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A New Approach to Evaluate Operations on Multi Granular Nano Topology

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Abstract

In this paper we propose a new concept on multi equivalence relation stimulated by nano topological space called as "Multi Granular Nano Topology (MGNT)". The well established set theoretical operations such as complement,union and intersection were defined and classified into four nano types in a table form based on MGNT. From the nano types of table we found that multiple answers indicating impreciseness and ambiguity in the information that is available with the user to classify object of a universe.

Mathematical Subject Classification: 54B05, 54C05

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1 Introduction

Lellis Thivagar et al[3] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations and boundary region of X.The elements of a nano topological space are called the nano open sets. But certain nano terms are satisfied simply to mean "very small". It originates from the Greek word "Nanos" which means "dwarf" in its modern scientific sense, an order to magnititude - one billionth of something. Nano car is an example. The topology recommended here is named so because of its size, since it has atmost five elements in it. In the granular computing point of view nano topological space is based on single granulation, its named as the indiscerniblity relation. This nano topological space model has been now extended to a multi granular nano topological space based on multiple equivalence relation, in where the set approximations are defined by using multiple equivalence relations on the universe. Several fundamental properties and an interesting classification of nano topological space have also been made.Further, we have studied the nano types of complement of a subset in multi granular nano topological space and also the nano types of union and intersection of two subsets in universe of multi granular nano topological spaces and also some examples are considered which are beneficial in solving practical problems.

2 Preliminary

Definition 2.1 *[3]: Let $\mathcal{U}$ be a non empty finite set of objects called the universe, R be an equivalence relation on $\mathcal{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be approximation space.Let $X \subseteq \mathcal{U}$.*

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{ \bigcup_{x\in\mathcal{U}} \{R(x) : R(x) \subseteq X\} \right\}$, where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \left\{ \bigcup_{X\in\mathcal{U}} \{R(x) : R(x) \cap X \neq \emptyset\} \right\}$.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is

denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 *[3]: Let* $\mathcal{U}$ *be the universe,* R *be an equivalence relation on* $\mathcal{U}$ *and* $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ *where* $X \subseteq \mathcal{U}$ *and* $\tau_R(X)$*satisfies the following axioms.*

(i) $\mathcal{U}$ *and* $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any sub collection$\tau_R(X)$ *is in* $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ *is in* $\tau_R(X)$.

That is, $\tau_R(X)$ *forms a topology on* $\mathcal{U}$ *and its called as the nano topology on* $\mathcal{U}$ *with respect to X.*

For more details we refer the reader to Abo-Tabl[1], Dubois et al[2], Pei[5], Pawlak[6-7], Qian et al[8]. Throughout this paper $\mathcal{U}$ is a non empty finite universe $X \subseteq \mathcal{U}$, then R and S are two equivalence relations on $\mathcal{U}$ respectively. Then $L_{R+S}(X)$,$\mathrm{U}_{R+S}(X)$, $B_{R+S}(X)$ are the lower,upper approximations and boundary region in $\tau_{R+S}(X)$ is called multi granular nano topological space based on multiple equivalence relation, and also through out this paper $\mathcal{U}/R, \mathcal{U}/S$, denote the multi granulation defined on the universe $\mathcal{U}$ and also $(\mathcal{U}, \tau_{R+S}(X \cup Y))$ and $(\mathcal{U}, \tau_{R+S}(X \cap Y))$ and $(\mathcal{U}, \tau_{R+S}(X^c))$, represent the multi granular nano topological space with respect to union and intersection,complement of two subsets X and Y with respect to 'R+S'.

3 Complement of Multi Granular Nano Topological Space(CMGNTS)

In this section, we will introduce the notion of complementation in multi granular nano topological space and investigate some of their properties.

Definition 3.1 *: Let* $\mathcal{U}$ *be a non empty finite set of objects called the universe and R, S be any two equivalence relations on* $\mathcal{U}$ *named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another.Let* $X^c \subseteq \mathcal{U}$

(i) The multi lower approximation of X^c *with respect to R and S is the set of all objects which can be for certain classified as* X^c *with respect to R and S and it is denoted by* $L_{R+S}(X^c)$. *That is,* $L_{R+S}(X^c) = \bigcup_{x \in \mathcal{U}} \{[x] : R(x) \subseteq X^c \text{ or } S(x) \subseteq X^c\}$, *where R(x) and S(x) denotes the equivalence class determined by x.*

(ii) *The multi upper approximation of X^c with respect to R and S is the set of all objects which can be possibly classified as X^c with respect to R and S and it is denoted by $U_{R+S}(X^c)$. That is, $U_{R+S}(X^c) = \bigcup_{x\in\mathcal{U}}\{[x] : R(x)\cap X^c \neq \emptyset$ and $S(x)\cap X^c \neq \emptyset\}$, where R(x) and S(x) denotes the equivalence class determined by x.*

(iii) *The multi boundary region of X^c with respect to R and S is the set of all objects which can be classified neither as X^c nor as not X^c with respect to R and S and it is denoted by $B_{R+S}(X^c)$. That is, $B_{R+S}(X^c)=U_{R+S}(X^c) - L_{R+S}(X^c)$.*

Definition 3.2 *: Let $\mathcal{U}$ be the universe and R, S be any two equivalence relations on $\mathcal{U}$ and $\tau_{R+S}(X^c) = \{\mathcal{U}, \emptyset, L_{R+S}(X^c), U_{R+S}(X^c), B_{R+S}(X^c)\}$ forms a topology on $\mathcal{U}$ called as the multi granular nano topology on $\mathcal{U}$ with respect to X^c. We call $(\mathcal{U}, \tau_{R+S}(X^c))$ as the multi granular nano topological space in terms of complement.*

Example 3.3 *: Let $\mathcal{U} = \{a, b, c, d, e\}$ and $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d, e\}\}$ and $\mathcal{U}/S = \{\{b\}, \{a, c\}, \{d, e\}\}$ be two equivalence classes on $\mathcal{U}$ and let $X = \{a, b\} \subseteq \mathcal{U}$ and $X^c = \{c, d, e\}$. Then $L_R(X^c) = \{d, e\}, U_R(X^c) = \{b, c, d, e\}$, $B_R(X^c) = \{b, c\}$ and also $L_S(X^c) = \{d, e\}, U_S(X^c) = \{a, c, d, e\}$ and $B_S(X^c) = \{a, c\}$. Therefore the nano topology $\tau_R(X^c) = \{\mathcal{U}, \emptyset, \{d, e\}, \{b, c, d, e\}, \{b, c\}\}$, $\tau_S(X^c) = \{\mathcal{U}, \emptyset, \{d, e\}, \{a, c, d, e\}, \{a, c\}\}$.Now $L_{R+S}(X^c) = \{d, e\}$, $U_{R+S}(X^c) = \{c, d, e\}$ and $B_{R+S}(X^c) = \{c\}$.Hence the multi granular nano topology in terms of complement $\tau_{R+S}(X^c) = \{\mathcal{U}, \emptyset, \{d, e\}, \{c, d, e\}, \{c\}\}$.*

Definition 3.4 *: Let $\mathcal{U}$ be a non empty finite universe and let $X \subseteq \mathcal{U}$, $\mathcal{U}/\mathcal{R}$,$\mathcal{U}/\mathcal{S}$ be two equivalence classes defined on $\mathcal{U}$.*

Nano Type-1 $(\mathcal{N}T_1)$ ***(Roughly Nano definable)****:*

If $L_{R+S}(X) \neq U_{R+S}(X)$ or $L_{R+S}(X)= U_{R+S}(X)$, where $L_{R+S}(X) \neq \emptyset$ and $U_{R+S}(X) \neq \mathcal{U}$, then either $\tau_{R+S}(X) = \{\mathcal{U}, \emptyset, L_{R+S}(X), U_{R+S}(X), B_{R+S}(X)\}$ or $\tau_{R+S}(X) = \{\mathcal{U}, \emptyset, L_{R+S}(X)\}$ then X is said to be Roughly nano definable.

Nano Type-2 $(\mathcal{N}T_2)$ ***(Internally Nano undefinable)****:*

If $L_{R+S}(X) = \emptyset$ and $U_{R+S}(X) \neq \mathcal{U}$, then $\tau_{R+S}(X) = \{\mathcal{U}, \emptyset, U_{R+S}(X)\}$ then X is said to be Internally nano undefinable.

Nano Type-3 $(\mathcal{N}T_3)$***(Externally Nano undefinable)****:*

If $L_{R+S}(X) \neq \emptyset$ and $U_{R+S}(X) = \mathcal{U}$, then $\tau_{R+S}(X) = \{\mathcal{U}, \emptyset, L_{R+S}(X), B_{R+S}(X)\}$ then X is said to be externally nano undefinable.

Nano Type-4 $(\mathcal{N}T_4)$***(Totally Nano undefinable)****:*

If $L_{R+S}(X) = \emptyset$ and $U_{R+S}(X) = \mathcal{U}$, then $\tau_{R+S}(X) = \{\mathcal{U}, \emptyset\}$ then X is said to be Totally nano undefinable.

Remark 3.5 : *Based on the classification of nano topological space, we classify the type of multi granular nano topological space. Hence the following table summaries it in a tabular form:*

Table-I

$\boldsymbol{X}$	$\boldsymbol{X^c}$
$\mathcal{N}T_1$	$\mathcal{N}T_1$
$\mathcal{N}T_2$	$\mathcal{N}T_3$
$\mathcal{N}T_3$	$\mathcal{N}T_2$
$\mathcal{N}T_4$	$\mathcal{N}T_4$

$Table\ for\ types\ of\ X\, and\, X^c\ with\ respect\ to\ R+S.$

The following example illustrates the entry of Row 1, and also the Justification for entry in Row 1.

Example 3.6 : *Let* $\mathcal{U} = \{a, b, c, d, e\}$ *and let* $\mathcal{U}/R = \{\{a, b\}, \{c\}, \{d, e\}\}$ *and* $\mathcal{U}/S = \{\{a\}, \{b\}, \{c, d, e\}\}$ *two equivalence classes defined on* $\mathcal{U}$ *and let* $X = \{b, c\} \subseteq \mathcal{U}$*, then* $\tau_{R+S}(X)= \{\mathcal{U}, \emptyset, \{b, c\}\}$*, So X is of type-1 with respect to R+S and also* $X^c = \{a, d, e\}$*, then* $\tau_{R+S}(X^c) = \{\mathcal{U}, \emptyset, \{a, d, e\}\}$*. Therefore*$X^c$ *is also of type-1 with respect to P+Q.*

Proposition 3.7 : *Let* $(\mathcal{U}, \tau_{R+S}(X^c))$ *be a multi granular nano topological space in terms of complement.Then, for all* $X^c \subseteq \mathcal{U}$

(i) $L_{R+S}(X^c)=\{U_{R+S}(X)\}^c$

(ii) $U_{R+S}(X^c)=\{L_{R+S}(X)\}^c$.

4 Union of Multi Granular Nano Topological Space (UMGNTS)

In this section, we establish the concept of union for X,Y in multi granular nano topological space and its properties. It is interesting to note that out of sixteen cases, as many as nine are unambiguous.

Definition 4.1 : *Let* $\mathcal{U}$ *be universe and R, S be any two equivalence relations defined on* $\mathcal{U}$ *named as the indiscernibility relation. Elements belonging to the same equivalence classes are said to be indiscernible with one another and* $\tau_{R+S}(X \cup Y) = \{\mathcal{U}, \emptyset, L_{R+S}(X \cup Y), U_{R+S}(X \cup Y), B_{R+S}(X \cup Y)\}$*,where* $L_{R+S}(X \cup Y), U_{R+S}(X \cup Y), B_{R+S}(X \cup Y)$ *defined as follows. Let* $X, Y \subseteq \mathcal{U}$

(i) *The multi lower approximation of* $X \cup Y$ *with respect to* R *and* S *is the set of all objects which can be for certain classified as* $X \cup Y$ *with respect to* R *or* S *and it is denoted by* $L_{R+S}(X \cup Y)$. *That is,* $L_{R+S}(X \cup Y) = \bigcup_{x \in \mathcal{U}} \{[x] : R[x] \subseteq X \cup Y \text{ or } S[x] \subseteq X \cup Y\}$, *where R[x] and S[x] denotes the equivalence class determined by x.*

(ii) *The multi upper approximation of* $X \cup Y$ *with respect to* R *and* S *is the set of all objects which can be possibly classified as* $X \cup Y$ *with respect to* R *and* S *and it is denoted by* $U_{R+S}(X \cup Y)$. *That is* $U_{R+S}(X \cup Y) = \bigcup_{x \in \mathcal{U}} \{[x] : R[x] \cap (X \cup Y) \neq \emptyset, S[x] \cap (X \cup Y) \neq \emptyset\}$.

(iii) *The multi boundary region of* $X \cup Y$ *with respect to* R *and* S *is the set of all objects which can be classified neither as* $X \cup Y$ *nor as* $X \cup Y$ *with respect to* R *and* S *and it is denoted by* $B_{R+S}(X \cup Y) = U_{R+S}(X \cup Y)$ - $L_{R+S}(X \cup Y)$.

That is, $\tau_{R+S}(X \cup Y)$ *forms a topology on* $\mathcal{U}$ *called as the multi granular nano topology on* $\mathcal{U}$ *with respect to* $X \cup Y$. *We call* $(\mathcal{U}, \tau_{R+S}(X \cup Y))$ *as the multi granular nano topological space in terms of union.*

Example 4.2 : *Let* $\mathcal{U} = \{a, b, c, d, e\}$ *with* $\mathcal{U}/R = \{\{a, b\}, \{c, d\}, \{e\}\}$ *and* $\mathcal{U}/S = \{\{a\}, \{b, d\}, \{c, e\}\}$ *be two equivalence classes defined on* $\mathcal{U}$. *Let* $X = \{a, b\} \subseteq \mathcal{U}$, *and* $Y = \{b, c\} \subseteq \mathcal{U}$, *then* $X \cup Y = \{a, b, c\}$. *Hence* $\tau_{R+S}(X \cup Y) = \{\mathcal{U}, \emptyset, \{a, b\}, \{a, b, c, d\}, \{c, d\}\}$.

Remark 4.3 : *This subsection provides the types of union of two multi granular nano topological space with respect to R+S in the following table. Proofs with examples for some of the entires in the table are then given.*

Table - II

$\mathbf{X \cup Y}$	$\mathcal{N}\mathbf{T_1}$	$\mathcal{N}\mathbf{T_2}$	$\mathcal{N}\mathbf{T_3}$	$\mathcal{N}\mathbf{T_4}$
$\mathcal{N}\mathbf{T_1}$	$\mathcal{N}T_1$ $\mathcal{N}T_3$	$\mathcal{N}T_1$ $\mathcal{N}T_3$	$\mathcal{N}T_3$	$\mathcal{N}T_3$
$\mathcal{N}\mathbf{T_2}$	$\mathcal{N}T_1$ $\mathcal{N}T_3$	$\mathcal{N}T_1$ $\mathcal{N}T_2$ $\mathcal{N}T_3$ $\mathcal{N}T_4$	$\mathcal{N}T_3$	$\mathcal{N}T_3$ $\mathcal{N}T_4$
$\mathcal{N}\mathbf{T_3}$	$\mathcal{N}T_3$	$\mathcal{N}T_3$	$\mathcal{N}T_3$	$\mathcal{N}T_3$
$\mathcal{N}\mathbf{T_4}$	$\mathcal{N}T_3$	$\mathcal{N}T_3$ $\mathcal{N}T_4$	$\mathcal{N}T_3$	$\mathcal{N}T_3$ $\mathcal{N}T_4$

Table for type of $X \cup Y$ *with respect to* $R + S$.

Example 4.4 : *Verification for the entry (1,2) in the above table.*
Case(i): *Let* $\mathcal{U} = \{a, b, c, d, e\}$, *and* $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d, e\}\}$ *and* $\mathcal{U}/S = \{\{b\}, \{a, d\}, \{c, e\}\}$ *be two equivalence classes defined on* $\mathcal{U}$. *Let* $X = \{a, b\} \subseteq \mathcal{U}$ *and* $Y = \{c, d\} \subseteq \mathcal{U}$ *then* $\tau_{R+S}(X) = \{\mathcal{U}, \emptyset, \{a, b\}\}$ *is of nano type-1, and*$\tau_{R+S}(Y) = \{\mathcal{U}, \emptyset, \{c, d, e\}\}$ *is of nano type-2. So,* $X \cup Y = \{a, b, c, d\}$, *thus* $\tau_{R+S}(X \cup Y) = \{\mathcal{U}, \emptyset, \{a, b, c, d\}, \{e\}\}$, *which is of nano type-3.*
Case(ii): *Now if* $X = \{b, c\} \subseteq \mathcal{U}$ *and* $Y = \{d\} \subseteq \mathcal{U}$, *then* $\tau_{R+S}(X) = \{\mathcal{U}, \emptyset, \{b, c\}\}$ *is of nano type-1, and* $\tau_{R+S}(Y) = \{\mathcal{U}, \emptyset, \{d\}\}$ *is of nano type-2. So* $X \cup Y = \{b, c, d\}$, *Thus* $\tau_{R+S}(X \cup Y) = \{\mathcal{U}, \emptyset, \{b, c\}, \{b, c, d, e\}, \{d, e\}\}$, *which is of nano type-1.*

5 Intersection of Multi Granular Nano Topological Space (IMGNTS)

In this section, we establish the intersection of two subsets in $\mathcal{U}$ for multi granular nano topological space(MGNT), and also we have discussed some of its properties. It is interesting to note that out of sixteen cases, as many as nine are unambiguous.

Definition 5.1 : *Let* $\mathcal{U}$ *be the universe, and* $\mathcal{U}/R$, $\mathcal{U}/S$ *be any two equivalence classes are defined on* $\mathcal{U}$. *Elements belonging to the same equivalence class are said to be indiscernible with one another and* $\tau_{R+S}(X \cap Y) = \{\mathcal{U}, \emptyset, L_{R+S}(X \cap Y),\ U_{R+S}(X \cap Y), B_{R+S}(X \cap Y)\}$, *where* $L_{R+S}(X \cap Y)$, $U_{R+S}(X \cap Y), B_{R+S}(X \cap Y)$ *defined as follows. Let* $X \cap Y \subseteq \mathcal{U}$.

(i) The multi lower approximation of $X \cap Y$ *with respect to R+S is denoted by* $L_{R+S}(X \cap Y)$. *That is,*
$L_{R+S}(X \cap Y) = \bigcup_{x \in \mathcal{U}} \{[x] : R[x] \subseteq X \cap Y \text{ or } S[x] \subseteq X \cap Y\}$.

(ii) The multi upper approximation of $X \cap Y$ *with respect to R+S is denoted by* $U_{R+S}(X \cap Y)$. *That is*
$U_{R+S}(X \cap Y) = \bigcup_{x \in \mathcal{U}} \{[x] : R[x] \cap (X \cap Y) \neq \emptyset, S[x] \cap (X \cap Y) \neq \emptyset\}$.

(iii) The multi boundary region of $X \cap Y$ *with respect to R+S is denoted by* $B_{R+S}(X \cap Y)$. *That is,*$B_{R+S}(X \cap Y) = U_{R+S}(X \cap Y)$-$L_{R+S}(X \cap Y)$.

That is, $\tau_{R+S}(X \cap Y)$ *forms a topology on* $\mathcal{U}$ *called as the multi granular nano topology on* $\mathcal{U}$ *with respect to* $X \cap Y$. *We call* $(\mathcal{U}, \tau_{R+S}(X \cap Y))$ *as the multi granular nano topological space in terms of intersection.*

Example 5.2 :*Let* $\mathcal{U} = \{a, b, c, d, e\}$ *with* $\mathcal{U}/R = \{\{a, b\}, \{c, d\}, \{e\}\}$ *and* $\mathcal{U}/S = \{\{a, c\}, \{b, e\}, \{d\}\}$ *and let* $X = \{a, b\} \subseteq \mathcal{U}$ *and* $Y = \{a, c\} \subseteq \mathcal{U}$ *then*

$\tau_{R+S}(X) = \{\mathcal{U}, \emptyset, \{a,b\}\}$ and $\tau_{R+S}(Y) = \{\mathcal{U}, \emptyset, \{a,c\}\}$ and so $X \cap Y = \{a\}$. Hence multi granular nano topological space $\tau_{R+S}(X \cap Y) = \{\mathcal{U}, \emptyset, \{a\}\}$.

Remark 5.3 *: This subsection of multi granular nano topological space provides the types of intersection of X and Y with respect to R+S in the following table. Proofs with examples for some of the entires in the table are then given.*

Table - III

$\mathbf{X \cap Y}$	$\mathcal{N}\mathbf{T_1}$	$\mathcal{N}\mathbf{T_2}$	$\mathcal{N}\mathbf{T_3}$	$\mathcal{N}\mathbf{T_4}$
$\mathcal{N}\mathbf{T_1}$	$\mathcal{N}T_1$ $\mathcal{N}T_2$	$\mathcal{N}T_2$	$\mathcal{N}T_1$ $\mathcal{N}T_2$	$\mathcal{N}T_2$
$\mathcal{N}\mathbf{T_2}$	$\mathcal{N}T_2$	$\mathcal{N}T_2$	$\mathcal{N}T_2$	$\mathcal{N}T_2$
$\mathcal{N}\mathbf{T_3}$	$\mathcal{N}T_1$ $\mathcal{N}T_2$	$\mathcal{N}T_2$	$\mathcal{N}T_1$ $\mathcal{N}T_2$ $\mathcal{N}T_3$ $\mathcal{N}T_4$	$\mathcal{N}T_2$ $\mathcal{N}T_4$
$\mathcal{N}\mathbf{T_4}$	$\mathcal{N}T_2$	$\mathcal{N}T_2$	$\mathcal{N}T_2$ $\mathcal{N}T_4$	$\mathcal{N}T_2$ $\mathcal{N}T_4$

Table for type of $X \cap Y$ with respect to $R+S$.

Remark 5.4 *:The following table provides the types of intersection of two multi granular nano topological space with respect to R+S in the following table.Proofs with examples for some of the entries in the table are given.*
Proof of entry(2,1)*5.3 : suppose X is of type-2 with respect to (R+S) and Y is type-1 with respect to (P+Q) then it follows that $X \cap Y$ is of type-2 with respect to R+S. Let $\mathcal{U} = \{a,b,c,d,e\}$ with $\mathcal{U}/R = \{\{a\},\{b,d\},\{c,e\}\}$ and $\mathcal{U}/S = \{\{b\},\{a,d\},\{c,e\}\}$, and if $X = \{c\}$, then $\tau_{R+S}(X) = \{\mathcal{U}, \emptyset, \{c,e\}\}$ is of type-2 and $Y = \{c,d,e\}$. $\tau_{R+S}(Y) = \{\mathcal{U}, \emptyset, \{c,e\}\}, \{d,c,e\}, \{d\}\}$ is of type-1, then $X \cap Y = \{c\}$, $\tau_{R+S}(X \cap Y) = \{\mathcal{U}, \emptyset, \{c,e\}\}$ is of type-2. Thus $X \cap Y$ ia of type-2 with respect to R+S.*

Theorem 5.5 *: If $(\mathcal{U}, \tau_{R+S}(X \cap Y))$ is a multi granular nano topological space in terms of intersection. Let $X, Y \subseteq \mathcal{U}$, then the following properties are hold:*

(i) $L_{R+S}(X \cap Y) \subseteq L_{R+S}(X) \cap L_{R+S}(Y)$.

(ii) $U_{R+S}(X \cup Y) \supseteq U_{R+S}(X) \cup U_{R+S}(Y)$.

(iii) $X \subseteq Y \Longrightarrow L_{R+S}(X) \subseteq L_{R+S}(Y) and U_{R+S}(X) \subseteq U_{R+S}(Y)$.

(iv) $L_{R+S}(X \cup Y) \supseteq L_{R+S}(X) \cup L_{R+S}(Y)$

(v) $U_{R+S}(X \cap Y) \supseteq U_{R+S}(X) \cap U_{R+S}(Y)$.

Proposition 5.6 : *If* $(\mathcal{U}, \tau_{R+S}(X \cap Y))$ *is a multi granular nano topological space. Let* $X, Y \subseteq \mathcal{U}$. *Then the following properties are hold:*

(i) $L_{R+S}(X \cap Y) = [L_R(X) \cap L_R(Y)] \cup [L_S(X) \cap L_S(Y)]$.

(ii) $U_{R+S}(X \cup Y) = [U_R(X) \cup U_R(Y)] \cap [U_S(X) \cup U_S(Y)]$.

Proof:

(i) *For any* $[x] \in L_{R+S}(X \cap Y)$ $,\Longleftrightarrow R[x] \subseteq X \cap Y$ *or* $S[x] \subseteq X \cap Y$ $\Longleftrightarrow [x] \in L_R(X \cap Y)$ *or* $[x] \in L_S(X \cap Y) \Longleftrightarrow [x] \in L_R(X \cap Y) \cup [x] \in L_S(X \cap Y) \Longleftrightarrow [x] \in [L_R(X) \cap L_R(Y)] \cup [L_S(X) \cap L_S(Y)]$. *Hence* $L_{R+S}(X \cap Y) = [L_R(X) \cap L_R(Y)] \cup [L_S(X) \cap L_S(Y)]$.

(ii) *For any* $[x] \in U_{R+S}(X \cup Y) \Longleftrightarrow R[x] \cap (X \cup Y) \neq \emptyset, S[x] \cap (X \cup Y) \neq \emptyset$ $\Longleftrightarrow [x] \in U_R(X \cup Y)$, $[x] \in U_S(X \cup Y) \Longleftrightarrow [x] \in U_R(X \cup Y) \cap [x] \in U_S(X \cup Y) \Longleftrightarrow [x] \in [U_R[X] \cup U_R(Y)] \cap [x] \in [U_S[X] \cup U_S[Y]$. *Hence* $U_{R+S}(X \cup Y) = [U_R(X) \cup U_R(Y)] \cup [U_S(X) \cup U_S(Y)]$.

6 Application

In this section, we have validated the basic set theoretic operations of complementation, union and intersection in MGNT by taking the Faculty details in a data base table:

Table - IV

ID NO.	Grade	H.D	N.S
ID_1	AP	$M.C.A$	$T.N$
ID_2	PR	$Ph.D$	$A.P$
ID_3	APJ	$M.Sc$	$A.P$
ID_4	ASP	$Ph.D$	$T.N$
ID_5	PR	$Ph.D$	$A.P$
ID_6	APJ	$M.Sc$	$T.N$
ID_7	APJ	$M.Sc$	$ORISSA$
ID_8	SP	$Ph.D$	$ORISSA$
ID_9	AP	$M.C.A$	$W.B$
ID_{10}	ASP	$Ph.D$	$ORISSA$

Here the above table denote that H.D - Highest Degree,N.S - Native State and it will contain that T.N - TAMILNADU, A.P - ANDHRAPRADESH, W.B-WEST BENGAL. Then grade contain is AP - Assistant Professor, PR - Professor, APJ - Assistant Professor (Junior), ASP - Associate Professor, SP - Senior Professor. Let $\mathcal{U} = \{ID_1, ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_9, ID_{10}\}$, be the universe of list of faculties of Computer Science Engineering Department of an Engineering College. Let $\mathcal{U}/P = \{\{ID_1, ID_9\}, \{ID_2, ID_4, ID_5, ID_8, ID_{10}\}, \{ID_3, ID_6, ID_7\}\}$, be an equivalence class based on their Highest degree, and let $\mathcal{U}/Q = \{\{ID_1, ID_4, ID_6\}, \{ID_2, ID_3\}, \{ID_5\}, \{ID_9\}, \{ID_7, ID_8, ID_{10}\}\}$ be an another equivalence class based on their Native State.

Validation of Complementation in MGNT

Proof of entry (3,2) in the Table - I. Let X=$\{ID_1, ID_2, ID_7, ID_9\}$, and $X^c = \{ID_3, ID_4, ID_5, ID_6, ID_8, ID_{10}\}$. Then $\tau_{P+Q}(X)$=$\{\mathcal{U}, \emptyset, \{ID_1, ID_9\}, \{ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}\}$. where $L_{P+Q}(X)$=$\{ID_1, ID_9\}$ and $U_{P+Q}(X)$=$\mathcal{U}$ $B_{p+Q}(X)$=$\{ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}$. Thus X is of Type-3 and also we can find X^C. Then $\tau_{P+Q}(X^c)$ =$\{\mathcal{U}, \emptyset, \{ID_1, ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}\}$ where $L_{P+Q}(X^c)$=$\emptyset$ and $U_{P+Q}(X^c)$=$\{ID_1, ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}$, $B_{p+Q}(X^c)$=$\{ID_1, ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}$.Thus X^c is of Type-2.

Validation of union in MGNT

Proof of entry (1,3) in the Table - II. Let X = $\{ID_1, ID_3, ID_9\}$ and Y = $\{ID_1, ID_2, ID_3, ID_8, ID_9\}$.Then $X \cup Y$=$\{ID_1, ID_2, ID_3, ID_8, ID_9\}$, $L_{P+Q}(X)$= $\{ID_1, ID_9\}$ and $U_{P+Q}(X)$=$\{ID_1, ID_3, ID_6, ID_9\}$, $B_{P+Q}(X)$=$\{ID_3, ID_6\}$.Hence $\tau_{P+Q}(X) = \{\mathcal{U}, \emptyset, \{ID_1, ID_9\}, \{ID_1, ID_3, ID_6, ID_9\}, \{ID_3, ID_6\}\}$. Thus X is of Type-1 also we can find $L_{P+Q}(Y)$=$\{ID_1, ID_9\}$ and $U_{P+Q}(Y) = \mathcal{U}$.,$B_{p+Q}(Y) = \{ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}$. Thus $\tau_{P+Q}(Y) = \{\mathcal{U}, \emptyset, \{ID_1, ID_9\}, \{ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}\}$. Hence Y is of Type-3. Then $L_{P+Q}(X \cup Y)$=$\{ID_1, ID_9\}$, $U_{P+Q}(X \cup Y)$ =$\mathcal{U}$ and $B_{P+Q}(X \cup Y) = \{ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}$,hence $\tau_{P+Q}(X \cup Y) = \{\mathcal{U}, \emptyset, \{ID_1, ID_9\}, \{ID_2, ID_3, ID_4, ID_5, ID_6, ID_7, ID_8, ID_{10}\}\}$. Thus $X \cup Y$ is of Type-3.

Validation of intersection in MGNT

Proof of entry (1,2) in the Table-III. Let X=$\{ID_1, ID_3, ID_9\}$ and Y=$\{ID_1\}$. and $X \cap Y$=$\{ID_1\}$, $L_{P+Q}(X) = \{ID_1, ID_9\}$ and $U_{P+Q}(X) = \{ID_1, ID_2, ID_3, ID_4, ID_6, ID_7, ID_9\}$, $B_{P+Q}(X) = \{ID_2, ID_3, ID_4, ID_6, ID_7\}$. Thus $\tau_{P+Q}(X) = \{\mathcal{U}, \emptyset, \{ID_1, ID_9\}, \{ID_1, ID_2, ID_3, ID_4, ID_6, ID_7, ID_9\}, \{ID_2, ID_3, ID_4, ID_6, ID_7\}\}$. Thus X is of Type-1.We have $L_{P+Q}(Y) = \emptyset$,$U_{P+Q}(Y) = \{ID_1, ID_4, ID_6, ID_9\}\}$. $B_{p+Q}(Y) = \{ID_1, ID_4, ID_6, ID_9\}\}$. Thus $\tau_{P+Q}(Y) = \{\mathcal{U}, \emptyset, \{ID_1, ID_4, ID_6, ID_9\}\}$. Thus Y is of Type-2.Then we have $L_{P+Q}(X \cap Y) = \emptyset$. and

$U_{P+Q}(X \cap Y) = \{ID_1, ID_4, ID_6, ID_9\}$. $B_{P+Q}(X \cap Y) = \{ID_1, ID_4, ID_6, ID_9\}$. $\tau_{P+Q}(X \cap Y) = \{\mathcal{U}, \emptyset, \{ID_1, ID_4, ID_6, ID_9\}\}$. Thus $X \cap Y$ is of Type-2.

Observation Hence with respect to basic set theoretic operations such as union, intersection and complementation in MGNT, there are multiple answers in the resulting table due to impreciseness and ambiguity in information.

7 Conclusion

In this paper we have studied the properties of multi granular nano topology with respect to set theoretic operations such as union,intersection and complementation. The table shows that are multiple answers to some of the nano types based on MGNT. Also,we provide examples in some nano types to illustrate the fact that multiple answers can actually occur.These results can be used for further studies in approximation of classification and rule generation.

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