International Mathematical Forum, Vol. 12, 2017, no. 3, 103 - 110
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/imf.2017.612169

Formula for Lucas Like Sequence of Fourth Step and Fifth Step

Rena Parindeni

Department of Mathematics
University of Riau
Pekanbaru 28293, Indonesia

Sri Gemawati

Department of Mathematics
University of Riau
Pekanbaru 28293, Indonesia

Abstract

This article discusses a formula to solve the n terms Lucas like sequence of fourth-step and fifth-step from the formula of Natividad [International Journal of Mathematics and Scientific Computing, 3.2 (2013), 38-40]. This formula is proved using the strong mathematical induction.

Keywords: Lucas sequence n step, Lucas like sequence 4-step, Lucas like sequence 5-step

1 Introduction

Fibonacci sequence (F_n) is defined as a sequence of number recurrence of order two are expressed in the form of [1]

$$F_n = F_{n-1} + F_{n-2}, \quad F_0 = 0, \quad F_1 = 1. \tag{1}$$

Generalizations of Fibonacci sequence is Tribonacci sequence defined by $T_0 = 0$, $T_1 = 0$, $T_2 = 1$ and reccurence equations $T_n = T_{n-1}+T_{n-2}+T_{n-3}$. This only means that the previous three terms are added to find the next term. On the other hand, generalizations of Fibonacci sequence is Lucas sequence. Lucas developes a sequence that has the properties like Fibonacci sequence. Lucas sequence is defined in the form of [1]

$$L_n = L_{n-1} + L_{n-2}, \quad L_1 = 1, \quad L_2 = 3. \tag{2}$$

Generalizations of the Lucas sequence is Lucas 3-step, which is derived from the sum of the previous three terms. Natividad and Policarpio [4] find a formula for finding the n^{th} terms of Tribonacci like sequence and followed by Singh [7] who discusses a formula for finding the n^{th} terms of Tetranacci like sequence. Further research was continued by Natividad [5], he finds a formula to find the n^{th} term of Fibonacci sequence of higher order, i.e. Tetranacci, Pentanacci and Hexanacci like sequences. In this paper, we derive a general formula to find the n^{th} term of the Lucas 3-step. Based on these studies we discuss a new formula to find for Lucas 4-step and Lucas 5-step whose proof uses mathematical induction.

2 Lucas *n*-Step Sequence

Lucas sequence n step is Lucas obtained from the sum of n terms before. By Lucas definition in [6] this sequence is generalized in higher order that is expressed in the form of

$$L_{k+1}^{(n)} = L_k^{(n)} + L_{k-1}^{(n)} + ... + L_{k-n+1}^{(n)}, \tag{3}$$

with $k < 0$, $L_k^{(n)} = -1$ and $L_0^{(n)} = n$.

In [4] and [5] Natividad found a formula to find n terms from of Fibonacci like sequence from third, fourth, fifth Order. This article discusses a formula to find the n^{th} term of Lucas 4-step and 5-step.

3 Formula For Lucas Like Sequence Of Fourth Step and Fifth Step

Considering the equation (4), we obtain some equations as follows :

$$L_5^{(4)} = L_1^{(4)} + L_2^{(4)} + L_3^{(4)} + L_4^{(4)}$$

$$L_6^{(4)} = L_1^{(4)} + 2L_2^{(4)} + 2L_3^{(4)} + 2L_4^{(4)}$$
$$L_7^{(4)} = 2L_1^{(4)} + 3L_2^{(4)} + 4L_3^{(4)} + 4L_4^{(4)}$$
$$L_8^{(4)} = 4L_1^{(4)} + 6L_2^{(4)} + 7L_4^{(6)} + 8L_4^{(4)}$$

$$L_9^{(4)} = 8L_1^{(4)} + 12L_2^{(4)} + 14L_3^{(4)} + 15L_4^{(4)}$$
$$L_{10}^{(4)} = 15L_1^{(4)} + 23L_2^{(4)} + 27L_3^{(4)} + 29L_4^{(4)}$$
$$L_{11}^{(6)} = 29L_1^{(4)} + 44L_2^{(4)} + 52L_3^{(4)} + 56L_4^{(4)} \quad (4)$$

All values for the coefficients $L_1^{(4)}$,$L_2^{(4)}$,$L_3^{(4)}$ and $L_4^{(4)}$ where a list of which is shown in Table 1.

Table 1: The coefficients of $L_1^{(4)}, L_2^{(4)}, L_3^{(4)}, L_4^{(4)}$

n^{th}	$L_n^{(4)}$	Coefficients			
		$L_1^{(4)}$	$L_2^{(4)}$	$L_3^{(4)}$	$L_4^{(4)}$
5	$L_1^{(4)}$	1	1	1	1
6	$L_8^{(6)}$	1	2	2	2
7	$L_9^{(6)}$	2	3	4	4
8	$L_{10}^{(6)}$	4	6	7	8
9	$L_{11}^{(6)}$	8	12	14	15
10	$L_{12}^{(6)}$	15	23	27	29
11	$L_{13}^{(6)}$	29	44	52	56
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	$L_n^{(4)}$	M_{n-2}^*	$(M_{n-2}^* + M_{n-3}^*)$	$(M_{n-2}^*$ $+M_{n-3}^* + M_{n-4}^*)$	$(M_{n-2}^* + M_{n-3}^*$ $+M_{n-4}^* + M_{n-5}^*)$

Table 1 shows that the coeficient $L_1^{(4)}$,$L_2^{(4)}$,$L_3^{(4)}$ and $L_4^{(4)}$ for $n \geq 5$ follows the Fibonacci sequence patterns that can be presented in the following Table 2.

Table 2: The first 12 of Fibonacci numbers

n term	1	2	3	4	5	6	7	8	9	10	11	12
Tetranacci numbers	0	0	1	1	2	4	8	15	29	56	108	208

After observing and examining of Table 1 and Table 2, it can be seen that all the coefficients $L_1^{(4)}$ is a pattern that relies on the number Tetranacci of Lucas 4-step to be searched.Then the coefficient $L_2^{(4)}$ is th sum of the two numbers before, while the coeficcient Tetranacci $L_3^{(4)}$ is the sum of three numbers Tetranacci earlier.The pattern will continue until 5-step to obtain the following theorem.

Theorem 3.1 *For any real number* $L_1^{(4)}$*,*$L_2^{(4)}$*,*$L_3^{(4)}$ *and* $L_4^{(4)}$ *Lucas 4-step then.*

$$L_n^{(4)} = (M_{n-2}^*)L_1^{(4)} + (M_{n-2}^* + M_{n-3}^*)L_2^{(4)} + (M_{n-2}^* + M_{n-3}^*$$

$$+M^*_{n-4})L^{(4)}_3 + (M^*_{n-2} + M^*_{n-3} + M^*_{n-4} + M^*_{n-5})L^{(4)}_4 \quad (5)$$

where $L^{(4)}_n$ *n term Lucas 4-step,* $L^{(4)}_1$ *first term,* $L^{(4)}_2$ *second term,* $L^{(4)}_3$ *third term,* $L^{(4)}_4$ *fourth term and* M^*_{n-2}, M^*_{n-3}, M^*_{n-4}, M^*_{n-5} *Tetranacci numbers.*

Bukti. We shall prove above theorem by strong mathematical induction for $n \in N$.

Basic step: First take $n = 5$, then we get

$$L^{(4)}_5 = (M^*_3)L^{(4)}_1 + (M^*_3 + M^*_2)L^{(4)}_2 + (M^*_3 + M^*_2 + M^*_1)L^{(4)}_3$$

$$+(M^*_3 + M^*_2 + M^*_1 + M^0_0)L^{(4)}_4$$

$$= (1)L^{(4)}_1 + (1 + 0)L^{(4)}_2 + (1 + 0 + 0)L^{(4)}_3 + (1 + 0 + 0 + 0)L^{(4)}_4$$

$$L^{(4)}_5 = L^{(4)}_1 + L^{(4)}_2 + L^{(4)}_3 + L^{(4)}_4 \quad (6)$$

which is true (by defenition of Lucas n step).

Induction step : Take $k \in N$ for $k \geq 4$ and $L^{(3)}_n$ true and assumed for $n = 5, 6, \cdots, (k-3), (k-2), (k-1), k$, then

$$L^{(4)}_{k-1} = (M^*_{k-3})L^{(4)}_1 + (M^*_{k-3} + M^*_{k-4})L^{(4)}_2 + (M^*_{k-3} + M^*_{k-4} + M^*_{k-5})L^{(4)}_3$$

$$+(M^*_{k-3} + M^*_{k-4} + M^*_{k-5} + M^*_{k-6})L^{(4)}_4 \quad (7)$$

$$L^{(4)}_{k-2} = (M^*_{k-4})L^{(4)}_1 + (M^*_{k-4} + M^*_{k-5})L^{(4)}_2 + (M^*_{k-4} + M^*_{k-5} + M^*_{k-6})L^{(4)}_3$$

$$+(M^*_{k-4} + M^*_{k-5} + M^*_{k-6} + M^*_{k-7})L^{(4)}_4 \quad (8)$$

$$L^{(4)}_{k-3} = (M^*_{k-5})L^{(4)}_1 + (M^*_{k-5} + M^*_{k-6})L^{(4)}_2 + (M^*_{k-5} + M^*_{k-6} + M^*_{k-7})L^{(4)}_3$$

$$+(M^*_{k-5} + M^*_{k-6} + M^*_{k-7} + M^*_{k-8})L^{(4)}_4 \quad (9)$$

$$L^{(4)}_{k} = (M^*_{k-2})L^{(4)}_1 + (M^*_{k-2} + M^*_{k-3})L^{(4)}_2 + (M^*_{k-2} + M^*_{k-3} + M^*_{k-4})L^{(4)}_3$$

$$+(M^*_{k-2} + M^*_{k-3} + M^*_{k-4} + M^*_{k-5})L^{(4)}_4 \quad (10)$$

It must be proved that $L^{(4)}_{k+1}$ true. From 3,it is known that $L^{(4)}_{k+1} = L^{(4)}_{k-3} + L^{(4)}_{k-2} + L^{(4)}_{k-1} + L^{(4)}_k$, so

$$L^{(4)}_{k+1} = L^{(4)}_{k-3} + L^{(4)}_{k-2} + L^{(4)}_{k-1} + L^{(4)}_k$$

$$= (M^*_{k-5})L_1^{(4)} + (M^*_{k-5} + M^*_{k-6})L_2^{(4)} + (M^*_{k-5} + M^*_{k-6} + M^*_{k-7})L_3^{(4)}$$
$$+ (M^*_{k-5} + M^*_{k-6} + M^*_{k-7} + M^*_{k-8})L_4^{(4)} + (M^*_{k-4})L_1^{(4)} + (M^*_{k-4} + M^*_{k-5})L_2^{(4)}$$
$$+ (M^*_{k-4} + M^*_{k-5} + M^*_{k-6})L_3^{(4)} + (M^*_{k-4} + M^*_{k-5} + M^*_{k-6} + M^*_{k-7})L_4^{(4)}$$
$$+ (M^*_{k-3})L_1^{(4)} + (M^*_{k-3} + M^*_{k-4})L_2^{(4)} + (M^*_{k-3} + M^*_{k-4} + M^*_{k-5})L_3^{(4)}$$
$$+ (M^*_{k-3} + M^*_{k-4} + M^*_{k-5} + M^*_{k-6})L_4^{(4)} + (M^*_{k-2})L_1^{(4)}$$
$$+ (M^*_{k-2} + M^*_{k-3})L_2^{(4)} + (M^*_{k-2} + M^*_{k-3} + M^*_{k-4})L_3^{(4)}$$
$$+ (M^*_{k-2} + M^*_{k-3} + M^*_{k-4} + M^*_{k-5})L_4^{(4)} \tag{11}$$

$$L_{k+1}^{(4)} = (M^*_{k-2} + M^*_{k-3} + M^*_{k-4} + M^*_{k-5})L_1^{(4)}$$
$$= +[(M^*_{k-2} + M^*_{k-3} + M^*_{k-5}) + (M^*_{k-3} + M^*_{k-4} + M^*_{k-5} + M^*_{k-6})L_2^{(4)} +$$
$$[(M^*_{k-2} + M^*_{k-3} + M^*_{k-4} + M^*_{k-5}) + (M^*_{k-3} + M^*_{k-4} + M^*_{k-5} + M^*_{k-6})$$
$$+ (M^*_{k-4} + M^*_{k-5} + M^*_{k-6} + M^*_{k-7})]L_3^{(4)} + [(M^*_{k-2} + M^*_{k-3} + M^*_{k-4} + M^*_{k-5})$$
$$+ (M^*_{k-3} + M^*_{k-4} + M^*_{k-5} + M^*_{k-6}) + (M^*_{k-4} + M^*_{k-5} + M^*_{k-6} + M^*_{k-7})$$
$$+ (M^*_{k-5} + M^*_{k-6} + M^*_{k-7} + M^*_{k-8}) \tag{12}$$

Because basic step and induction step have been proved true, therefore the statement given is true.

Theorem 3.2 *For any real number* $L_1^{(5)}$, $L_2^{(5)}$, $L_3^{(5)}$, $L_4^{(5)}$ *and* $L_5^{(5)}$ *Lucas 5-step then*

$$L_n^{(5)} = (P^*_{n-2})L_1^{(5)} + (P^*_{n-2} + P^*_{n-3})L_2^{(5)} + (P^*_{n-2} + P^*_{n-3}$$
$$+ P^*_{n-4})L_3^{(4)} + (P^*_{n-2} + P^*_{n-3} + P^*_{n-4} + P^*_{n-5})L_4^{(5)} + (P^*_{n-2}$$
$$+ P^*_{n-3} + P^*_{n-4} + P^*_{n-5} + P^*_{n-6})L_5^{(5)} \tag{13}$$

where $L_n^{(5)}$ *n terms ,* $L_1^{(5)}$ *first term,* $L_2^{(5)}$ *second term,* $L_3^{(5)}$ *third term,* $L_4^{(5)}$ *fourth term,* $L_4^{(5)}$ *fifth term and* P^*_{n-2}, P^*_{n-3}, P^*_{n-4}, P^*_{n-5}, P^*_{n-5} *Pentanacci number.*

Bukti. We shall prove above theorem by strong mathematical induction $n \in N$

Basic Step: First we take $n = 6$, then we get

$$L_6^{(5)} = (P^*_{6-2})L_1^{(5)} + (P^*_{6-2} + P^*_{6-3})L_2^{(5)} + (P^*_{6-2} + P^*_{6-3} + P^*_{6-4})L_3^{(4)}$$

$$+(P^*_{6-2}+P^*_{6-3}+P^*_{6-4}+P^*_{6-5})L_4^{(5)}$$
$$+(P^*_{6-2}+P^*_{6-3}+P^*_{6-4}+P^*_{6-5}+P^*_{6-6})L_5^{(5)} \quad (14)$$

$$L_6^{(5)}=(P_4^*)L_1^{(5)}+(P_4^*+P_3^*)L_2^{(5)}+(P_4^*+P_3^*+P_2^*)L_3^{(4)}$$
$$+(P_2^*+P_3^*+P_2^*+P_1^*)L_4^{(5)}+(P_4^*+P_3^*+P_2^*+P_1^*+P_0^*)L_5^{(5)} \quad (15)$$

$$L_6^{(5)}=(1)L_1^{(5)}+(1+0)L_2^{(5)}+(1+0+0)L_3^{(4)}+(1+0+0+0)$$
$$L_4^{(5)}(1+0+0+0+0)L_5^{(5)} \quad (16)$$

$$L_6^{(5)}=L_1^{(5)}+L_2^{(5)}+L_3^{(4)}+L_4^{(5)}+L_5^{(5)} \quad (17)$$

which is true (by defenition of Lucas n step.

Induction step : Take $k\in N$ for $k\geq 5$ dan $L_n^{(5)}$ correctly assumed for $n=6,7,\cdots,(k-4),(k-3),(k-2),(k-1),k$, then

$$L_{k-4}^{(5)}=(P^*_{k-6})L_1^{(5)}+(P^*_{k-6}+P^*_{k-7})L_2^{(5)}+(P^*_{k-6}+P^*_{k-7}$$
$$+P^*_{k-8})L_3^{(5)}+(P^*_{k-6}+P^*_{k-7}+P^*_{k-8}+P^*_{k-9})L_4^{(5)}$$
$$+(P^*_{k-6}+P^*_{k-7}+P^*_{k-8}+P^*_{k-9}+P^*_{k-10})L_5^{(5)} \quad (18)$$

$$L_{k-3}^{(5)}=(P^*_{k-5})L_1^{(5)}+(P^*_{k-5}+P^*_{k-6})L_2^{(5)}+(P^*_{k-5}+P^*_{k-6}$$
$$+P^*_{k-7})L_3^{(5)}+(P^*_{k-5}+P^*_{k-6}+P^*_{k-7}+P^{*]_{k-8})L_4^{(5)}}$$
$$+(P^*_{k-5}+P^*_{k-6}+P^*_{k-7}+P^*_{k-8}+P^*_{k-9})L_5^{(5)} \quad (19)$$

$$L_{k-2}^{(5)}=(P^*_{k-4})L_1^{(5)}+(P^*_{k-4}+P^*_{k-5})L_2^{(5)}+(P^*_{k-4}+P^*_{k-5}$$
$$+P^*_{k-6})L_3^{(5)}+(P^*_{k-4}+P^*_{k-5}+P^*_{k-6}+P^*_{k-7})L_4^{(5)}$$
$$+(P^*_{k-4}+P^*_{k-5}+P^*_{k-7}+P^*_{k-7}+P^*_{k-8})L_5^{(5)} \quad (20)$$

$$L_{k-1}^{(5)}=(P^*_{k-3})L_1^{(5)}+(P^*_{k-3}+P^*_{k-4})L_2^{(5)}+(P^*_{k-3}+P^*_{k-4}$$
$$+P^*_{k-5})L_3^{(5)}+(P^*_{k-3}+P^*_{k-4}+P^*_{k-5}+P^*_{k-6})L_4^{(5)}+(P^*_{k-3}+P^*_{k-4}$$
$$+P^*_{k-5}+P^*_{k-6}+P^*_{k-7})L_5^{(5)} \quad (21)$$

$$L_k^{(5)} = (P^*_{k-2})L_1^{(5)} + (P^*_{k-2} + P^*_{k-3})L_2^{(5)} + (P^*_{k-2} + P^*_{k-3} + P^*_{k-4})L_3^{(5)} + (P^*_{k-2} + P^*_{k-3} + P^*_{k-4} + P^*_{k-5})L_4^{(5)} + (P^*_{k-2} + P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6})L_5^{(5)} \tag{22}$$

It must be proved that $L_{k+1}^{(5)}$ true. Based on equation of 3, it is known that $L_{k+1}^{(5)} = L_{k-4}^{(5)} + L_{k-3}^{(5)} + L_{k-2}^{(5)} + L_{k-1}^{(5)} + L_k^{(5)}$, so

$$\begin{aligned} L_{k+1}^{(5)} = {} & (P^*_{k-6})L_1^{(5)} + (P^*_{k-6} + P^*_{k-7})L_2^{(5)} + (P^*_{k-6} + P^*_{k-7} + P^*_{k-8})L_3^{(5)} + (P^*_{k-6} \\ & + P^*_{k-7} + P^*_{k-8} + P^*_{k-9})L_4^{(5)} + (P^*_{k-6} + P^*_{k-7} + P^*_{k-8} + P^*_{k-9} + P^*_{k-10})L_5^{(5)} \\ & + (P^*_{k-5})L_1^{(5)} + (P^*_{k-5} + P^*_{k-6})L_2^{(5)} + (P^*_{k-5} + P^*_{k-6} + P^*_{k-7})L_3^{(5)} + (P^*_{k-5} \\ & + P^*_{k-6} + P^*_{k-7} + P^*_{k-8})L_4^{(5)} + (P^*_{k-5} + P^*_{k-6} + P^*_{k-7} + P^*_{k-8} + P^*_{k-9})L_5^{(5)} \\ & + (P^*_{k-4})L_1^{(5)} + (P^*_{k-4} + P^*_{k-5})L_2^{(5)} + (P^*_{k-4} + P^*_{k-5} + P^*_{k-6})L_3^{(5)} + (P^*_{k-4} \\ & + P^*_{k-5} + P^*_{k-6} + P^*_{k-7})L_4^{(5)} + (P^*_{k-4} + P^*_{k-5} + P^*_{k-7} + P^*_{k-7} + P^*_{k-8})L_5^{(5)} \\ & + (P^*_{k-3})L_1^{(5)} + (P^*_{k-3} + P^*_{k-4})L_2^{(5)} + (P^*_{k-3} + P^*_{k-4} + P^*_{k-5})L_3^{(5)} \\ & + (P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6})L_4^{(5)} + (P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6} \\ & + P^*_{k-7})L_5^{(5)} + (P^*_{k-2})L_1^{(5)} + (P^*_{k-2} + P^*_{k-3})L_2^{(5)} + (P^*_{k-2} + P^*_{k-3} \\ & + P^*_{k-4})L_3^{(5)} + (P^*_{k-2} + P^*_{k-3} + P^*_{k-4} + P^*_{k-5})L_4^{(5)} + (P^*_{k-2} + P^*_{k-3} \\ & + P^*_{k-4} + P^*_{k-5} + P^*_{k-6})L_5^{(5)} \end{aligned} \tag{23}$$

$$\begin{aligned} L_{k+1}^{(5)} = {} & (P^*_{k-2} + P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6})L_1^{(5)} + [P^*_{k-2} + P^*_{k-3} + P^*_{k-4} \\ & + P^*_{k-5} + P^*_{k-6}) + (P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6} + P^*_{k-7})]L_2^{(5)} + [(P^*_{k-2} \\ & + P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6}) + (P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6} + P^*_{k-7}) \\ & + (P^*_{k-4} + P^*_{k-5} + P^*_{k-6} + P^*_{k-7} + P^*_{k-8})]L_3^{(5)} + [(P^*_{k-2} + P^*_{k-3} + P^*_{k-4} \\ & + P^*_{k-5} + P^*_{k-6}) + (P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6} + P^*_{k-7}) + (P^*_{k-4} + P^*_{k-5} \\ & + P^*_{k-6} + P^*_{k-7} + P^*_{k-8}) + (P^*_{k-5} + P^*_{k-6} + P^*_{k-7} + P^*_{k-8} + P^*_{k-9})L_4^{(5)} \\ & + [(P^*_{k-2} + P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6}) + (P^*_{k-3} + P^*_{k-4} + P^*_{k-5} + P^*_{k-6} \\ & + P^*_{k-7}) + (P^*_{k-4} + P^*_{k-5} + P^*_{k-6} + P^*_{k-7} + P^*_{k-8}) + (P^*_{k-5} + P^*_{k-6} + P^*_{k-7} \\ & + P^*_{k-8} + P^*_{k-9}) + (P^*_{k-6} + P^*_{k-7} + P^*_{k-8} + P^*_{k-9} + P^*_{k-10})]L_1^{(5)} \end{aligned} \tag{24}$$

Because basic step and induction step have been proved true, then the statement given is true.

4 Conclusion

From the results of this article can be concluded that the formula to find the n^{th} terms of the generalizations Fibonacci sequence is not only Tribonacci, Tetranacci and Pentanacci but also presents in Lucas 3-step, 4-step and 5-step. This formula can be proved by mathematical induction.

References

[1] T. Koshy, *Fibonacci and Lucas Number with Applications*, Jhon Wiley and Sons, Inc., New York, 2011. https://doi.org/10.1002/9781118033067

[2] P. E. Mendelshon, The Pentanacci Number, *The Fibonacci Quarterly*, (1980), 31-33.

[3] L. R. Natividad, Deriving A Formula in Solving Fibonacci Like Sequence, *International Journal of Mathematics and Scientific Computing*, **1** (2011), 19-21.

[4] L. R. Natividad and Policarpio, A Novel Formula in Solving Tribonacci Like Sequence, *Gen. Math. Notes*, **17** (2013), 82-87.

[5] L. R. Natividad, On Solving Fibonacci Like Sequence of Fourth, Fifth, Sixth Order, *International Journal of Mathematics and Scientific Computing*, **3** (2013), 38–40.

[6] T. D. Noe and J. V. Post, Primes in Fibonacci n-Step and Lucas n-Step Sequences, *Journal of Integer Sequence*, **8** (2005), 1-12.

[7] B. Singh, P. Bhaduria, O. Sikhwal and K. Sisodiya, A Formula For Tetranacci-Like Sequence, *Gen. Math. Notes*, **20** (2014), 136-141.

[8] M. E. Waddill, The Tetranacci Sequence and Generalizations, *Fibonacci Quartely*, **30** (1992), 9-19.

Received: December 27, 2016; Published: January 18, 2017