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Lehmer - 3 Mean Labeling of Graphs

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Abstract

A graph G=(V,E) with P vertices and q edges is called Lehmer - 3 mean graph, if it is possible to label vertices x Є V with distinct label f(x) from 1,2,3,…………q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=$\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then the edge labels are distinct. In this case f is called Lehmer - 3 mean labeling of G. In this paper we investigate Lehmer - 3 mean labeling of some standard graphs.

Keywords: Path, Cycle, Comb, Ladder, Crown, Triangular snake, Quadrilateral snake

1. Introduction

A graph considered here are finite, undirected and simple. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. A path of length n is denoted by P_n. A cycle of length n is denoted by C_n. For standard terminology and notations we follow Harrary [2] and for the detailed survey of graph labeling we follow J.A. Gallian [1]. S. Somasundaram and S.S. Sandhya introduced the concept of Harmonic Mean Labeling of Graphs in [3] and its basic results was proved in [3].
In this paper we investigate Lehmer – 3 Mean Labeling behavior of some standard graphs. The following definitions are used here:

Definition 1.1 A graph G=(V,E) with P vertices and q edges is called Lehmer - 3 mean graph. If it is possible to label vertices x ЄV with distinct labels f(x) from 1,2,3,…………q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=$\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then the edge labels are distinct. In this case f is called Lehmer - 3 mean labeling of G.

Definition 1.2 A path P_n is obtained by joining u_i to the consecutive vertices u_{i+1} for $1 \leq i \leq n$.

Definition 1.3 Comb is a graph obtained by joining a single pendant edge to each vertex of a path.

Definition 1.4 A closed path is called a cycle of G.

Definition 1.5 A product graph $P_m x P_n$ is called a planar grid $P_2 x P_n$ is called a ladder.

Definition 1.6 Crown is a graph obtained by joining a single pendant edge to each vertex of a cycle.

Definition 1.7 A star graph is a graph obtained from a complete bipartite graph $k_{1,n}$.

Definition 1.8 A triangular snake T_n is obtained from a path v_1, v_2,……..v_n by joining v_i to a new vertex w_i for $1 \leq i \leq n-1$. That is every edge of a path is replaced by a triangle c_3.

Definition 1.9 A quadrilateral snake Q_n is obtained from a path u_1,u_2,………..u_n by joining u_i,u_{i+1} to a new vertices v_i and w_i respectively and joining v_i and w_i .That is every edge of a path is replaced by a cycle C_4.

2. Main Results

Theorem 2.1 Any path is a Lehmer -3 mean graph.

Proof: Let P_n be a path v_1, v_2,……..v_n.
Define a function f:(V(P_n))→{1,2,…..q+1} by f(v_i)=i ; $1 \leq i \leq n$.

Then we get distinct edge labels clearly f is a Lehmer -3 mean labeling of G.

Example 2.2 A Lehmer -3mean labeling of P_n is given below.

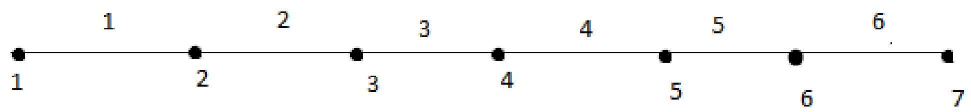

Figure-1

Theorem 2.3 Any cycle is a Lehmer -3 mean graph.

Proof: Let C_n be a cycle of length n. Let the vertices of C_n be $u_1, u_2, \ldots\ldots\ldots u_n, u_1$.
Define a function $f:V(C_n)\rightarrow\{1,2,\ldots\ldots.q+1\}$ by $f(u_i)=i; 1\leq i\leq n$.
The edges are labeled with $f(u_i,u_{i+1})=i;\ 1\leq i\leq n$.

Hence f is a Lehmer -3 mean labeling of graph G.

Example 2.4 A Lehmer -3 mean labeling of C_{10} is given below.

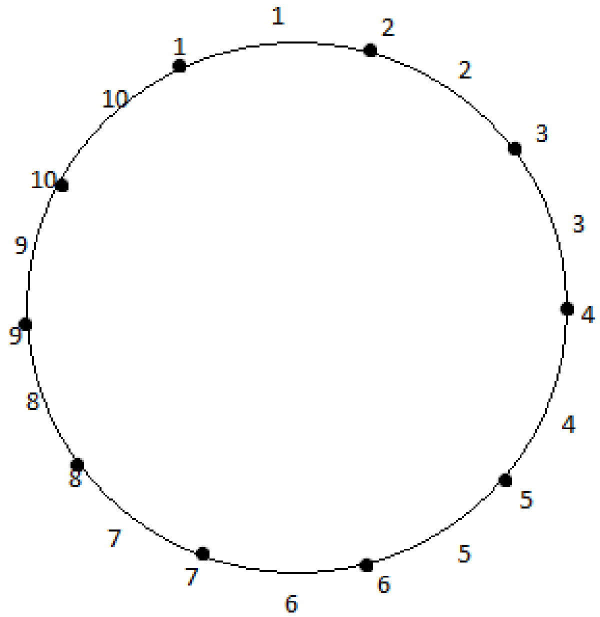

Figure-2

Theorem 2.5 Combs are Lehmer -3 mean graph.

Proof: Let C_n be a comb with $V(G)=\{v_1,v_2,\ldots\ldots v_n,u_1,u_2,\ldots\ldots u_n\}$.
Let P_n be a path. Let us label $P_n=v_1, v_2,\ldots\ldots v_n$ and join a vertex u_i to v_i , $1\leq i\leq n$.

Define a function $f:V(G)\rightarrow\{1,2,3,\ldots\ldots.q+1\}$ by $f(v_i)=2i-1$; $1\leq i\leq n$ and $f(u_i)=2i$, $1\leq i\leq n$.

The label of the edge u_iv_i is $2i-1$; $1\leq i\leq n$ and the label of the edge u_iv_{i+1} is $2i$, $1\leq i\leq n$.

Clearly the edge labels are distinct and hence f is a Lehmer -3 men labeling of graph G.

Example 2.6 A Lehmer -3 mean labeling of $P_6 \odot k_1$, is given below.

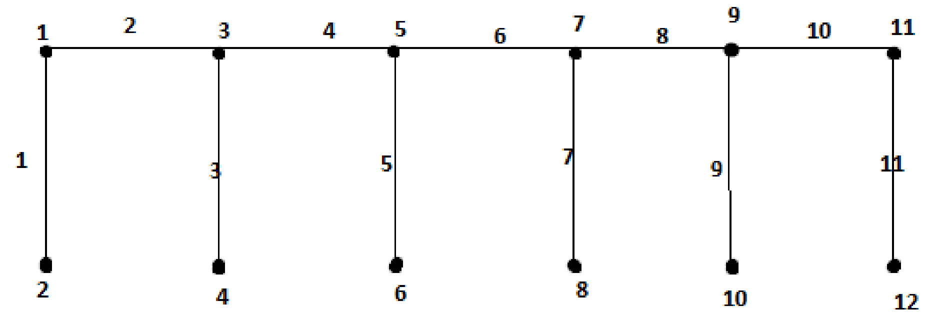

Figure -3

Theorem 2.7 Any Triangular snake T_n is a Lehmer -3 mean graph.

Proof: Let T_n be a Triangular snake.
Define a function $f: V(T_n)\rightarrow\{1,2,3,\ldots\ldots\ldots.q+1\}$ by

$f(v_i)=1$

$f(v_i)=3i-3$; $2\leq i\leq n$

$f(w_i)=3i-1$; $1\leq i\leq n$

The label of the edges $f(v_1v_2)=2$

The label of the edges $f(v_iv_{i+1})$ is $3i-1$, $2\leq i\leq n$

The label of the edge $f(v_iw_i)$ is $3i-2$, $1\leq i\leq n$

The label of the edge $f(w_iv_{i+1})=3i$, $1\leq i\leq n$

Hence this T_n forms a Lehmer -3 mean graph.

Example 2.8 The Lehmer -3 mean labeling of T_5 is given below.

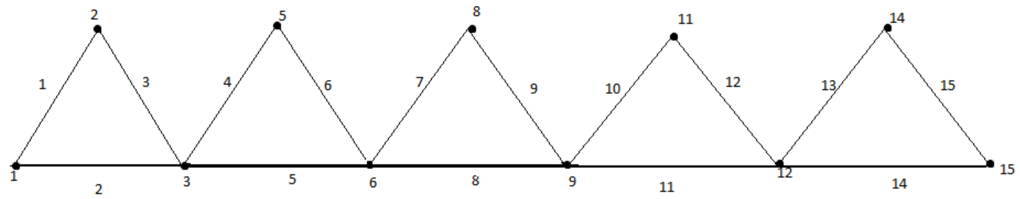

Figure-4

Theorem 2.9 Any Quadrilateral snake Q_n is a Lehmer -3 mean graph.

Proof: Let Q_n be the Quadrilateral snake as in definition.
Define f: $V(Q_n)\rightarrow\{1,2,\ldots..q+1\}$ by

$f(u_1)=1$,

$f(u_i)=4i-4;\ 2\leq i\leq n$

$f(v_i)=4i-2;\ 1\leq i\leq n$

$f(w_i)=4i-1;\ 1\leq i\leq n$

The label of the edge u_1u_2 is 3

The label of the edge u_iu_{i+1} is 4i-1, $2\leq i\leq n$

The label of the edge u_iv_i is 4i-3, $1\leq i\leq n$

The label of the edge v_i,w_i is 4i-2, $1\leq i\leq n$

The label of the edge u_iw_i is 4i, $1\leq i\leq n$

This gives a Lehmer -3 mean labeling Q_n.

Example 2.10 The Lehmer -3 mean labeling of Q_4 is given below.

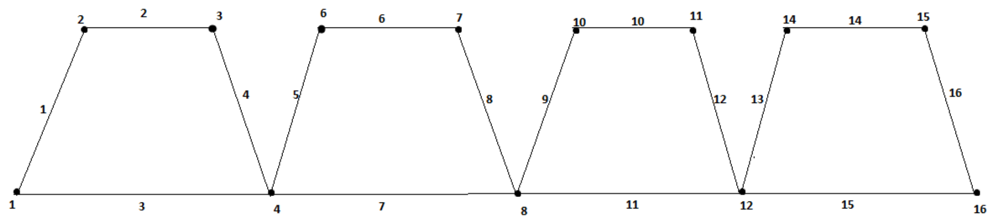

Figure-5

Theorem 2.11 Any Ladder is a Lehmer -3 mean graph.

Proof: Let L_n denote a ladder graph. Define $f:V(L_n)\rightarrow\{1,2,\ldots..q+1\}$ by $f(u_1)=1$ $f(u_i)=3i-3$; $i=2,3,\ldots.n$ and $f(v_i)=f(u_i)+1$; $i\leq i\leq n$

The label of the edge u_iu_{i+1} is $3i-1$, $1\leq i\leq n$

The label of the edge u_iv_i is $3i-2$, $1\leq i\leq n$

The label of the edge $v_i v_{i+1}$ is $3i$, $1\leq i\leq n$

This makes L_n as a Lehmer -3 mean graph.

Example 2.12 Lehmer -3 mean labeling of $L=P_qxP_2$ is given below.

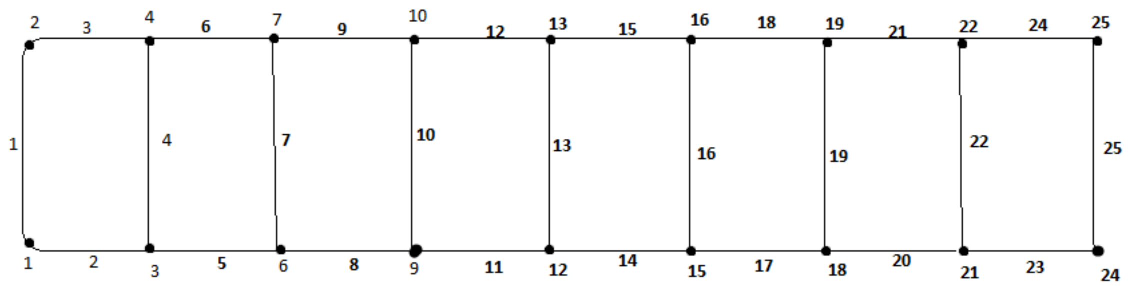

Figure-6

Theorem 2.13 A Crown $C_n \Theta k_1$ is a Lehmer 3 mean graph for all $n\geq 3$

Proof: Let C_n be a cycle $u_1,u_2,\ldots\ldots\ldots.u_n,u_1$ and v_i be the pendent vertices adjacent to u_i $1\leq i\leq n$.

Define a function $f: V(C_n\Theta K_1)\rightarrow\{1,2,\ldots..q+1\}$ by

$f(u_i)=2i-1$; $1\leq i\leq n$

$f(v_i)=2i$; $1\leq i\leq n$

Then the edge labels are all distinct.

Obviously f is a Lehmer -3 mean labeling.

Example 2.14 A Lehmer -3 mean labeling of $C_5\Theta K_1$ is given below.

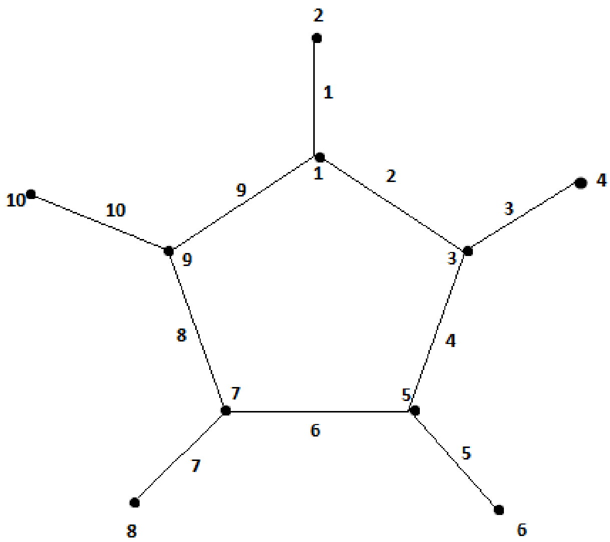

Figure-7

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