on the set \{\( B_1, B_2 \)\} defined by \( \phi(B_1) = B_2 \) and \( \phi(B_2) = B_1 \), then

\[ \mu_{B_1}^{\phi}(u_0,u_1) = 0; \quad \nu_{B_1}^{\phi}(u_0,u_1) = 0.1 \quad \text{and} \quad \mu_{B_2}^{\phi}(u_1,u_2) = 0.1; \quad \nu_{B_2}^{\phi}(u_1,u_2) = 0, \mu_{B_1}^{\phi}(u_0,u_4) = 0.1; \quad \nu_{B_1}^{\phi}(u_0,u_4) = 0.2 \quad \text{and} \quad \mu_{B_2}^{\phi}(u_1,u_2) = 0; \quad \nu_{B_2}^{\phi}(u_1,u_2) = 0.1, \mu_{B_1}^{\phi}(u_0,u_5) = 0.1; \quad \nu_{B_1}^{\phi}(u_0,u_5) = 0.1.

\textbf{Remark (3.3):} Here in the above example, we can check that \( (\tilde{G}^{\phi})^{\phi} = \tilde{G} \) i.e., the \( \phi \) - complement of \( \phi \) - complement of \( \tilde{G} \) is \( \tilde{G} \).

\textbf{Theorem (3.4):} If \( \phi \) is a cyclic permutation on \{\( B_1, B_2,...,B_k \)\} of order \( m (1 \leq m \leq k) \), then \( \tilde{G}^{\phi^m} = \tilde{G} \).
Proof. Since \( \phi^n \) = identity permutation. Hence, \( \tilde{G}^{\phi^n} = (A, B_1^{\phi^n}, B_2^{\phi^n}, \ldots, B_k^{\phi^n}) = (A, B_1, B_2, \ldots, B_k) = \tilde{G}.

**Proposition (3.5):** Let \( \tilde{G} = (A, B_1, B_2, \ldots, B_k) \) be an intuitionistic fuzzy graph structure of graph structure \( G = (V, R_1, R_2, \ldots, R_k) \) and let \( \phi \) and \( \psi \) be two permutations on \( \{B_1, B_2, \ldots, B_k\} \), then
\[
(\tilde{G}^{\phi})^{\psi} = \tilde{G}^{(\phi \circ \psi)}.
\]
In particular \( \tilde{G}^{(\phi \circ \psi)} = \tilde{G} \) if and only if \( \phi \) and \( \psi \) are inverse of each other.

**Proof:** Straight forward.

**Definition (3.6):** Let \( \tilde{G} = (A, B_1, B_2, \ldots, B_k) \) and \( \tilde{G}' = (A', B'_1, B'_2, \ldots, B'_k) \) be two IFGSs on graph structures \( G = (V, R_1, R_2, \ldots, R_k) \) and \( G' = (V', R'_1, R'_2, \ldots, R'_k) \) respectively, then \( \tilde{G} \) is isomorphic to \( \tilde{G}' \) if there exists a bijective mapping \( f : V \rightarrow V' \) and a permutation \( \phi \) on \( \{B_1, B_2, \ldots, B_k\} \) such that \( \phi(B_i) = B'_i \) and

1. \( \forall u \in V, \ \mu_A(u) = \mu_{A'}(f(u)) \) and \( \nu_A(u) = \nu_{A'}(f(u)) \)
2. \( \forall (uv) \in R_i, \ \mu_{R_i}(uv) = \mu_{R'_i}(f(u)f(v)) \) and \( \nu_{R_i}(uv) = \nu_{R'_i}(f(u)f(v)) \).

In particular, if \( V = V', A = A' \) and \( B_i = B'_i \) for all \( i = 1, 2, 3, \ldots, k \), then the above two IFGSs \( \tilde{G} \) and \( \tilde{G}' \) are identical.

**Remark (3.7):** Note that identical IFGSs are always isomorphic, but converse is not true. (In example (3.10), IFGSs \( \tilde{G} \) and \( \tilde{G}' \) are isomorphic but not identical).

**Example (3.8):** Consider the two intuitionistic fuzzy graph structures \( \tilde{G} = (A, B_1, B_2) \) and \( \tilde{G}' = (A', B'_1, B'_2) \) such that \( V = \{u_0, u_1, u_2, u_3, u_4\} \) and \( V' = \{u'_0, u'_1, u'_2, u'_3, u'_4\} \). Let \( A = \{(u_0, 0.8, 0.2), (u_1, 0.9, 0.1), (u_2, 0.6, 0.3), (u_3, 0.5, 0.4), (u_4, 0.6, 0.1)\} \) be IFS on \( V \) and \( A' = \{(u'_0, 0.7, 0.2), (u'_1, 0.8, 0.2), (u'_2, 0.9, 0.1), (u'_3, 0.6, 0.3), (u'_4, 0.5, 0.4), (u'_5, 0.6, 0.1)\} \) be IFS on \( V' \). Let \( B_1 = \{(u_0, u_1), (u_0, u_2), (u_0, u_3), (u_0, u_4), (u_1, u_2), (u_1, u_3), (u_1, u_4), (u_2, u_3), (u_2, u_4), (u_3, u_4)\} \) be IFS on \( V \) and \( B'_1 = \{(u'_0, u'_1), (u'_0, u'_2), (u'_0, u'_3), (u'_1, u'_2), (u'_1, u'_3), (u'_2, u'_3), (u'_3, u'_4), (u'_4, u'_5)\} \) be IFS on \( V' \) as shown in Fig 1 and Fig 2 respectively.
Then it can be easily verified that $\tilde{G} = (A, B_1, B_2)$ and $\tilde{G}' = (A', B_1', B_2')$ are IFGSs. Let $\phi$ be a permutation on $\{B_1, B_2\}$ such that $\phi(B_i) = B_i'$ and $h : V \rightarrow V'$ be a map defined by

$$h(u_k) = \begin{cases} u_{k+1}' & \text{if } k = 0, 1, 2, 3, 4 \\ u_k' & \text{if } k = 5. \end{cases}$$

Then it can be easily checked that

(1) $\mu_A(u_k) = \mu_A'(h(u_k))$ and $\nu_A(u_k) = \nu_A'(h(u_k)) \forall u_k \in V$

(2) $\mu_{B_1}(uv) = \mu_{B_1'}(h(u)h(v))$ and $\nu_{B_1}(uv) = \nu_{B_1'}(h(u)h(v)), \forall (u, v) \in V \times V$, and $i = 1, 2$.

Hence $\tilde{G} \cong \tilde{G}'$.

**Definition (3.9):** Consider an IFGS $G$ of graph structure $G$ and $\phi$ is a permutation on the set $\{B_1, B_2, ..., B_k\}$ then $\tilde{G}$ is $\phi$-self complementary if $\tilde{G}$ is isomorphic to $\tilde{G}^\phi$ and $\tilde{G}$ is strong $\phi$-self complementary if $\tilde{G}$ is identical to $\tilde{G}^\phi$.

**Example (3.10):** Consider an IFGS $\tilde{G} = (A, B_1, B_2)$ such that $V = \{u_1, u_2, u_3, u_4\}$. Let $A = \{(u_1, 0.6, 0.1), (u_2, 0.8, 0.2), (u_3, 0.6, 0.1), (u_4, 0.8, 0.2)\}$ and $B_1 = \{(u_1, u_2), (u_4, u_1), (u_3, 0.3, 0.1), (u_4, u_2)\}$, $B_2 = \{(u_2, u_3), (u_3, 0.1), (u_4, u_3), (u_3, 0.1)\}$.

Let $\phi$ be a permutation on the set $\{B_1, B_2\}$ defined by $\phi(B_1) = B_2$ and $\phi(B_2) = B_1$, then

$\mu_{B_1}(u_1u_2) = 0.3$ ; $\nu_{B_1}(u_1u_2) = 0.1$, and $\mu_{B_1}(u_4u_1) = 0.3$ ; $\nu_{B_1}(u_4u_1) = 0.1$

and $\mu_{B_2}(u_1u_3) = 0.3$ ; $\nu_{B_2}(u_1u_3) = 0.1$ and $\mu_{B_2}(u_4u_1) = 0.3$ ; $\nu_{B_2}(u_4u_1) = 0.1$.

Let there exists a one-to-one and onto map $h : V \rightarrow V$ defined by $h(u_1) = u_3$; $h(u_2) = u_4$; $h(u_3) = u_1$ and $h(u_4) = u_2$. Then

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\[ \mu_A(h(u_1)) = \mu_A(u_3) = 0.6 = \mu_A(u_1) \quad \text{and} \quad \nu_A(h(u_1)) = \nu_A(u_3) = 0.1 = \nu_A(u_1); \]
\[ \mu_A(h(u_2)) = \mu_A(u_4) = 0.8 = \mu_A(u_2) \quad \text{and} \quad \nu_A(h(u_2)) = \nu_A(u_4) = 0.2 = \nu_A(u_2); \]
\[ \mu_A(h(u_3)) = \mu_A(u_5) = 0.6 = \mu_A(u_3) \quad \text{and} \quad \nu_A(h(u_3)) = \nu_A(u_5) = 0.1 = \nu_A(u_3); \]
\[ \mu_A(h(u_4)) = \mu_A(u_6) = 0.8 = \mu_A(u_4) \quad \text{and} \quad \nu_A(h(u_4)) = \nu_A(u_6) = 0.2 = \nu_A(u_4). \]

\[ \mu_{h^\phi}(h(u_1)) = \mu_{h^\phi}(u_3) = 0.3 = \mu_{h^\phi}(u_5) \quad \text{and} \quad \nu_{h^\phi}(h(u_1)) = \nu_{h^\phi}(u_3) = 0.1 = \nu_{h^\phi}(u_5); \]
\[ \mu_{h^\phi}(h(u_2)) = \mu_{h^\phi}(u_4) = 0.3 = \mu_{h^\phi}(u_6) \quad \text{and} \quad \nu_{h^\phi}(h(u_2)) = \nu_{h^\phi}(u_4) = 0.1 = \nu_{h^\phi}(u_6); \]
\[ \mu_{h^\phi}(h(u_3)) = \mu_{h^\phi}(u_5) = 0.3 = \mu_{h^\phi}(u_6) \quad \text{and} \quad \nu_{h^\phi}(h(u_3)) = \nu_{h^\phi}(u_5) = 0.1 = \nu_{h^\phi}(u_6); \]
\[ \mu_{h^\phi}(h(u_4)) = \mu_{h^\phi}(u_6) = 0.3 = \mu_{h^\phi}(u_5) \quad \text{and} \quad \nu_{h^\phi}(h(u_4)) = \nu_{h^\phi}(u_6) = 0.1 = \nu_{h^\phi}(u_5). \]

\[ \therefore \, \bar{G} \text{ is } \phi\text{-self complementary.} \]

**Definition (3.11):** Consider an IFGS \( \bar{G} \) of graph structure \( G \) then

1. \( \bar{G} \) is self complementary (SC) if \( \bar{G} \) is isomorphic to \( \bar{G}^\phi \) for some permutation \( \phi \).
2. \( \bar{G} \) is strong self complementary (SSC) if \( \bar{G} \) is identical to \( \bar{G}^\phi \) for some permutation \( \phi \) other than the identity permutation.
3. \( \bar{G} \) is totally self complementary (TSC) if \( \bar{G} \) is isomorphic to \( \bar{G}^\phi \) for every permutation \( \phi \).
4. \( \bar{G} \) is totally strong self complementary (TSSC) if \( \bar{G} \) is identical to \( \bar{G}^\phi \) for every permutation \( \phi \), where \( \phi \) is a permutation on the set \( \{B_1, B_2, \ldots, B_k\} \).

**Remark (3.12):** Totally self complementarity \( \Rightarrow \) self complementarity and totally strong self complementarity \( \Rightarrow \) strong self complementarity, but converse is not true, as is obvious from the following example (3.14).

**Remark (3.13):**

\[
\begin{array}{c c c c c c}
\text{TSSC} & \longrightarrow & \text{SSC} \\
\downarrow & & & & \downarrow \\
\text{TSC} & \longrightarrow & \text{SC}
\end{array}
\]

**Example (3.14):** Consider an intuitionistic fuzzy graph structure \( \bar{G} = (A, B_1, B_2) \) such that \( V = \{ u_1, u_2, u_3, u_4 \} \). Let \( R_1 = \{(u_1, u_2), (u_4, u_1)\}, R_2 = \{(u_2, u_3), (u_4, u_3)\}, A = \{< u_1, 0.4, 0.2 >, < u_2, 0.5, 0.1 >, < u_3, 0.4, 0.2 >, < u_4, 0.5, 0.1 >\}, B_1 = \{< (u_1, u_2), 0.2, 0.1 >, < (u_4, u_1), 0.2, 0.1 >\}, B_2 = \{< (u_2, u_3), 0.2, 0.1 >, < (u_4, u_3), 0.2, 0.1 >\}. \) Let \( \phi \) be a permutation on the set \( \{B_1, B_2\} \) defined by \( \phi(B_1) = B_2 \) and \( \phi(B_2) = B_1 \), then
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\[
\mu_{B_i}(u, u_2) = 0.2 \; ; \; v_{B_i}(u, u_2) = 0.1 \; , \; \text{and} \; \mu_{B_i}(u, u_4) = 0.2 \; ; \; v_{B_i}(u, u_4) = 0.1 \\
\mu_{B_i}(u_2, u_3) = 0.2 \; ; \; v_{B_i}(u_2, u_3) = 0.1 \; \text{and} \; \mu_{B_i}(u_4, u_3) = 0.2 \; ; \; v_{B_i}(u_4, u_3) = 0.1.
\]

Let there exists a one - one and onto map \( h: V \to V \) defined by
\[
h(u_1) = u_5 \; ; \; h(u_2) = u_1 \; \text{and} \; h(u_4) = u_2
\]
\[
\mu_\phi(h(u_i)) = \mu_\phi(u_i) = 0.4 = \mu_\phi(u_1) \; \text{and} \; \nu_\phi(h(u_i)) = \nu_\phi(u_i) = 0.1 = \nu_\phi(u_1);
\]
\[
\mu_\phi(h(u_2)) = \mu_\phi(u_2) = 0.5 = \mu_\phi(u_2) \; \text{and} \; \nu_\phi(h(u_2)) = \nu_\phi(u_2) = 0.2 = \nu_\phi(u_2);
\]
\[
\mu_\phi(h(u_3)) = \mu_\phi(u_3) = 0.4 = \mu_\phi(u_3) \; \text{and} \; \nu_\phi(h(u_3)) = \nu_\phi(u_3) = 0.1 = \nu_\phi(u_3);
\]
\[
\mu_\phi(h(u_4)) = \mu_\phi(u_4) = 0.5 = \mu_\phi(u_4) \; \text{and} \; \nu_\phi(h(u_4)) = \nu_\phi(u_4) = 0.2 = \nu_\phi(u_4).
\]

\[ \therefore \; \tilde{G} \text{ is } \phi \text{- self complementary and hence } \tilde{G} \text{ is self complementary.} \]

Let \( \phi \) be another permutation on the set \( B_1, B_2 \) defined by \( \phi(B_1) = B_1 \) and \( \phi(B_2) = B_2 \).

Let there exists a one - one and onto map \( h: V \to V \) such that
\[
h(u_1) = u_1 \; , \; h(u_2) = u_4 \; , \; h(u_3) = u_1 \; \text{and} \; h(u_4) = u_2.
\]

\[ \therefore \; \tilde{G} \text{ is not totally self complementary.} \]

**Theorem (3.15):** Let \( \tilde{G} \) be self complementary IFGS, for some permutation \( \phi \) on the set \( B_1, B_2, \ldots, B_k \) then for each \( i = 1, 2, 3, \ldots, k \), we have

\[
\sum_{u \neq v} \mu_{B_i}(uv) + \sum_{u \neq v} \sum_{j \neq i} (\phi \mu_{B_j})(uv) = \sum_{u \neq v} (\mu_\phi(u) \wedge \mu_\phi(v)) \quad \text{and}
\]
\[
\sum_{u \neq v} v_{B_i}(uv) + \sum_{u \neq v} \sum_{j \neq i} (\phi v_{B_j})(uv) = \sum_{u \neq v} (\nu_\phi(u) \vee \nu_\phi(v)).
\]

**Proof:** Given \( \tilde{G} = (A, B_1, B_2, \ldots, B_k) \) is \( \phi \)- self complementary IFGS. Therefore, there exists a one - one and onto map \( h: V \to V \) such that
\[
\mu_\phi(h(u)) = \mu_\phi(u) \quad \text{and} \quad \nu_\phi(h(u)) = \nu_\phi(u), \quad \forall u, v \in V \quad \text{and} \quad j = 1, 2, \ldots, k.
\]

By definition of \( \phi \)-complement of IFGS, we have
\[ \mu_{B_i}(h(u)h(v)) = \mu_A(h(u)) \land \mu_A(h(v)) - \sum_{j \neq i} (\phi \mu_{B_j})(h(u)h(v)) \]
\[ \nu_{B_i}(h(u)h(v)) = \nu_A(h(u)) \lor \nu_A(h(v)) - \sum_{j \neq i} (\phi \nu_{B_j})(h(u)h(v)) \]

\[ \Rightarrow \mu_{B_i}(uv) = \mu_A(u) \land \mu_A(v) - \sum_{j \neq i} (\phi \mu_{B_j})(h(u)h(v)) \]
\[ \nu_{B_i}(uv) = \nu_A(u) \lor \nu_A(v) - \sum_{j \neq i} (\phi \nu_{B_j})(h(u)h(v)) . \]

Now, \[ \sum_{u \in v} \mu_{B_i}(uv) = \sum_{u \in v} (\mu_A(u) \land \mu_A(v)) - \sum_{u \in v} \sum_{j \neq i} (\phi \mu_{B_j})(h(u)h(v)) \] and
\[ \sum_{u \in v} \nu_{B_i}(uv) = \sum_{u \in v} (\nu_A(u) \lor \nu_A(v)) - \sum_{u \in v} \sum_{j \neq i} (\phi \nu_{B_j})(h(u)h(v)) \]

\[ \Rightarrow \sum_{u \in v} \mu_{B_i}(uv) = \sum_{u \in v} (\mu_A(u) \land \mu_A(v)) - \sum_{u \in v} \sum_{j \neq i} (\phi \mu_{B_j})(uv) \] and
\[ \sum_{u \in v} \nu_{B_i}(uv) = \sum_{u \in v} (\nu_A(u) \lor \nu_A(v)) - \sum_{u \in v} \sum_{j \neq i} (\phi \nu_{B_j})(uv) \]

\[ \Rightarrow \sum_{u \in v} \mu_{B_i}(uv) + \sum_{u \in v} \sum_{j \neq i} (\phi \mu_{B_j})(uv) = \sum_{u \in v} (\mu_A(u) \land \mu_A(v)) \] and
\[ \sum_{u \in v} \nu_{B_i}(uv) + \sum_{u \in v} \sum_{j \neq i} (\phi \nu_{B_j})(uv) = \sum_{u \in v} (\nu_A(u) \lor \nu_A(v)). \]

**Remark (3.16):** The result of Theorem (3.15) holds for a strong self complementary IFGS \( \bar{G} \), by using the identity mapping as the isomorphism.

**Corollary (3.17):** If an IFGS \( \bar{G} \) is totally self complementary, then
\[ \sum_{u \in v} \sum_{j} (\mu_{B_j})(uv) = \sum_{u \in v} (\mu_A(u) \land \mu_A(v)) \text{ and } \sum_{u \in v} \sum_{j} (\nu_{B_j})(uv) = \sum_{u \in v} (\nu_A(u) \lor \nu_A(v)) \]

**Proof:** By Theorem (3.15), we have
\[ \sum_{u \in v} \mu_{B_i}(uv) + \sum_{u \in v} \sum_{j \neq i} (\phi \mu_{B_j})(uv) = \sum_{u \in v} (\mu_A(u) \land \mu_A(v)) \] and
\[ \sum_{u \in v} \nu_{B_i}(uv) + \sum_{u \in v} \sum_{j \neq i} (\phi \nu_{B_j})(uv) = \sum_{u \in v} (\nu_A(u) \lor \nu_A(v)) , \text{ hold for every permutation } \phi. \]

Using the identity permutation \( \phi \), we have
\[ \sum_{u \in v} \sum_{j} (\phi \mu_{B_j})(uv) = \sum_{u \in v} (\mu_A(u) \land \mu_A(v)) \text{ and } \sum_{u \in v} \sum_{j} (\phi \nu_{B_j})(uv) = \sum_{u \in v} (\nu_A(u) \lor \nu_A(v)) \]

i.e., the sum of the membership (non-membership) of all \( B_i \)-edges \( i = 1, 2, 3, \ldots, k \), is equal to the sum of the minimum (maximum) of the membership (non-membership) of the corresponding vertices.
Corollary (3.18): If IFGS $\bar{G}$ is totally strong self complementary, then the above result also holds.

**Theorem (3.19):** In an IFGS $\bar{G}$, if for all $u, v \in V$, we have
$$\mu\_b(uv) + \sum\limits_{j \in I} (\phi \mu\_b)(uv) = \mu\_\lambda(u) \wedge \mu\_\lambda(v)$$ and $$\nu\_b(uv) + \sum\limits_{j \in I} (\phi \nu\_b)(uv) = \nu\_\lambda(u) \vee \nu\_\lambda(v),$$
then $\bar{G}$ is self complementary for a permutation $\phi$ on the set $\{B_1, B_2, ..., B_k\}$.

**Proof:** Let $h: V \to V$ be the identity map. Therefore, $\mu\_\lambda(h(u)) = \mu\_\lambda(u)$ and $\nu\_\lambda(h(u)) = \nu\_\lambda(u)$.

By definition of $\phi$-complement of IFGS, we have
$$\mu\_b(h(u)h(v)) = \mu\_\lambda(h(u)) \wedge \mu\_\lambda(h(v)) - \sum\limits_{j \in I} (\phi \mu\_b)(h(u)h(v)) = \mu\_\lambda(u) \wedge \mu\_\lambda(v) - \sum\limits_{j \in I} (\phi \mu\_b)(uv)$$
$$= (\mu\_b(uv) + \sum\limits_{j \in I} (\phi \mu\_b)(uv)) - \sum\limits_{j \in I} (\phi \mu\_b)(uv) = \mu\_b(uv)$$
and $$\nu\_b(h(u)h(v)) = \nu\_\lambda(h(u)) \wedge \nu\_\lambda(h(v)) - \sum\limits_{j \in I} (\phi \nu\_b)(h(u)h(v)) = \nu\_\lambda(u) \wedge \nu\_\lambda(v) - \sum\limits_{j \in I} (\phi \nu\_b)(uv)$$
$$= (\nu\_b(uv) + \sum\limits_{j \in I} (\phi \nu\_b)(uv)) - \sum\limits_{j \in I} (\phi \nu\_b)(uv) = \nu\_b(uv).$$

$\therefore$ $\bar{G}$ is $\phi$-self complementary. Hence $\bar{G}$ is self complementary for some permutation $\phi$.

**Corollary (3.20):** In an IFGS $\bar{G}$, if $\forall u, v \in V$, we have
$$\mu\_b(uv) + \sum\limits_{j \in I} (\phi \mu\_b)(uv) = \mu\_\lambda(u) \wedge \mu\_\lambda(v)$$ and $$\nu\_b(uv) + \sum\limits_{j \in I} (\phi \nu\_b)(uv) = \nu\_\lambda(u) \vee \nu\_\lambda(v),$$
for every permutation $\phi$ on the set $\{B_1, B_2, ..., B_k\}$ then $\bar{G}$ is totally self complementary.

4. Conclusion

The complement of intuitionistic fuzzy graph plays an important role in the further development of the theory. Similarly the concept of $\phi$-complementary IFGS is significant.

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**References**


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