International Mathematical Forum, Vol. 12, 2017, no. 20, 973 - 982
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/imf.2017.7977

On Locating Domination Number of Boolean Graph BG_2 (G)

M. Bhanumathi

Government Arts College for Women (Autonomous)
Pudukkottai - 622001, TamilNadu, India

M. Thusleem Furjana

Dept. of Mathematics
Government Arts College for Women (Autonomous)
Pudukkottai - 622001, TamilNadu, India

Abstract

Let G(V, E) be a simple, finite and undirected connected graph. A non-empty set $S \subseteq V$ of a graph G is a dominating set, if every vertex in $V - S$ is adjacent to at least one vertex in S. A dominating set $S \subseteq V$ is called a locating dominating set, if for any two vertices $v, w \in V - S$, $N_G(v) \cap S$ and $N_G(w) \cap S$ are not empty and distinct. In this paper, we give some general bounds for $\gamma_L(BG_2(G))$ and characterize graphs for which $\gamma_L(BG_2(G)) = 3$.

Keywords: Dominating set, Locating dominating set, Boolean graph $BG_2(G)$

1 Introduction

Let G be a (p, q) simple, undirected graph with vertex set V(G) and edge set E(G). For $v \in V(G)$, the set of all vertices adjacent to v in G is called the neighbourhood $N_G(v)$ of v. The concept of domination in graphs was introduced by Ore[4]. A non empty set $S \subseteq V(G)$ of a graph G is a dominating set, if every vertex in $V(G) - S$ is adjacent to some vertex in S. A special case of dominating set S is called a locating dominating set. It was defined by D.F Rall and P.J Slater [5]. A dominating set S in a graph G is called a locating dominating set in G, if for any two vertices $v, w \in V(G) - S$, $N_G(v) \cap S$ and $N_G(w) \cap S$ are not empty and distinct. The location domination number of G is defined as the minimum number of vertices in a locating dominating set in G and denoted by $\gamma_L(G)$.

In 2004, Janakiraman and Bhanumathi defined Boolean Graphs. The Boolean graph $BG_2(G)$ has vertex set $V(G) \cup E(G)$ and two vertices in $BG_2(G)$ are adjacent if and only if they correspond to two adjacent vertices of G or to a vertex and an edge incident to it in G or two non-adjacent edges of G. The vertices of $BG_2(G)$, which are in V(G) are called point vertices and those in E(G) are called line vertices of $BG_2(G)$. $V(BG_2(G)) = V(G) \cup E(G)$ and $E(BG_2(G)) = [E(T(G)) - E(L(G))] \cup E(\overline{L(G)})$, where T(G) is the total graph of G and L(G) is the line graph of G.

Notation: In this paper $N_{BG_2(G)}(x)$ is denoted by N(x), degree of vertex v in $BG_2(G)$ is denoted by d(v) and degree of v in G is denoted by $d_G(v)$.

Theorem: 1.1 [1] If $G = K_{m,n}$ then $\gamma_L(BG_2(K_{m,n})) = m + n - 2$.
Theorem: 1.2 [3] If $G = K_n$, n > 1 then $\gamma_L(K_n) = n - 1$.
Theorem: 1.3 [3] If $G = K_{1, n-1}$, n > 2 then $\gamma_L(K_{1, n-1}) = n - 1$.
Theorem: 1.4 [3] If $G = K_{r, n-r}$, $1 < r \leq n - r$ then $\gamma_L(K_{r, n-r}) = n - 2$.
Theorem: 1.5 [1] If $G = \overline{K_m} + K_1 + K_1 + \overline{K_n}$, n > 1 then $\gamma_L(BG_2(G)) = m + n - 1$.
Theorem: 1.6 [1] Let $G \neq C_3$ be any connected graph with atleast three vertices then $\gamma_L(BG_2(G)) \leq p - 1$.

2 Locating domination of BG_2 (G)

First, we shall find the bounds for $\gamma_L(BG_2(G))$.
Theorem: 2.1 $\gamma_L(G) \leq \gamma_L(BG_2(G)) \leq \gamma_L(G) + q$.

Proof: Let S be a γ_L-set of $BG_2(G)$.
If $S \subseteq V(G)$, S is also a locating dominating set of G. This implies that $\gamma_L(G) \leq \gamma_L(BG_2(G))$. If S contains line vertices, let $W \subseteq S$ be set of line vertices of $BG_2(G)$ in S. Let $e \in W$ and $e = xy \in E(G)$. Deleting e from S and adding one incident vertex of e, that is, x or y to S for all $e \in W$, we will get a locating dominating set of G. Hence $\gamma_L(G) \leq \gamma_L(BG_2(G))$. On the other hand, let S be a γ_L-set of G. S need not be a locating dominating set of $BG_2(G)$. But $S \cup E(G)$ is a locating dominating set of $BG_2(G)$. Hence, $\gamma_L(BG_2(G)) \leq \gamma_L(G) + q$.

Lemma: 2.1 Let G be a connected graph with r(G) = 1, d(G) = 2. Let v be a central vertex of G. If $V(G) - \{v\}$ is a γ_L-set of $BG_2(G)$, then $p \geq 3$ and $\delta(G) \geq 3$.

Proof: Let $S = V(G) - \{v\}$ is a γ_L-set of $BG_2(G)$. Suppose $x \in V(G)$ is a vertex of G. Then let $e_1 = vx \in E(G)$, $N(e_1) \cap S = \{x\}$ and $N(v) \cap S = S$. Since S is a locating dominating set, this implies that $S \neq \{x\}$. Hence S contains more than one vertex and hence $|V(G)| \geq 3$. If G has a vertex x of degree two and $e_1 = xv$, $e_2 = xy$,

$e_3 = vy \in E(G)$ then let $N_G(x) = \{v, y\}$ and $N(x) = \{v, y, e_1, e_2\}$ and $N(e_3) = \{v, y\}$. Also, $N(x) \cap S = N(e_3) \cap S = \{y\}$, which is a contradiction. Hence, G has no vertex of degree two. If G has a vertex y′ of degree one, then in G, y′ is adjacent to v only and in $BG_2(G)$, y′ is adjacent to v and the line vertex e′ = vy′. Therefore, S is a dominating set and $S \subseteq V(G)$ implies that S must contain v. Hence, G cannot have a vertex of degree one or two. This implies that, $\delta(G) \geq 3$.

Lemma: 2.2 If G has a pendant vertex v, incident with an edge e, then v must be in any locating dominating set S of $BG_2(G)$, where $S \subseteq V(G)$.

Proof: Let $S \subseteq V(G)$ be a locating dominating set not containing v. Let $e = uv \in E(G)$. Since S is a dominating set, if must contain u to dominate v. Now, if $S \subseteq V(G)$, then $N(v) \cap S = N(e) \cap S = \{u\}$, which is a contradiction to S as a locating dominating set. So, v must be in S.

Lemma: 2.3 Let G be a connected graph with r(G) = 1, d(G) = 2. Let e(v) = 2 in G. If $V(G) - \{v\}$ is a γ_L-set of $BG_2(G)$, then $d_G(v) \geq 3$.

Proof: Let $S = V(G) - \{v\}$ is a γ_L-set of $BG_2(G)$, Suppose degree of v in G is one, Then v is adjacent to u, where u is the only central vertex of G and $e_G(u) = 1$. Let $e = uv \in E(G)$. In $BG_2(G)$, $N(e) \cap S = N(v) \cap S = \{u\}$, which is a contradiction.

Suppose $d_G(v) = 2$.

Case: i $N_G(v) = \{u, x\}$, where $e_G(u) = e_G(x) = 1$.

In this case, $N_G(v) = \{u, x\}$ and let $e_1 = ux \in E(G)$. In $BG_2(G)$, $N(e_1) \cap S = \{u, x\}$ and $N(v) \cap S = \{u, x\}$, which is a contradiction.

Case: ii $N_G(v) = \{u, y\}$, where $e_G(u) = 1$ and $e_G(y) = 2$.

Let $e_2 = uy$. Again in $BG_2(G)$, $N(e_2) \cap S = \{u, y\}$ and $N(v) \cap S = \{u, y\}$, which is a contradiction. Hence $d_G(v) \geq 3$.

Lemma: 2.4 If G has a pendant edge e, incident with a vertex v, then e must be in any locating dominating set S of $BG_2(G)$, where $S \subseteq E(G)$.

Proof: Let $S \subseteq E(G)$ be a locating dominating set not containing e. Let $e = uv \in E(G)$, then S is not a dominating set and also $N(v) \cap S = \phi = N(u) \cap S$, which is a contradiction to S as a locating dominating set. So, e must be in S.

Theorem: 2.2 If G is a connected graph with r(G) = 1, d(G) = 2, Then $S = V(G) - \{v\}$ cannot be a γ_L-set of $BG_2(G)$ if $d_G(v) \leq 2$.

Proof: Proof follows from Lemma 2.1, Lemma 2.2 and Lemma 2.3.

Proposition: 2.1 Let G be a connected graph with r(G) = 1and d(G) = 2. Let $S \subseteq E(G)$. If G[S] has K_2 as a component then S is not a locating dominating set of $BG_2(G)$.

Proof: Let e = uv ∈ S form a K_2 in G[S]. Then in $BG_2(G)$, N(u) ∩ S = N(v) ∩ S = {e}, which is a contradiction to S is a locating dominating set. This proves the result.

Proposition: 2.2 Let S ⊆ V(G) and let v ∈ S such that e_1 = vx and e_2 = vy ∈ E(G) and x, y ∉ S, then S cannot be a locating dominating set of $BG_2(G)$.

Proof: In $BG_2(G)$, $N(e_1) \cap S = N(e_2) \cap S = \{v\}$, which is a contradiction to S as a locating dominating set, This proves the result.

Remark: 2.1 If S ⊆ V(G) is a locating dominating set and if v ∈ S such that d(v) = m > 1, then at least (m – 1) neighbours of v is also in S.

Proposition: 2.3 Let G be a connected graph with r(G) = 1, d(G) = 2. Let v be a central vertex of G. Let S ⊆ V(G) be a locating dominating set of $BG_2(G)$ containing a central vertex of G. Then |S| = p – 1.
Proof: Proof follows from the previous remark.

Theorem: 2.3 Let G be a graph with radius one. If there exists a γ_L-set S of $BG_2(G)$ such that S ⊆ V(G), then $\gamma_L(BG_2(G)) = p - 1$.

Proof: We know that $\gamma_L(BG_2(G)) \leq p - 1$. So, it is enough to prove that $\gamma_L(BG_2(G)) \nless p - 1$. Let V(G) ={$v_1$, v_2, ..., v_p} and let v = v_1 such that e(v) = 1. Suppose $\gamma_L(BG_2(G)) < p - 1$. Then there exists at least two vertices x, y ∈ V(G) such that x, y ∉ S. Let S = V(G) – {x, y}.
Case: i Let v ≠ x, y, v ∈ S. Let e_1 = vx, e_2 = vy ∈ E(G), Then $N(e_1) \cap S = \{v\} = N(e_2) \cap S$ in $BG_2(G)$ which is a contradiction to S is a γ_L-Set . Hence $\gamma_L(BG_2(G)) \nless p - 1$. Similarly, if S = V(G) – {x, y, z}, x, y, z ∈ V(G), then also, $N(e_x) \cap S = N(e_y) \cap S = N(e_z) \cap S = \{v\}$, which is a contradiction where e_x = vx, e_y = vy, e_z = vz. Hence |S| must be p – 1.
Case: ii suppose S contains no central vertices. S has at least two vertices. Suppose |S| < p – 1, V – S has at least two vertices. Also, V – S contains atleast one central vertex. Suppose V – S contains two central vertices v_1, v_2. Then the line vertex e = v_1v_2 is not dominated by S in $BG_2(G)$. So, assume that V – S contains exactly one central vertex v, Thus v ∉ S and G is a unicentral graph with radius one. Let v, x ∉ S such that $e_G(v)$ = 1and $e_G(x)$ = 2. Then the edge vx = e_1 ∈ E(G) is not dominated by S in $BG_2(G)$, which is again a contradiction. Hence, $|S| \nless p - 1 \Rightarrow |S| = p - 1$ and hence $\gamma_L(BG_2(G)) = p - 1$.

Remark: 2.2
(1) If G is a connected graph with radius one and has a unique central vertex v, then V(G) – {v} is a locating dominating set of $BG_2(G)$.

(2) If G is a connected graph with radius one and has more than one central vertex then any locating dominating set $S \subseteq V(G)$ of $BG_2(G)$ must contain a central vertex of G.
(3) If G is a Graph with radius one and $\gamma_L(BG_2(G)) < p - 1$, then every γ_L-set of $BG_2(G)$ must contain line vertices.
(4) There may exists graphs with radius one such that $\gamma_L(BG_2(G)) < p - 1$.

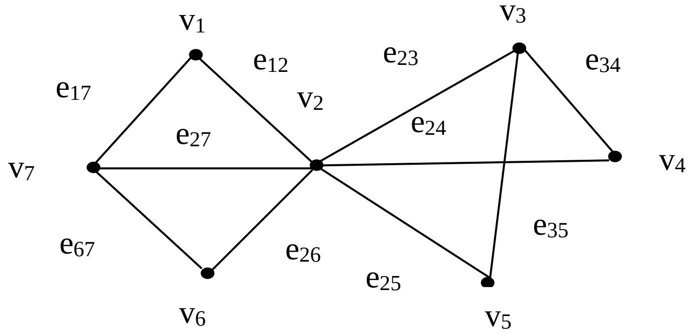

Figure: 2.1

Let G be a graph in Figure 2.1. $S \subseteq E(G)$ and $S = \{e_{17}, e_{25}, e_{35}, e_{34}, e_{67}\}$ form a minimum locating dominating set of $BG_2(G)$. Hence, $\gamma_L(BG_2(G)) = p - 2$.

Theorem: 2.4 Let G be a disconnected graph without isolated vertices with components $G_1, G_2, G_3, ..., G_n$ $(n \geq 2)$ then $\gamma_L(BG_2(G)) \leq \gamma_L(BG_2(G_1)) + \gamma_L(BG_2(G_2)) + \ldots + \gamma_L(BG_2(G_n)) = \sum_{i=1}^{n} \gamma_L(BG_2(G_i))$.

Proof: Let S_i be γ_L-set of $BG_2(G_i)$, i = 1, 2, ..., n. Then $S = \bigcup_{i=1}^{n} S_i$ is a locating dominating set of $BG_2(G)$. Hence $\gamma_L((BG_2(G)) \leq |S| \leq \sum_{i=1}^{n} \gamma_L(BG_2(G_i))$

Theorem: 2.5 If G is any one of K_n, $K_{1,n}$ and $K_{m,n}$ then $\gamma_L(BG_2(G)) = \gamma_L(G)$.

Proof: Assume G is a connected graph with p vertices and S is the minimum locating dominating set of G. Let $A = S - N(u) = \phi$, where $u \not\subseteq S$.
(i) Let $G = K_n$. Let $V(G) = \{v_1, v_2, v_3, ... , v_n\}$. Then by Theorem 2.3, $S = \{v_1, v_2, v_3, ... , v_{n-1}\}$ is a γ_L-set of $BG_2(G)$. Also $\gamma_L(G) = p - 1$ by Theorem 1.2. Hence the proof follows.
(ii) If G is a star graph $K_{1,n}$ with $p = n + 1$ and by Theorem 2.3, $S = \{v_1, v_2, v_3, ... , v_n\}$, then S is independent and S is a γ_L-set of $BG_2(G)$, $|S| = p - 1 = \gamma_L(BG_2(G))$, Also, $\gamma_L(K_{1,n}) = p - 1$ by Theorem 1.3. Therefore $\gamma_L(G) = \gamma_L(BG_2(G))$.
(iii) If $G = K_{m,n}$. Let $V(G) = V_1 \cup V_2$, $V_1 = \{u_1, u_2, \ldots, u_m\}$, $V_2 = \{v_1, v_2, v_3, \ldots, v_n\}$ and $u_i v_j = e_{ij}$; i = 1, 2, 3, ..., m; j = 1, 2, 3, ..., n. Then $S = \{e_{12}, e_{13}, ..., e_{1n}, e_{22}, e_{31}, ..., e_{m-11}, e_{mn}\}$ is the minimum locating dominating set of $BG_2(G)$ containing $m + n - 2$ elements by Theorem 1.1. Also $\gamma_L(G) = m + n - 2$ by Theorem 1.4. Therefore, we get $\gamma_L(BG_2(G)) = \gamma_L(G)$.

Lemma: 2.5 Let G be any connected graph. Then $\gamma_L(BG_2(G)) = 3$ if and only if G $\in A'$, where A' is the set of all graphs K_4, $K_4 - e$, W_3, $K_{2,2}$, P_4, P_5, C_n (n = 3, 4, 5), $C_4 - e$, $C_5 - e$, $K_{1,3}$, $K_{1,3} + e$, $K_1 + K_1 + 2K_1 + K_1$.

Proof: If $G \in A'$ then $\gamma_L(BG_2(G)) = 3$. Conversely, assume that G is connected and S be the minimum locating dominating set of $BG_2(G)$, with |S| = 3. Let S = {u, v, w}. The non - empty subsets of S are {u}, {v}, {w}, {u, v}, {u, w}, {v, w} and {u, v, w}. Since, $\gamma_L(BG_2(G)) = 3$, for any two vertices $x, y \in V(BG_2(G)) - S$, $N(x) \cap S \neq N(y) \cap S \neq \phi$. Since $N(x) \cap S$ and $N(y) \cap S$ are any one of the seven distinct sets, $BG_2(G)$ is a graph which contain at most ten vertices. Hence |V(G)| = $p \leq 5$, since if $|V(G)| \geq 6$, number of vertices of $BG_2(G)$ is greater than ten. Among the connected graphs with $p \leq 5$ the following are the graphs with $\gamma_L(BG_2(G)) = 3$. K_4, $K_4 - e$, W_3, $K_{2,2}$, P_4, P_5, C_n (n = 3, 4, 5), $C_4 - e$, $C_5 - e$, $K_{1,3}$, $K_{1,3} + e$, $K_1 + K_1 + 2K_1 + K_1$.

Lemma: 2.6 Let G be any disconnected graph. Then $\gamma_L(BG_2(G) = 3$ if and only if G is any one of the following graphs $K_{1,2} \cup K_2$ and $2K_2$.

Proof: If $G = K_{1,2} \cup K_2$ or $2K_2$ then $\gamma_L(BG_2(G)) = 3$. Conversely, Assume G is a disconnected graph and S be the minimum locating dominating set of $BG_2(G)$ with |S| = 3. Let S = {u, v, w}. The non-empty subsets of S are {u}, {v}, {w}, {u, v}, {u, w}, {v, w} and {u, v, w}. Since, $\gamma_L(BG_2(G)) = 3$, for any two vertices x, y $\in V(BG_2(G)) - S$, $N(x) \cap S \neq N(y) \cap S \neq \phi$. Since $N(x) \cap S$ and $N(y) \cap S$ are any one of the seven distinct sets, $BG_2(G)$ is a graph which contain at most ten vertices. If p > 6, p + q > 10. Hence $p \leq 6$. Among the disconnected graphs with p ≤ 6, having no isolated vertices $\gamma_L(BG_2(G)) = 3$ for $K_{1,2} \cup K_2$ or $2K_2$.

Theorem: 2.6 Let G be any graph. Then $\gamma_L(BG_2(G)) = 3$ if and only if G is any one of the following graphs K_4, $K_4 - e$, W_3, $K_{2,2}$, P_4, P_5, C_n (n = 3, 4, 5), $C_4 - e$, $C_5 - e$, $K_{1,3}$, $K_{1,3} + e$, $K_1 + K_1 + 2K_1 + K_1$, $K_{1,2} \cup K_2$ or $2K_2$.
Proof: Proof follows from the Lemma 2.5 and Lemma 2.6.

Corollary: 2.6.1 Let G be any connected graph then $\gamma_L(BG_2(G)) = 3$ and any γ_L-set contains only point vertices if and only if $G \in A'$, where A' is the set of all graphs K_4, $K_4 - e$, W_3, $K_{1,3}$, $K_{1,3}$+e, C_3.
Proof: Proof follows from Theorem 2.6.

Corollary: 2 Let G be any connected graph then $\gamma_L(BG_2(G)) = 3$ and any γ_L-set contains only line vertices or point vertices and line vertices if and only if $G \in A'$, where A' is any one of the following graphs P_4, P_5, $C_4 - e$, $C_5 - e$, C_n (n = 3, 4, 5), $K_{1,2} \cup K_2$, $2K_2$.
Proof: Proof follows from Theorem 2.6.

Theorem: 2.7 Let G be a connected graph with non-adjacent vertices v_1, $v_p \in V(G)$ such that $d_G(v_1) = p - 2$ and $d_G(v_p) = 1$. If $BG_2(G)$ has a γ_L-set S such that $S \subseteq V(G)$, then $\gamma_L(BG_2(G)) = p - 1$.

Proof: $d_G(v_1) = p - 2$. Let $N_G(v_1) = \{v_2, v_3, \ldots, v_{p-1}\}$, Since G is connected v_p is adjacent to some v_i, $1 \leq i \leq p - 1$. Let $N_G(v_p) = \{v_2\}$, assuming $d_G(v_2) < p - 1$ and G is a graph with radius two and diameter three. Let S be a γ_L-set of $BG_2(G)$ such that $S \subseteq V(G)$, We know that $\gamma_L(BG_2(G)) \leq p - 1$. Hence $|S| \leq p - 1$. Since v_p is a pendant vertex in G, v_p must be in S by Lemma 2.2.

Case: i $v_2 \notin S$. We claim that all other point vertices are in S. If $v_1 \notin S$, for $e_{12} = v_1v_2 \in E(G)$ in $BG_2(G)$, $N(e_2) \cap S = \phi$ which is a contradiction to S is a dominating set of $BG_2(G)$. Hence v_1 must be in S. Thus v_1, $v_p \in S$ and $v_2 \notin S$, Again if there exists any other $v_i \in V(G)$ such that $v_i \notin S$ let $e_i = v_1 v_i \in E(G)$ and $e_2 = v_1v_2 \in E(G)$. Then in $BG_2(G)$, $N(e_2) \cap S = N(e_i) \cap S = \{v_1\}$, which is a contradiction to S is a locating dominating set. Hence $S = V(G) - \{v_2\}$, This implies that $|S| = p - 1$. That is, $\gamma_L(BG_2(G)) = p - 1$.

Case: ii $v_2 \in S$. Vertices v_2 and $v_p \in S$. Let $e_2 = v_1 v_2$, $e = v_2 v_p \in E(G)$. If $v_1 \notin S$, then in $BG_2(G)$, $N(e_2) \cap S = N(e) \cap S = \{v_2\}$, which is a contradiction to S is a locating dominating set of $BG_2(G)$. Hence $v_1 \in S$. So, v_1, v_2 and $v_p \in S$. But we know that $\gamma_L(BG_2(G)) \leq p - 1$. Hence there exists a vertex v_i, $3 \leq i \leq p - 1$ such that $v_i \notin S$. If there exists any other $v_j \notin S$, $3 \leq j \leq p - 1$, $i \neq j$ then in $BG_2(G)$, $N(e_i) \cap S = N(e_j) \cap S = \{v_i\}$where $e_i = v_1v_i$, $e_j = v_1v_j \in E(G)$, which is again a contradiction. Hence $|S| = p - 1$, $\gamma_L(BG_2(G)) = p - 1$.

Remark: 2.3

If G is a connected graph with adjacent vertices v_1 and v_p such that $d_G(v_1) = p - 2$ and $d_G(v_p) = 1$, then $\gamma_L(BG_2(G))$ need not be $p - 1$, where $S \subseteq V(G) \cup E(G)$.

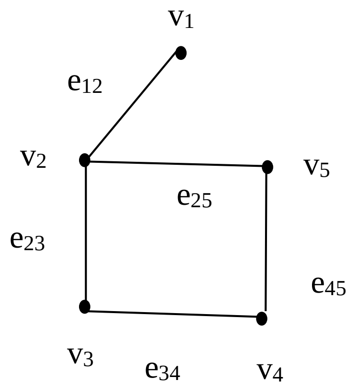

Figure: 2.2

In Figure 2.2, Let G be a connected graph with adjacent vertices v_1 and v_2 such that $d_G(v_1) = 1$, $d_G(v_2) = 3$ then $S = \{v_4, v_5, e_{12}\}$ forms a minimum locating dominating set of $BG_2(G)$. Hence $\gamma_L(BG_2(G)) = p - 2$.

Corollary to Theorem: 1.12 If G is a connected graph with atleast (p – 2) pendant vertices then $\gamma_L(BG_2(G)) = p - 1$.

Proof: G has either p – 1 or p – 2 pendant vertices. Hence G is either a star or a double star. By Theorems 1.3 and 1.5 in both the cases $\gamma_L(BG_2(G)) = p - 1$.

Theorem: 2.8 If there exists an edge $e \in E(G)$ such that e is adjacent to all other edges of G then $\gamma_L(BG_2(G)) = p - 1$.

Proof: By the given condition either $G = K_{1,n}$, double star $\overline{K_m} + K_1 + K_1 + \overline{K_n}$ or G is of the following type:

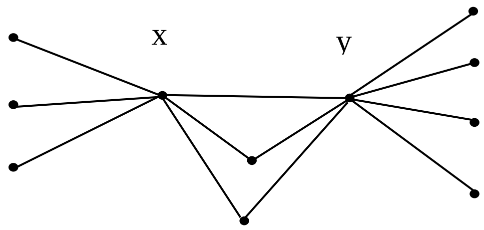

Figure: 2.3

If G is a star or double star $\gamma_L(BG_2(G)) = p - 1$ by Theorems 1.3 and 1.5. So it is enough to prove the result for the graph in Figure 2.3 only. Let e = xy be the edge in G, which is adjacent to all other edges of G. Let $S \subseteq V(G) \cup E(G)$ be the locating dominating set of $BG_2(G)$.

Case: i $S \subseteq V(G)$. Since $S \subseteq V(G)$, all the pendant vertices of G are in S by Lemma 2.2. Suppose $z \in V(G)$ such that z is adjacent to both x and y in G. let $e_1 = xz$, $e_2 = yz \in E(G)$. Suppose $z \notin S$. Then x and y must be in S to dominate e_1 and e_2 in $BG_2(G)$. In this case, $N(e_1) \cap S = N(z) \cap S = \{x\}$ and $N(e_2) \cap S = N(z) \cap S = \{y\}$ which is again a contradiction to S is a locating dominating set of $BG_2(G)$. So z must be in S and to dominate e = xy in $BG_2(G)$, x or y must be in S. Hence $S = V(G) - \{x\}$ or $S = V(G) - \{y\}$, So $|S| = p - 1$.

Case: ii $S \subseteq E(G)$. S must contain all the line vertices which are pendant edges in G. Consider $e = xy \in E(G)$. The line vertex e is not adjacent to any other line vertices in $BG_2(G)$. Hence e must be in S by Theorem 2.1. Now, consider $e_1 = xz$, $e_2 = yz \in E(G)$. To dominate z in $BG_2(G)$, any one of e_1 or e_2 must be in S. Thus, we see that S is a set of edges which form a spanning tree of G and S contains p – 1 line vertices of $BG_2(G)$ by Theorem 1.6. This implies that $|S| = p - 1$.

Case: iii S contains both point and line vertices. S must contain pendant vertices of G or the pendant edges of G. Let N(x) contains m pendant vertices and N(y) contains n pendant vertices and let k vertices are adjacent to both x and y.

Therefore p = m + n + 2 + k then |S| = m + n + k +1 = p – 1-------------------------I.

In G, e is adjacent to all other edges. Hence in $BG_2(G)$, e is adjacent to x and y only. Hence any one of x, y, e is in S. -- II.

Case: i Let $x \in S$ (or $y \in S$). Now, consider $z \in V(G)$ which is adjacent to both x and y in G. Suppose z and line vertices incident with z are not in S. Consider z and e = xy. In $BG_2(G)$, $N(z) \cap S = x = N(e) \cap S$. so z or e must be in S. -------III.

Case: ii $e \in S$, If $e \in S$ and $x, y \notin S$, z is not dominated by S.

Sub case: i z be the only vertex adjacent to both x and y. So at least one of z, $e_1 = xz$, $e_2 = yz$, $N(e_1) \cap S = N(e_2) \cap S = \phi$. x and y must be in S. -------------------IV.

Sub case: ii If there exists more than one vertex adjacent to both x and y. Let z_1, $z_2 \in V(G)$ such that $e_1 = xz_1$, $e_1' = yz_1$, $e_2 = xz_2$, $e_2' = yz_2 \in E(G)$. If z_1, z_2 and the incident edges are not in S then $N(e_1) \cap S = N(e_2) \cap S$ or $N(e_1') \cap S = N(e_2') \cap S$. So, among z_i , e_i, e_i' any one must be in S.-------------------------------------- V.

From I, II, III, IV and V, it is clear that |S| = p – 1, This proves the result.

Theorem: 2.9 If G has a pendant vertex v, which is adjacent to the central vertex u and incident with an edge e = uv, then v or e must be in any locating dominating set of $BG_2(G)$.

Proof: Proof follows from Lemma 2.2 and Lemma 2.4.

References

[1] M. Bhanumathi, M. Thusleem Furjana, Locating Domination in Boolean Graph $BG_2(G)$, *Aryabhatta Journal of Mathematics and Informatics [AJMI]*, **8** (2016), no. 2.

[2] T. N. Janakiraman, M. Bhanumathi and S. Muthammai, Domination Parameters of the Boolean Graph $BG_2(G)$ and its Complement, *International Journal of Engineering Science, Advanced Computing and Bio-Technology*, **3** (2012), no. 3, 115-135.

[3] Jose Caceres, Carmen Hernando, Merce Mora, Ignacio M. Pelayo and Maria Luz Puertas, Locating dominating codes: Bounds and extremal cardinalities *Applied Mathematics and Computation*, **220** (2013), 38-45. https://doi.org/10.1016/j.amc.2013.05.060

[4] O. Ore, *Theory of Graphs*, Coll. Publ., Vol. 38, Amer. Math. Soc., Providence, Rhode Island, 1962. https://doi.org/10.1090/coll/038

[5] D.F. Rall, P.J. Slater, On location domination number for certain classes of graphs, *Congrences Numerantium*, **45** (1984), 77-106.

Received: September 15, 2017; Published: December 20, 2017