International Mathematical Forum, Vol. 12, 2017, no. 3, 133 - 143
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/imf.2017.610131

The Ruled Surfaces According to Type-2 Bishop Frame in E^3

Esra Damar

Department of Automotive Technologies
Hitit University, Çorum, Turkey

Nural Yüksel

Department of Mathematics
Erciyes University, Kayseri, Turkey

Aysel Turgut Vanlı

Department of Mathematics
Gazi University, Ankara, Turkey

Abstract

In this paper, we focus on the theory of the ruled surfaces with respect to type-2 Bishop frame. Firstly, type-2 Bishop motion is defined for space curve and then Darboux vector of this motion is calculated for fixed and moving spaces in $\mathbb{E}^3$. We obtained the distribution parameter of a ruled surfaces generated by a darboux vector in type-2 Bishop trihedron moving along a curve and we show that the ruled surfaces whose generated by a darboux vector is developable but according to the type-2 Bishop frame, there is no developable ruled surfaces generated by the straight line in type-2 Bishop trihedron moving along a curve.

Mathematics Subject Classification: 53A04, 53A05

Keywords: Ruled surfaces, type-2 Bishop frame, Darboux vector, distribution parameter, developable ruled surfaces

1 Introduction

Researchers aimed to determine moving frame for regular curve. In 1975, L.R. Bishop introduced Bishop Frame or parallel transport frame. This is an alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative[1]. Nowadays a good deal of research has been done on Bishop frame in Euclidean space see [4],[5]; in Minkowski space, see [3]; and dual space, see [7]. And recently, this special frame is exenteded to study of ruled surfaces we refer [9].

In [8], the authors introduced a new version of the Bishop frame and called it as "type-2 Bishop frame". They also researched spherical images of a reguler curve which correspond to each vector fields of the new trihedra. In this work we focus on the theory of the ruled surfaces with respect to type-2 Bishop frame. Firstly, type-2 Bishop motion is defined for space curve and then darboux vector of this motion is calculated for fixed and moving spaces in $\mathbb{E}^3$. We obtained the distribution parameter of a ruled surfaces generated by a darboux vector in type-2 Bishop trihedron moving along a curve. We show that the ruled surfaces whose generated by a darboux vector is developable but according to the type-2 Bishop frame, there is no developable ruled surfaces generated by the straight line in type-2 Bishop trihedron moving along a curve. The Euclidean 3-space $\mathbb{E}^3$provided with the standard flat metric given by

$$\langle , \rangle = dx_1^2 + dx_2^2 + dx_3^2$$

where (x_1, x_2, x_3) is a rectangular coordinate system of $\mathbb{E}^3$. To remember, the norm of an arbitrary vector a $\in \mathbb{E}^3$ is given by $\|a\| = \sqrt{\langle a, a\rangle}$. α is called an unit speed curve if velocity vector v of α satisfies $\|v\| = 1$. For vectors v, w $\in \mathbb{E}^3$ it is said to be orthogonal if and only if $\langle u, v\rangle = 0$. Let $\alpha = \alpha(s)$ regular curve in $\mathbb{E}^3$. In three dimensional Euclidean space $\{T, N, B\}$ denote Frenet -Serret frame along the curve α. For an arbitrary curve α with first and second curvature, κ and τ in $\mathbb{E}^3$, the following Frenet-Serret formulae is given in [6].

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

Here, curvatures functions are defined by $\kappa(s) = \left\|T'(s)\right\|$ and $\tau(s) = -\left\langle N, B'\right\rangle$. In [8], the authors introduced a new version of the Bishop frame with the following statements. Let $\alpha = \alpha(s)$ be a unit speed regular curve in $\mathbb{E}^3$. The type-2 Bishop frame of the $\alpha(s)$ is defined by

$$\begin{bmatrix} \zeta_1' \\ \zeta_2' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\epsilon_1 \\ 0 & 0 & -\epsilon_2 \\ \epsilon_1 & \epsilon_2 & 0 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ B \end{bmatrix} \tag{1}$$

The relation matrix between Frenet-Serret and type-2 Bishop frames can be expressed

$$\begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ B \end{bmatrix} \tag{2}$$

Here, the type-2 Bishop curvatures are defined by

$$\begin{aligned} \epsilon_1 &= -\tau\cos\theta(s) \\ \epsilon_2 &= -\tau\sin\theta(s) \end{aligned} \tag{3}$$

It can be also deduced as $\theta(s) = \arctan\frac{\epsilon_2}{\epsilon_1}$, $\kappa(s) = \theta'(s)$. The frame $\{\zeta_1, \zeta_2, B\}$ is properly oriented, and τ and $\theta(s) = \int \kappa(s)\, ds$ are polar coordinates for the curve $\alpha = \alpha(s)$. We shall call the set $\{\zeta_1, \zeta_2, B, \epsilon_1, \epsilon_2\}$ as type-2 Bishop invariants of the curve α.

2 Basic Concept

A one - parameter motion of a body in $\mathbb{E}^3$ is generated by the transformation

$$\begin{bmatrix} Y \\ 1 \end{bmatrix} = \begin{bmatrix} A & C \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix} \tag{4}$$

where A is a orthogonal matrix, $A \in SO(3)$, and C is the displacement vector of the origin. A and C are C^∞ functions of a real parameter s. X, Y are $n \times 1$ matrices and

$SO(3) = \left\{A = [a_{ij}] \in R_3^3 : A^{-1} = A^T, a_{ij} \in R\right\}$ [2]. X and Y respectively correspond to the positon vector of the same point, with respect to the orthogonal coordinate systems of the moving space H and the fixed space H'. At the same time $s = s_0$ we consider the coordinate system H and $H'$are coincident. We will use the arc length, s of α as the motion parameter and use primes to denote derivatives with respect to s. Denote by $\{\zeta_1, \zeta_2, B\}$ the moving type-2 Bishop frame along the curve $\alpha = \alpha(s)$ parameterized by arc-length parameter s, i.e, $\langle \alpha'(s), \alpha'(s)\rangle = 1$. Let $\zeta_1 = (n_1, n_2, n_3)$, $\zeta_2 = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3)$ be the unit vectors along α curve. Then we have

$$A = \begin{bmatrix} n_1 & a_1 & b_1 \\ n_2 & a_2 & b_2 \\ n_3 & a_3 & b_3 \end{bmatrix} \tag{5}$$

It can be defined that an one parameter special motion of a body in Euclidean 3-space is generated by the transformation by

$$Y = AX + C \tag{6}$$

where $A \in SO(3)$ and X, Y, C are 3×1 real matrices [2].

3 Type-2 Bishop Darboux vector of Bishop Motion

Theorem 1 *Let the motion H/H' be represented by the equation* (4). *Then the component of darboux vectors of the motion H/H', respectively are*

$$\overrightarrow{\Omega} = (\epsilon_1 a_1 - \epsilon_2 n_2, \epsilon_2 a_2 - \epsilon_2 n_2, \epsilon_1 a_3 - \epsilon_2 n_3)$$

and

$$\overrightarrow{\omega} = (-\epsilon_2, \epsilon_1, 0)$$

Proof. From (6) we obtain

$$X = A^{-1}(Y - C) \tag{7}$$

and as

$$A^{-1} = A^T$$

and $\det A = +1$ we have (6) and (7) eliminating Y

$$Y'(s) = \Omega(Y - C) + C'(s) \tag{8}$$

with $\Omega = A'A^{-1}$, or explicity, by means of (5)

$$\Omega = A'A^{-1} = A'A^T = \begin{bmatrix} n_1' & a_1' & b_1' \\ n_2' & a_2' & b_2' \\ n_3' & a_3' & b_3' \end{bmatrix} \begin{bmatrix} n_1 & n_2 & n_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \tag{9}$$

One can find the following equalities

$$\begin{aligned} \zeta_1 &= \zeta_2 \times B = (a_2 b_3 - b_2 a_3, b_1 a_3 - a_1 b_3, a_1 b_2 - b_1 a_2) \\ \zeta_2 &= B \times \zeta_1 = (b_2 n_3 - n_2 b_3, n_1 b_3 - b_1 n_3, b_1 n_2 - n_1 b_2) \\ B &= \zeta_1 \times \zeta_2 = (n_2 a_3 - a_2 n_3, a_1 n_3 - n_1 a_3, n_1 a_2 - a_1 n_2) \end{aligned} \tag{10}$$

$$\begin{aligned} \zeta_1' &= \left(n_1', n_2', n_3'\right) = (-\epsilon_1 b_1, -\epsilon_1 b_{2,} - \epsilon_1 b_3) \\ \zeta_2' &= \left(a_1', a_2', a_3'\right) = (-\epsilon_2 b_1, -\epsilon_2 b_{2,} - \epsilon_2 b_3) \\ B' &= \left(b_1', b_2', b_3'\right) = (-\epsilon_1 n_1 + \epsilon_2 a_{1,} \epsilon_1 n_2 + \epsilon_2 a_{2,} \epsilon_1 n_3 + \epsilon_2 a_3) \end{aligned} \tag{11}$$

By using (9) and (11) ,we get

$$\Omega = \begin{bmatrix} n_1' n_1 + a_1' a_1 + b_1' b_1 & n_1' n_2 + a_1' a_2 + b_2' b_2 & n_1' n_3 + a_2' a_3 + b_1' b_3 \\ n_2' n_1 + a_2' a_1 + b_2' b_1 & n_2' n_2 + a_2' a_2 + b_2' b_2 & n_2' n_3 + a_2' a_3 + b_2' b_3 \\ n_3' n_1 + a_3' a_1 + b_3' b_1 & n_3' n_2 + a_3' a_2 + b_3' b_2 & n_3' n_3 + a_3' a_3 + b_3' b_3 \end{bmatrix}$$

So we obtain

$$\Omega = \begin{bmatrix} 0 & -(\epsilon_1 a_3 - \epsilon_2 n_3) & (\epsilon_1 a_2 - \epsilon_2 n_2) \\ (\epsilon_1 a_3 - \epsilon_2 n_3) & 0 & -(\epsilon_1 a_1 - \epsilon_2 n_1) \\ -(\epsilon_1 a_2 - \epsilon_2 n_2) & (\epsilon_1 a_1 - \epsilon_2 n_1) & 0 \end{bmatrix} \tag{12}$$

Since Ω is skew symmetric matrix from equality $\Omega\overrightarrow{\Omega} = 0$ we have $\Omega = (\epsilon_1 a_1 - \epsilon_2 n_1, \epsilon_1 a_2 - \epsilon_2 n_2, \epsilon_1 a_3 - \epsilon_2 n_3)$. Its component with respect to the moving frame follow from $\overrightarrow{\Omega} = A\overrightarrow{\varpi}$ [2],and we obtain for the vector

$$\begin{aligned} \overrightarrow{\varpi} &= A^{-1}\overrightarrow{\Omega} \\ &= \begin{bmatrix} n_1 & n_2 & n_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} \epsilon_1 a_1 - \epsilon_2 n_1 \\ \epsilon_1 a_2 - \epsilon_2 n_2 \\ \epsilon_1 a_3 - \epsilon_2 n_3 \end{bmatrix} \\ &= \begin{bmatrix} \epsilon_1 (n_1 a_1 + n_2 a_2 + n_3 a_3) - \epsilon_2 \left(n_1^2 + + n_2^2 + n_3^2\right) \\ \epsilon_1 \left(a_1^2 + + a_2^2 + a_3^2\right) - \epsilon_2 (n_1 a_1 + n_2 a_2 + n_3 a_3) \\ \epsilon_1 (b_1 a_1 + b_2 a_2 + b_3 a_3) - \epsilon_2 (b_1 n_1 + b_2 n_2 + b_3 n_3) \end{bmatrix} = \begin{bmatrix} \epsilon_1 \langle \zeta_1, \zeta_2 \rangle - \epsilon_2 \langle \zeta_1, \zeta_1 \rangle \\ \epsilon_1 \langle \zeta_2, \zeta_2 \rangle - \epsilon_2 \langle \zeta_2, \zeta_1 \rangle \\ \epsilon_1 \langle B, \zeta_2 \rangle - \epsilon_2 \langle B, \zeta_1 \rangle \end{bmatrix} = \\ &= \begin{bmatrix} -\epsilon_2 \\ \epsilon_1 \\ 0 \end{bmatrix} \end{aligned}$$

and so thus it holds

$$\overrightarrow{\varpi} = (-\epsilon_2, \epsilon_1, 0)\,. \tag{13}$$

■

4 Ruled surfaces according to type-2 Bishop frame

A ruled surface is a surface swept out by a straight line X moving along a curve α. The various position of the generating line X are called the rullings of the surface. Such a surface has a parametrization in ruled form as follows

$$\phi(s,v) = \alpha(s) + vX(s),$$

where α is the base curve and X is the director vector along α. If the tangent plane is constant along a fixed rulling, then the ruled surface is called developable surface. The ruled surface M in $\mathbb{E}^3$ is given by the parametrization

$$\begin{aligned} \phi &: I \times \mathbb{R} \to \mathbb{R}^3 \\ (s,v) &\to \phi(s,v) = \alpha(s) + vX(s) \end{aligned} \tag{14}$$

where $\alpha : I \to \mathbb{R}^3$ is adifferentiable curve parametrized by its arc-length in $\mathbb{R}^3$ and $X(s)$ is the director vector of the director curve such that X is ortogonal the tangent vector field T of the base curve α.

Remark 1 *The distribution parameter of the ruled surfaces $\phi(s,v)$ are given by*

$$P_x = \frac{\det(T, X, D_T X)}{\langle D_T X, D_T X \rangle} \tag{15}$$

where D is Levi-Civita connection on $\mathbb{R}^3$.

Theorem 2 *A ruled surfaces is developable surface if and only if the distrubition parameter of the ruled surface is zero[6].*

The foot on the main rulling of the common perpendicular of two constructive rullings in the ruled surfaces is called a central point .The locus of the central point is called striction curve.The parametrization of the striction curve on the ruled surface is given by

$$\overline{\alpha}(s) = \alpha(s) - \frac{\langle T, D_T X \rangle}{\langle D_T X, D_T X \rangle} X(s). \tag{16}$$

Theorem 3 *The ruled surfaces generated by a darboux vector in type-2 Bishop trihedron moving along a curve is always developable.*

Proof. From (13) we can write type-2 darboux vector as following

$$\overrightarrow{\omega} = -\epsilon_2 \overrightarrow{\zeta_1} + \epsilon_1 \overrightarrow{\zeta_2} \quad and \quad \|\overrightarrow{\omega}\| = \sqrt{\epsilon_1^2 + \epsilon_2^2} = \tau$$

So the director vector X as following

$$\overrightarrow{X} = \frac{\overrightarrow{\omega}}{\|\overrightarrow{\omega}\|} = \left(\frac{-\epsilon_2}{\tau}\right)\zeta_1 + \left(\frac{\epsilon_1}{\tau}\right)\zeta_2 \tag{17}$$

and differention (17) we have

$$\overrightarrow{X}' = \left(\frac{-\epsilon_2}{\tau}\right)'\zeta_1 + \left(\frac{\epsilon_1}{\tau}\right)'\zeta_2 = \frac{\epsilon_2^2\epsilon_1\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)^{\frac{3}{2}}}\zeta_1 + \frac{\epsilon_2^3\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)^{\frac{3}{2}}}\zeta_2 \tag{18}$$

If substituting (17)and(18) into (15) we get

$$\det(T, X, D_T X) = \begin{vmatrix} \sin\theta & -\cos\theta & 0 \\ \frac{-\epsilon_2}{\tau} & \frac{\epsilon_1}{\tau} & 0 \\ \left(\frac{-\epsilon_2}{\tau}\right)' & \left(\frac{\epsilon_1}{\tau}\right)' & 0 \end{vmatrix} = 0$$

hence we obtain from (15) $P_x = 0$. Thus ruled surfaces generated by a darboux vector in type-2 Bishop trihedron is developable. ■

Corollary 4 *In a type-2 Bishop motion, line of striction of ruled surface drawn by the unit darboux vector X can not be base curve.*
Proof. *Let's examine that striction curve for ruled surface generated by the unit darboux vector X can be taken as base curve or not ın type-2 Bishop motion $\overline{\alpha}$ be a striksiyon curve and we calculate from (18) following*

$$\left\|X'\right\|^2 = \left[\frac{\epsilon_2^2\epsilon_1\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)^{\frac{3}{2}}}\right]^2 + \left[\frac{\epsilon_2^3\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)^{\frac{3}{2}}}\right]^2 = \left[\frac{\epsilon_2^2\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)}\right]^2 \tag{19}$$

and from (2) and (18) we have

$$\left\langle X', T\right\rangle = \frac{\epsilon_2^2\epsilon_1\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)^{\frac{3}{2}}}\sin\theta - \frac{\epsilon_2^3\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)^{\frac{3}{2}}}\cos\theta \tag{20}$$

If substituting (17) , (19)and (20) into (16)

$$\begin{aligned}\overline{\alpha} &= \alpha - \left[\frac{\frac{\epsilon_2^2\epsilon_1\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)^{\frac{3}{2}}}\sin\theta - \frac{\epsilon_2^3\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)^{\frac{3}{2}}}\cos\theta}{\left[\frac{\epsilon_2^2\left(\frac{\epsilon_1}{\epsilon_2}\right)'}{(\epsilon_1^2+\epsilon_2^2)}\right]^2}\right]\left[\left(\frac{-\epsilon_2}{\tau}\right)\zeta_1 + \left(\frac{\epsilon_1}{\tau}\right)\zeta_2\right] \\ & \alpha + \frac{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin\theta - \cos\theta}{\left(\frac{\epsilon_1}{\epsilon_2}\right)'}\left(\zeta_1 - \left(\frac{\epsilon_1}{\epsilon_2}\right)\zeta_2\right), \quad q = \left(\frac{\epsilon_1}{\epsilon_2}\right) \neq cons\tan t\end{aligned}$$

$$\alpha = (a, b, c) = a\zeta_1 + b\zeta_2 + cB, q = \left(\frac{\epsilon_1}{\epsilon_2}\right)$$

$$\overrightarrow{\overline{\alpha}} = (a\zeta_1 + b\zeta_2 + cB) + \frac{q\sin\theta - \cos\theta}{q'}(\zeta_1 - q\zeta_2)$$

So we obtain

$$\overrightarrow{\overline{\alpha}} = \left(a + \frac{q\sin\theta - \cos\theta}{q'}\right)\zeta_1 + \left(b - \frac{q^2\sin\theta - \cos\theta}{q'}\right)\zeta_2 + cB$$

We show $x = \left(a + \frac{q\sin\theta - \cos\theta}{q'}\right)$ $y = \left(b - \frac{q^2\sin\theta - \cos\theta}{q'}\right)$ $z = c$.

Therefore $\overrightarrow{\overline{\alpha}} = (x, y, z)$. ■

5 One Parameter Spatial Motion in $\mathbb{E}^3$

Let $\alpha : I \to \mathbb{R}^3$ be a spacelike curve and $\{\zeta_1, \zeta_2, B\}$ be its type-2 Bishop frame where ζ_1, ζ_2, B first , second and binormal vector field of the curve α , respectively. The two coordinate system in $\mathbb{R}^3$which represent the moving space H and the fixed space H' respectively. Let X be a a unit vector

$$X \in Sp\{\zeta_1(s), \zeta_2(s), B(s)\} \text{ and } \overrightarrow{X} = x_1\zeta_1 + x_2\zeta_2 + x_3B, \tag{21}$$

Such that $\langle X, X\rangle = 1$. We can obtain the distrubition parameter of the ruled surface generated by a straight line X of the moving space H. Differentiating (21) with respect to s, we get

$$D_TX = x_1\zeta_1' + x_2\zeta_2' + x_3B', \qquad x_1^2 + x_2^2 + x_3^2 = 1 \tag{22}$$

By using the type-2 Bishop frame in (22) , we obtain

$$D_TX = x_3\epsilon_1\zeta_1 + x_3\epsilon_2\zeta_2 - (x_1\epsilon_1 + x_2\epsilon_2)B$$

From (16) we get

$$P_x = \frac{(x_1^2\epsilon_2 - x_1x_2\epsilon_1 - \epsilon_2)\sin\theta + (x_2^2\epsilon_1 - x_1x_2\epsilon_2 - \epsilon_1)\cos\theta}{-x_1^2\epsilon_2^2 - x_2^2\epsilon_1^2 + 2x_1x_2\epsilon_1\epsilon_2 + \epsilon_1^2 + \epsilon_2^2} \tag{23}$$

Substuting (3) into (23) we obtain

$$P_x = \frac{1}{\sqrt{\epsilon_1^2 + \epsilon_2^2}}$$

Corollary 5 *According to the type-2 Bishop frame , there is no developable ruled surface generated by a straight line* X *in* $\mathbb{E}^3$.

Corollary 6 *In type-2 Bishop motion striction curve for ruled surface generated by the vector* X *can be taken as base curve.*

Proof. *From Equation* (21) and (22) proof is clear. ■

5.1 Special Cases

Let M be a ruled surface given by the paremetrization (14) and X be the director vector of the base curve α in $\mathbb{E}^3$.

5.1.1 The Case $X = \zeta_1$

In this case , $x_1 = 1, x_2 = x_3 = 0$, thus from (23)

$$P_{\zeta_1} = \frac{-\cos\theta}{\epsilon_1}$$

$P_{\zeta_1} = 0$ if and only if $\cos\theta = 0$. Thus we have $\epsilon_1 = 0$. Which is a contradiction. Hence the follwing theorem is hold:

Theorem 7 *During the one-parameter spatial motion $H/H.'$ There is no developable ruled surface in the fixed space H' generated by the first vector field ζ_1 line of the curve $\alpha(s)$ in the moving space H .*

5.1.2 The Case X=ζ_2

In this case , $x_2 = 1, x_1 = x_3 = 0$, thus from (23)

$$P_{\zeta_2} = \frac{-\sin\theta}{\epsilon_2}$$

$P_{\zeta_2} = 0$ if and only if $\sin\theta = 0$. Thus we have $\epsilon_2 = 0$. Which is a contradiction. Hence the follwing theorem is hold:

Theorem 8 *During the one-parameter spatial motion $H/H.'$ There is no developable ruled surface in the fixed space H' generated by the first vector field ζ_2 line of the curve $\alpha(s)$ in the moving space H.*

5.1.3 The Case X=B

In this case , $x_3 = 1, x_1 = x_2 = 0$, thus from (23)

$$P_B = \frac{-(\cos\theta\epsilon_1 + \sin\theta\epsilon_2)}{\epsilon_1^2 + \epsilon_2^2}$$

$P_B = 0$ if and only if $(\cos\theta\epsilon_1 + \sin\theta\epsilon_2) = 0$. Thus we have

$$\frac{\epsilon_1}{\epsilon_2} = -\frac{\sin\theta}{\cos\theta} \tag{24}$$

Using type-2 Bishop curvatures $\epsilon_1 = -\tau\cos\theta$ and $\epsilon_2 = -\tau\sin\theta$ in (24), we have $\cot\theta + \tan\theta = 0$. Which is a contradiction. Hence the follwing theorem is hold:

Theorem 9 *During the one-parameter spatial motion $H/H^{'}$. There is no developable ruled surface in the fixed space $H^{'}$ generated by the first vector field B line of the curve $\alpha(s)$ in the moving space H .*

5.1.4 The Case $\mathbf{X} \in Sp\{\zeta_1(s), \zeta_2(s)\}$

In this case , x_3 is zero. So director vector is given by $X = x_1\zeta_1 + x_2\zeta_2$, $x_1^2 + x_2^2 = 1$. The distrubition parameter of ruled surface given by

$$P_x = \frac{1}{\sqrt{\epsilon_1^2 + \epsilon_2^2}}$$

Therefore according to the type-2 Bishop frame , there is no developable ruled surface generated by a straight line X in $\mathbb{E}^3$.

5.1.5 The Case $\mathbf{X} \in Sp\{\zeta_1(s), B(s)\}$

In this case , x_2 is zero. So director vector is given by $X = x_1\zeta_1 + x_3\zeta_2$, $x_1^2 + x_3^2 = 1$. The distrubition parameter of ruled surface given by

$$P_x = \frac{1}{\sqrt{\epsilon_1^2 + \epsilon_2^2}}$$

Therefor according to the type-2 Bishop frame , there is no developable ruled surface generated by a straight line X in $\mathbb{E}^3$.

5.1.6 The Case $\mathbf{X} \in Sp\{\zeta_2(s), B(s)\}$

From Theorem(2) its obvious that the according to type-2 Bishop frame, there is no developable ruled surface.

6 References

[1] R. L. Bishop, There is More than One Way to Frame a Curve, *The American Mathematical Monthly*, **3** (1975), 246-251. https://doi.org/10.2307/2319846

[2] O. Bottema, B. Roth, *Theoretical Kinematics*, North Holland Publ. Com., 1979.

[3] B. Bükcü, M.K. Karacan, On the slant helices according to Bishop frame of the timelike curve in Lorentzian space, *Tamkang J. Math.*, **39** (2008), 255-262.

[4] B. Bükcü, M.K. Karacan, Special Bishop motion and Bishop Darboux rotation axis of the space curve, *J. Dyn. Syst. Geom. Theor.*, **6** (2008), 27-34. https://doi.org/10.1080/1726037x.2008.10698542

[5] B. Bükçü, M.K. Karacan, The slant helices according to Bishop frame, *Int. J. Comput. Math. Sci.*, **3** (2009), 67-70.

[6] M.P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice Hall, Englewood Cliffs, NJ, 1976.

[7] M.K. Karacan, B. Bükcü, N. Yuksel, On the dual Bishop Darboux rotation axis of the dual space curve, *Appl. Sci.*, **10** (2008), 115-120.

[8] S. Yılmaz, M. Turgut, A new version of Bishop frame and an application to spherical images, *J. Math. Anal. Appl.*, **371** (2010), 764-776. https://doi.org/10.1016/j.jmaa.2010.06.012

[9] N. Yuksel, The Ruled Surfaces according to Bishop Frame in Minkowski 3-Space, *Abstract and Applied Analysis*, **2013** (2013), 1-5. https://doi.org/10.1155/2013/810640

Received: October 21, 2016; Published: January 19, 2017