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Soft βc-Open Sets and Soft βc-Continuity

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Abstract

In this paper, a new class of generalized soft open sets in soft topological spaces, called soft βc-open sets, is introduced and studied. In particular, the class of βc-open sets is contained propely in the class of soft β-open sets. This class is incomparable with the classes of soft open sets, soft pre-open sets and soft semi-open sets. We also introduce and study the concepts of soft βc-interior, soft βc-closure and soft βc-continuous function.

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1 Introduction and preliminaries

Molodtsove [5] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainly. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement, and so on. Soft set theory and its applications have shown great development in recent years. This is because of the general nature of parametrization expressed by a soft set [11].

Definition 1.1. [5] Let X be an initial universe set and E a set of parameters. Let $\mathcal{P}(X)$ be the power set of X and A a nonempty subset of E. A pair (F, A), denoted by F_A, is called *a soft set* over X if F is a mapping given by $F : A \to \mathcal{P}(X)$. Shortly, a soft set over X is a parameterized family of subsets of the universe X. The family of all these soft sets over X is denoted by $SS(X)_A$. For a particular $e \in A$, the collection $\{F(e) : F\ is\ a\ soft\ set\}$ is considered to be the *set of e-approximate elements* of the soft sets. If $e \notin A$, then $F(e) = \emptyset$.

Let I be an arbitrary indexed set and $L = \{(F_i, A) : i \in I\}$ be a subfamily of $SS(X)_A$. *The union* of L is the soft set (H, A)[15], where $H(e) = \bigcup_{i\in I} F_i(e)$ for each $e \in A$. We write $\tilde{\bigcup}_{i\in I}(F_i, A) = (H, A)$. *The intersection* of L is the soft set (M, A)[15], where $M(e) = \bigcap_{i\in I} F_i(e)$ for each $e \in A$. We write $\tilde{\bigcap}_{i\in I}(F_i, A) = (M, A)$. If (G, A), (F, A) are two soft sets over X, we say (F, A) is *a soft subset* of (G, B) (or (G, B) is said to be *a soft superset* of (F, A))[15], denoted by $(F, A)\tilde{\subseteq}(G, B)$, if $A \subseteq B$ and $F(e) \subseteq G(e)$, $\forall e \in A$. Also (F, A) and (G, B) are called *soft equal*, if $A = B$ and $F(e) = G(e)$, $\forall e \in A$ [15]. For a soft set (F, A) over X, we define $(F, A)^c = (F^c, A)$ to be the soft complement of (F, A) such that $F^c(e) = X - F(e)$[12]. $(F, A)\tilde{\subseteq}(G, A)$ iff $(G, A)^c\tilde{\subseteq}(F, A)^c$. A soft set (F, A) over X is called *a null soft set* [15](resp. *absolute soft set*)[15], denoted by Φ_A (resp. denoted by X_A), if for all $e \in A$, $F(e) = \emptyset$ (resp. if for all $e \in A$, $F(e) = X$). Clearly, $X_A^c = \Phi_A$. *A soft point* [16], denoted by x_e, is a soft set where $x \in X$ and $e \in A$ defined by $x_e(e) = \{x\}$ and $x_e(e') = \emptyset$ $\forall$ $e' \neq e$ in A. $x_e\tilde{\in}(G, A)$ if for the element $e \in A$, $\{x\} \subseteq G(e)$. Note that any soft point $x_e\tilde{\in}X_A$.

Theorem 1.2. *[8](De Morgan's Law) Let I be an arbitrary index set and $\{(F_i, A) : i \in I\}$ be a subfamily of $SS(X)_A$. Then*

(1) $[\tilde{\bigcup}_{i\in I}(F_i, A)]^c = \tilde{\bigcap}_{i\in I}(F_i, A)^c$,

(2) $[\tilde{\bigcap}_{i\in I}(F_i, A)]^c = \tilde{\bigcup}_{i\in I}(F_i, A)^c$.

Definition 1.3. [1] Let $SS(X)_A$ and $SS(Y)_B$ be two families of soft sets on X and Y respectively, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping defined as: if $(F, A) \in SS(X)_A$, then the image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(b) = \begin{cases} \cup_{a \in p^{-1}(b)} \ u(F(a)) & , p^{-1}(b) \neq \emptyset \,, \\ \emptyset & , \text{otherwise} \,. \end{cases}$$

for all $b \in B$. *The soft function* f_{pu} will be injective if p and u are injectives, and surjective if p and u are surjectives.

Definition 1.4. [1] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets on X and Y respectively, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft mapping. If $(G, B) \in SS(Y)_B$, then the inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}, p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))) & , p(a) \in B \,, \\ \emptyset & , \text{otherwise} \,. \end{cases}$$

for all $a \in A$.

Theorem 1.5. *[1] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$, the following statements hold:*

(a) $f_{pu}(X_A)\tilde{\subseteq}Y_B$. If f_{pu} is surjective, then the equality holds.

(b) If $(F, A)\tilde{\subseteq}(G, A)$ in $SS(X)A$, then $f_{pu}(F, A)\tilde{\subseteq}f_{pu}(G, A)$.

(c) $f_{pu}[(F, A)\tilde{\cup}(G, A)] = f_{pu}(F, B)\tilde{\cup}f_{pu}(G, A)$ and $f_{pu}[(F, A)\tilde{\cap}(G, A)]\tilde{\subseteq}f_{pu}(F, A)$ $\tilde{\cap}f_{pu}(G, A)$ $\forall$ (F, A),$(G, A) \in SS(X)_A$. If f_{pu} is injective, then the equality holds.

2 Soft topological spaces

Definition 2.1. [13] Let τ be a collection of soft sets over a universe X with a fixed set of parameters A. Then τ is said to be *a soft topology* on X, if

(1) Φ_A, X_A belong to τ.

(2) The union of any number of soft sets in τ belongs to τ.

(3) The intersection of any two soft sets in τ belongs to τ.

The triple (X, τ, A) (briefly, $\widetilde{X}$) is called *a soft topological space over* X. The members of τ are called *soft open sets*. A soft complement of a soft open set (F, A) is called *a soft closed set* in $\widetilde{X}$. If (F, A) belongs to τ, we write $(F, A)\tilde{\in}\tau$. A soft set (F, A) which is both soft open and soft closed is called *soft clopen set*.

Definition 2.2. [17] Let $\widetilde{X}$ be a soft topological space over X and $x_e, y_{e'} \tilde{\in} X_A$ such that $x_e \neq y_{e'}$. If there exist soft open sets (F_1, A) and (F_2, A) such that $x_e \tilde{\in} (F_1, A)$, $y_{e'} \tilde{\notin} (F_1, A)$ and $y_{e'} \tilde{\in} (F_2, A)$, $x_e \tilde{\notin} (F_2, A)$, then $\widetilde{X}$ is called a *soft T_1-space*.

Definition 2.3. [7] A soft topological space $\widetilde{X}$ is called a *soft locally indiscrete*, if every soft open set over X is a soft closed set over X.

Theorem 2.4. *[17] Let $\widetilde{X}$ be a soft topological space over X. Then each soft point is soft closed if and only if $\widetilde{X}$ is a soft T_1-space.*

Definition 2.5. [13] Let $\widetilde{X}$ be a soft topological space and (F, A) a soft set over X. The *soft interior of a soft set* (resp. *soft closure*) (F, A) is denoted by $(F, A)^\circ$ (resp. $\overline{(F, A)}$) and is defined as the union of all soft open sets contained in (F, A) (resp. is the intersection of all soft closed supersets of (F, A)) Clearly $(F, A)^\circ$ (resp. $\overline{(F, A)}$) is the largest (resp. the smallest) soft open set contained in (F, A) (resp. the smallest soft closed set contains (F, A)).

Theorem 2.6. *[8] Let $\widetilde{X}$ be a soft topological space and let (F, A) and (G, A) be soft sets over X. Then $\left(\overline{(G, A)}\right)^c = ((G, A)^c)^\circ$ and $((G, A)^\circ)^c = \overline{((G, A)^c)}$.*

Theorem 2.7. *[13],[3] Let $\widetilde{X}$ be a soft topological space and (F, A),(G, A) are soft sets over X. Then*

(1) $(X_A)^\circ = X_A$, $(\phi_A)^\circ = \phi_A$ and $\overline{\phi_A} = \phi_A$, $\overline{X_A} = X_A$.

(2) $(G, A)^\circ \tilde{\subseteq} (G, A)$ and $(G, A) \tilde{\subseteq} \overline{(G, A)}$.

(3) $\left((G, A)^\circ\right)^\circ = (G, A)^\circ$ and $\overline{\overline{(G, A)}} = \overline{(G, A)}$.

(4) $(G, A) \tilde{\subseteq} (F, A)$ implies $(G, A)^\circ \tilde{\subseteq} (F, A)^\circ$ and $\overline{(G, A)} \tilde{\subseteq} \overline{(F, A)}$.

(5) $[(G, A) \tilde{\cap} (F, A)]^\circ = (G, A)^\circ \tilde{\cap} (F, A)^\circ$ and $\overline{(F, A) \tilde{\cap} (G, A)} \tilde{\subseteq} \overline{(F, A)} \tilde{\cap} \overline{(G, A)}$.

(6) $[(G, A) \tilde{\cup} (F, A)]^\circ \tilde{\supseteq} (G, A)^\circ \tilde{\cup} (F, A)^\circ$ and $\overline{(F, A) \tilde{\cup} (G, A)} = \overline{(F, A)} \tilde{\cup} \overline{(G, A)}$.

Theorem 2.8. *[14] Let $\widetilde{X}$ be a soft topological space and (F, A),(G, A) are soft sets in $SS(X)_A$. Then $x_e \tilde{\in} \overline{(F, A)}$ if and only if every soft open (G, A) containing x_e intersect (F, A).*

Theorem 2.9. *Let $\widetilde{X}$ be a soft topological space. For any soft set over X,*

(1) if (G, A) is a soft open set, then $\overline{(F, A)} \tilde{\cap} (G, A) \tilde{\subseteq} \overline{((F, A) \tilde{\cap} (G, A))}$.

(2) If (M, A) is a soft open set, then $((F, A) \tilde{\cup} (M, A))^\circ \tilde{\subseteq} (F, A)^\circ \tilde{\cup} (M, A)$.

Proof. (1) Let $x_e\tilde{\in}\overline{(F,A)}\tilde{\cap}(G,A)$, then $x_e\tilde{\in}\overline{(F,A)}$ and $x_e\tilde{\in}(G,A)$. Let (O,A) be any soft open set containing x_e, then $(O,A)\tilde{\cap}(G,A)$ is a soft open set containing x_e. Since $x_e\tilde{\in}\overline{(F,A)}$, $((O,A)\tilde{\cap}(G,A))\tilde{\cap}(F,A) \neq \phi_A$ which implies $(O,A)\tilde{\cap}((G,A)\tilde{\cap}(F,A)) \neq \phi_A$. Hence, $x_e\tilde{\in}\overline{((F,A)\tilde{\cap}(G,A))}$.

(2) Using Theorem 2.6 and part (1). □

Definition 2.10. [6] Let $\widetilde{X}$ be a soft topological space over X and $Y \subseteq X$. Then $\tau_Y = \{(F_Y, A) = Y_A\tilde{\cap}(F,A) | (F,A)\tilde{\in}\tau\}$ is said to be the *soft relative topology* on Y, where $F_Y(e) = Y \cap F(e)$, for all $e \in A$. (Y, τ_Y, A) is called a *soft subspace* of $\widetilde{X}$. We can easily verify that τ_Y is, in fact, a soft topology on Y.

Theorem 2.11. *[6] Let $\widetilde{Y}$ be a soft subspace of a soft topological space $\widetilde{X}$ and (F,A) a soft set over X. Then (F,A) is a soft closed in $\widetilde{Y}$ if and only if $(F,A) = Y_A\tilde{\cap}(K,A)$, for some soft closed set (K,A) in $\widetilde{X}$.*

Definition 2.12. Let $\widetilde{X}$ be a soft topological space and (F,A) a soft set over X. Then (F,A) is called soft pre-open [2](resp. a soft semi-open [2], a soft α-open [2], a soft β-open [2], a soft regular closed [18]) set if $(F,A)\tilde{\subseteq}\left(\overline{(F,A)}\right)^{\circ}$ (resp. $(F,A)\tilde{\subseteq}\overline{(F,A)^{\circ}}$, $(F,A)\tilde{\subseteq}\left(\overline{(F,A)^{\circ}}\right)^{\circ}$, $(F,A)\tilde{\subseteq}\overline{\left(\overline{(F,A)}\right)^{\circ}}$, $(F,A) = \overline{(F,A)^{\circ}}$).

Theorem 2.13. *[2] In a soft topological space $\widetilde{X}$, the following statements hold:*

(1) Every soft open set is a soft semi-open set.

(2) Every soft open set is a soft β-open set.

(3) Every soft α-open set is a soft semi-open set.

Theorem 2.14. *Let (Y, τ_Y, A) be a soft subspace of a soft topological space $\widetilde{X}$ and $(F,A)\tilde{\subseteq}Y_A$ such that Y_A is a soft β-open set over X. If (F,A) is a soft β-open set over X, then (F,A) is a soft β-open set over Y.*

Proof. Since (F,A) is a soft β-open set ove X, $(F,A)\tilde{\subseteq}\overline{\left(\overline{(F,A)}\right)^{\circ}}$. Since $(F,A)\tilde{\cap}Y_A = (F,A)$, $((F,A)\tilde{\cap}Y_A)\tilde{\subseteq}\overline{\left(\overline{(F,A)\tilde{\cap}Y_A}\right)^{\circ}}$. Hence (F,A) is a soft β-open set over Y. □

Definition 2.15. Let (X, τ_1, A) and (Y, τ_2, B) be two soft topological spaces. A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is called: *soft-continuous* [4] (resp. soft β-continuous [19], soft α-continuous [10], soft semi-continuous [9]) if for every soft open set (G,B) over Y, $f_{pu}^{-1}(G,B)$ is a soft open (resp. $f_{pu}^{-1}(G,A)$ is a soft β-open, $f_{pu}^{-1}(G,B)$ is a soft α-open, $f_{pu}^{-1}(G,B)$ is a soft semi-open) set over X

Theorem 2.16. *[19] Arbitrary union of soft β-open sets is a soft β-open set.*

Theorem 2.17. *Let $\widetilde{X}$ be a soft topological space and (F, A), (G, A) are soft sets over X. If (F, A) is a soft open set and (G, A) is a soft β-open set, then $(F, A)\tilde{\cap}(G, A)$ is a soft β-open set.*

Proof. $(F, A)\tilde{\cap}(G, A)\tilde{\subseteq}(F, A)\tilde{\cap}\overline{\left(\overline{(G, A)}\right)^{\circ}}\tilde{\subseteq}\overline{\left((F, A)\tilde{\cap}\left(\overline{(G, A)}\right)^{\circ}\right)} = \overline{\left((F, A)\tilde{\cap}\overline{(G, A)}\right)^{\circ}}$ $\tilde{\subseteq}\overline{\left(\overline{(F, A)\tilde{\cap}(G, A)}\right)^{\circ}}$. Hence, $(F, A)\tilde{\cap}(G, A)$ is a soft β-open set. □

3 Soft βc-open sets

Definition 3.1. Let $\widetilde{X}$ be a soft topological space and $(F, A) \in SS(X)_A$. Then (F, A) is called a *soft βc-open* if (F, A) is a soft β-open set and for each $x_e\tilde{\in}(F, A)$ there is a soft closed set (H, A) in $SS(X)_A$ such that $x_e\tilde{\in}(H, A)\tilde{\subseteq}(F, A)$. The complement of a soft βc-open set is called a *soft βc-closed set.*

Theorem 3.2. *A soft set (F, A) in a soft topological space $\widetilde{X}$ is a soft βc-open if and only if (F, A) is a soft β-open set and it is a union of soft closed set.*

Proof. For each $x_e\tilde{\in}(F, A)$ there exists soft closed set (H_{x_e}, A) such that $x_e\tilde{\in}(H_{x_e}, A)$ $\tilde{\subseteq}(F, A)$. This implies that $(F, A) = \tilde{\bigcup}_{x_e\tilde{\in}(F,A)}(H_{x_e}, A)$. The Converse is direct from Definition 3.1. □

Corollary 3.3. *Let $\widetilde{X}$ be a soft topological space and (F, A) a soft β-open set over X. If (F, A) is a soft closed set, then (F, A) is a soft βc-open set.*

Remark 3.4. *(a) Every soft βc-open set is soft β-open set. But the converse need not be true in general.*

(b) The family of soft open sets is incomparable with the family of soft βc-open sets.

(c) The family of soft pre-open sets is incomparable with the family of soft βc-open sets.

(d) The family of soft closed sets is incomparable with the family of soft βc-open sets.

(e) The family of soft semi-open sets is incomparable with the family of soft βc-open sets.

The following examples illustrate the previous remark:

Example 3.5. *Let $X = \{a, b, c\}$ and $A = \{e\}$, with a soft topology $\tau = \{X_A, \phi_A, (e, \{a\}), (e, \{b\}), (e, \{a, b\})\}$. Then*

(a) the soft set $(e, \{a\})$ is soft β-open set which is not soft βc-open set.

(b) The soft set $(e, \{b\})$ is soft open set but is not soft βc-open set and $(e, \{a, c\})$ is soft βc-open set which is not soft open set.

(c) The soft set $(e, \{b, c\})$ is soft βc-open set which is not soft pre-open set. Also $(e, \{b\})$ is soft pre-open set which is not soft βc-open set.

Example 3.6. *Let $X = \{x_1, x_2, x_3\}$ and $A = \{e_1, e_2, e_3\}$ with the soft topology $\tau = \{\phi_A, X_A, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$ where (F_1, A), (F_2, A), (F_3, A) and (F_4, A) are soft sets over X, define as follows:*

$$\begin{aligned}(F_1, A) &= \{(e_1, X), (e_2, \{x_2, x_3\}), (e_3, \{x_1, x_2\})\},\\ (F_2, A) &= \{(e_2, \{x_1\}), (e_3, \{x_3\})\},\\ (F_3, A) &= \{(e_1, \{x_2\}), (e_2, \{x_1, x_3\}), (e_3, \{x_3\})\},\\ (F_4, A) &= \{(e_1, \{x_2\}), (e_2, \{x_3\})\}.\end{aligned}$$

Then the soft set $(G, A) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_3\}), (e_3, \{x_1, x_3\})\}$ is soft βc-open which is not soft semi-open. The soft set $(F, A) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2\})$ $, (e_3, \{x_1, x_2\})\}$ is soft closed which is not soft βc-open. The soft set $(M, A) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_3\}), (e_3, \{x_1, x_2\})\}$ is soft βc-open which is not soft semi-open. The soft set $(H, A) = \{(e_1, \{x_1, x_2\}, (e_2, \{x_3\}), (e_3, \{x_2\})\}$ is soft semi-open which is not soft βc-open.

Theorem 3.7. *An arbitrary union of soft βc-open sets is a soft βc-open set.*

Proof. Let $\{(F_\alpha, A) : \alpha \in \Delta\}$ be a family of soft βc-open sets in a soft topological space $\widetilde{X}$. Then (F_α, A) is a soft β-open set for each $\alpha \in \Delta$ and by Theorem 2.16, $\tilde{\bigcup}_{\alpha\in\Delta}(F_\alpha, A)$ is a soft β-open set. If $x_e \tilde{\in} \tilde{\bigcup}_{\alpha\in\Delta}(F_\alpha, A)$, then there exists $\gamma \in \Delta$ such that $x_e \tilde{\in} (F_\gamma, A)$. Since (F_γ, A) is soft βc-open set, there exists a soft closed set (H_{x_e}, A) such that $x_e \tilde{\in} (H_{x_e}, A) \tilde{\subseteq} (F_\gamma, A) \tilde{\subseteq} \tilde{\bigcup}_{\alpha\in\Delta}(F_\alpha, A)$. Hence, $\tilde{\bigcup}_{\alpha\in\Delta}(F_\alpha, A)$ is a soft βc-open set. □

Remark 3.8. *The intersection of even two soft βc-open sets need not be a soft βc-open as shown in the following example:*

Example 3.9. *In Example 3.5, $(e, \{a, c\})$ and $(e, \{b, c\})$ are soft βc-open sets but $(e, \{a, c\}) \tilde{\cap} (e, \{b, c\}) = (e, \{c\})$ is not soft βc-open set.*

Corollary 3.10. *An arbitrary intersection of soft βc-closed sets is soft βc-closed set.*

Proof. Direct from Theorem 3.1 and De Morgen's law. □

Remark 3.11. *The union of even two soft βc-closed sets need not be a soft βc-closed set.*

Theorem 3.12. *Every soft clopen set is a soft βc-open set.*

Proof. Direct from Theorem 2.13 part (2) and Corollary 3.3. □

Theorem 3.13. *A soft set (F, A) in a soft topological space $\widetilde{X}$ is a soft βc-open set if and only if for each $x_e \tilde{\in} (F, A)$, there exists a soft βc-open set (H_{x_e}, A) such that $x_e \tilde{\in} (H_{x_e}, A) \tilde{\subseteq} (F, A)$.*

Proof. Direct from Theorem 3.2 and Definition 3.1. □

Theorem 3.14. *Let $\widetilde{X}$ be a soft topological space and (F, A) a soft set over X. If $\widetilde{X}$ is a soft T_1-space and (F, A) is a soft β-open set, then (F, A) is a soft βc-open set.*

Proof. If $(F, A) = \phi_A$, then ϕ_A is a soft βc-open set. If not, then for each $x_e \tilde{\in} (F, A)$, x_e is a soft closed set by Theorem 2.4. But $x_e \tilde{\in} \{x_e\} \tilde{\subseteq} (F, A)$. Therefore, (F, A) is a soft βc-open set. □

Theorem 3.15. *Let $\widetilde{X}$ be a soft topological space, (F, A) a soft open set and (G, A) a soft βc-open set. If (F, A) is a union of soft closed sets, then $(F, A) \tilde{\cap} (G, A)$ is a soft βc-open set.*

Proof. By Theorem 2.17, $(F, A) \tilde{\cap} (G, A)$ is a soft β-open set. If $x_e \tilde{\in} (F, A) \tilde{\cap} (G, A)$, then there exists a soft closed set (C, A) over X such that $x_e \tilde{\in} (C, A) \tilde{\subseteq} (F, A)$. Moreover there exists a soft closed set (M, A) over X such that $x_e \tilde{\in} (M, A) \tilde{\subseteq} (G, A)$. Hence, $x_e \tilde{\in} (C, A) \tilde{\cap} (M, A) \tilde{\subseteq} (F, A) \tilde{\cap} (G, A)$ where $(C, A) \tilde{\cap} (M, A)$ is a soft closed set over X. Therefore, $(F, A) \tilde{\cap} (G, A)$ is a soft βc-open set. □

Corollary 3.16. *Let $\widetilde{X}$ be a soft topological space, (F, A) a soft clopen set and (G, A) a soft βc-open set. Then $(F, A) \tilde{\cap} (G, A)$ is a soft βc-open set.*

Theorem 3.17. *Every soft regular closed set is a soft βc-open set.*

Proof. $(F, A) = \overline{(F, A)^\circ} \tilde{\subseteq} \overline{\left(\overline{(F, A)}\right)^\circ}$. So, (F, A) is a soft β-open set. Since (F, A) is a soft closed set, by Corollary 3.3 (F, A) is a soft βc-open set. □

Theorem 3.18. *If $\widetilde{X}$ is a soft locally indiscrete space, then every soft semi-open set is a soft βc-open set.*

Proof. $(F, A) \tilde{\subseteq} \overline{(F, A)^\circ} \tilde{\subseteq} \overline{\left(\overline{(F, A)}\right)^\circ}$. So (F, A) is a soft β-open set. Since $\widetilde{X}$ is a soft locally indiscrete, $(F, A)^\circ$ is a soft closed set and $(F, A) \tilde{\subseteq} \overline{(F, A)^\circ} = (F, A)^\circ$ which implies that, (F, A) is a soft open set and for any $x_e \tilde{\in} (F, A)$, $x_e \tilde{\in} (F, A)^\circ \tilde{\subseteq} (F, A)$. Hence (F, A) is a soft βc-open set. □

Corollary 3.19. *Let (X, τ, A) be a soft locally indiscrete space. Then:*

(1) every soft open set is a soft βc-open set .

(2) Every soft α-open set is a soft βc-open set .

Theorem 3.20. *Let $\widetilde{Y}$ be a soft subspace of a soft topological space $\widetilde{X}$. If (F, A) is a soft βc-open set over X and $(F, A)\tilde{\subseteq}Y_A$ such that Y_A is a soft β-open set over X, then (F, A) is a soft βc-open set over Y.*

Proof. (F, A) is a soft β-open set over X and Y_A is a soft β-open set over X, so that, by Theorem 2.14, (F, A) is a soft β-open set over Y. Also, for each $x_e\tilde{\in}(F, A)$ there exists a soft closed set (C, A) over X such that $x_e\tilde{\in}(C, A)\tilde{\subseteq}(F, A)$. Since (C, A) is a soft closed set over X and $(F, A)\tilde{\subseteq}Y_A$, by Theorem 2.11, (C, A) is a soft closed set over Y. Hence (F, A) is a soft βc-open set over Y. □

Remark 3.21. *The condition of being Y_A a soft β-open set over X is necessary in Theorem 3.20 as shown in the following example:*

Example 3.22. *Let $X = \{x_1, x_2, x_3\}$ and $A = \{e_1, e_2\}$ with a soft topology $\tau = \{\phi_A, X_A, (F_1, A), (F_2, A), (F_3, A)\}$ where:*

$$\begin{aligned}(F_1, A) &= \{(e_1, X), (e_2, \{x_1, x_2\}), (e_3, X)\},\\ (F_2, A) &= \{(e_1, \{x_1, x_3\}), (e_2, X), (e_3, \{x_2, x_3\})\},\\ (F_3, A) &= \{(e_1, \{x_1, x_3\}), (e_2, \{x_1, x_2\}), (e_3, \{x_2, x_3\})\}.\end{aligned}$$

If $(F, A) = \{(e_1, \{x_2\}), (e_2, \{x_3\})\}$, then (F, A) is a soft βc-open set over X. If $Y_A = \{(e_1, \{x_2\}), (e_2, \{x_3\}), (e_3, \{x_1\})\}$, then Y_A is not a soft β-open set over X. Clearly $(F, A)\tilde{\subseteq}Y_A$. Now $\tau_Y = \{\phi_A, Y_A, (F_Y, A), (G_Y, A)\}$ where $(F_Y, A) = \{(e_1, \{x_2\}), (e_3, \{x_1\})\}$ and $(G_Y, A) = \{(e_2, \{x_3\})\}$. Since $(e_1, \{x_2\})\tilde{\in}(F, A)$ and there is no soft closed set (C_Y, A) over Y such that $(e_1, \{x_2\})\tilde{\in}(C_Y, A)\tilde{\subseteq}(F, A)$, (F, A) is not a soft βc-open set over Y.

Theorem 3.23. *Let $\widetilde{Y}$ be a soft subspace of a soft topological space $\widetilde{X}$. If (F, A) is a soft βc-open set over X and Y_A is a soft clopen set over X, then $(F, A)\tilde{\cap}Y_A$ is a soft βc-open set over Y.*

Proof. Since Y_A is a soft clopen set over X, by Theorem 2.17, $(F, A)\tilde{\cap}Y_A$ is a soft β-open set over X. Since (F, A) is a soft βc-open set over X, then for each $x_e\tilde{\in}(F, A)$, there exists a soft closed set (C, A) over X such that $x_e\tilde{\in}(C, A)\tilde{\subseteq}(F, A)$. Hence $x_e\tilde{\in}(C, A)\tilde{\cap}Y_A\tilde{\subseteq}(F, A)\tilde{\cap}Y_A$ and therefore, $(F, A)\tilde{\cap}Y_A$ is a soft βc-open set over X such that $(F, A)\tilde{\cap}Y_A\tilde{\subseteq}Y_A$. Thus, by Theorem 3.20, $(F, A)\tilde{\cap}Y_A$ is a soft βc-open set over Y. □

Remark 3.24. *The condition of being Y_A a soft clopen set over X is necessary in Theorem 3.23 as illustrated in the following example:*

Example 3.25. *In Example 3.22, (F, A) is a soft βc-open set over X and Y_A is not a soft clopen set over X. But $(F, A)\tilde{\cap}Y_A = (F, A)$ is not a soft βc-open set over Y.*

4 Soft βc-interior and soft βc-closure

Definition 4.1. Let $\widetilde{X}$ be a soft topological space and (F, A) a soft set over X. Then the *soft βc-interior* of (F, A), denoted by $Int_{\beta c}(F, A)$, is the union of all soft βc-open sets contained in (F, A). (F, A) is called a *soft βc-neighborhood* (briefly, soft βc-nhood) of a soft point $x_e\tilde{\in}X_A$ if there exists a soft open set (G, A) over X such that $x_e\tilde{\in}(G, A)\tilde{\subseteq}(F, A)$. A soft point $x_e\tilde{\in}X_A$ is called a *soft βc-interior point* of (F, A) if there exists a soft βc-open set (G, A) over X such that $x_e\tilde{\in}(G, A)\tilde{\subseteq}(F, A)$. Clearly $Int_{\beta c}(F, A)$ is the largest soft βc-open set contained in (F, A). Moreover, (F, A) is βc-nhood of x_e if $x_e\tilde{\in}Int_{\beta c}(F, A)$.

Theorem 4.2. *Let $\widetilde{X}$ be a soft topological space and let (F, A) and (G, A) be a soft sets over X. Then*

(1) (F, A) is a soft βc-open set if and only if $(F, A) = Int_{\beta c}(F, A)$.

(2) $Int_{\beta c}(\phi_A) = \phi_A$ and $Int_{\beta c}(X_A) = X_A$.

(3) $Int_{\beta c}(Int_{\beta c}(F, A)) = Int_{\beta c}(F, A)$.

(4) $(F, A)\tilde{\subseteq}(G, A)$ implies $Int_{\beta c}(F, A)\tilde{\subseteq}Int_{\beta c}(G, A)$.

(5) If $(F, A)\tilde{\cap}(G, A) = \phi_A$, then $Int_{\beta c}(F, A)\tilde{\cap}Int_{\beta c}(G, A) = \phi_A$

(6) $Int_{\beta c}((F, A)\tilde{\cap}(G, A))\tilde{\subseteq}Int_{\beta c}(F, A)\tilde{\cap}Int_{\beta c}(G, A)$.

(7) $Int_{\beta c}(F, A)\tilde{\cup}Int_{\beta c}(G, A)\tilde{\subseteq}Int_{\beta c}((F, A)\tilde{\cup}(G, A))$.

Proof. The proofs of (1)... (4) are direct and easy.

(5) If $Int_{\beta c}(F, A)\tilde{\cap}Int_{\beta c}(G, A) \neq \phi_A$, then there is $x_e\tilde{\in}Int_{\beta c}(F, A)\tilde{\cap}Int_{\beta c}(G, A)$. So there exist soft βc-open sets (M, A) and (V, A) such that $x_e\tilde{\in}(M, A)\tilde{\subseteq}(F, A)$ and $x_e\tilde{\in}(V, A)\tilde{\subseteq}(G, A)$ which implies that $x_e\tilde{\in}(M, A)\tilde{\cap}(V, A)\tilde{\subseteq}(M, A)\tilde{\subseteq}(F, A)$ and $x_e\tilde{\in}(M, A)\tilde{\cap}(V, A)\tilde{\subseteq}(V, A)\tilde{\subseteq}(G, A)$. Hence, $x_e\tilde{\in}(F, A)\tilde{\cap}(G, A)$ and therefore, $(F, A)\tilde{\cap}(G, A) \neq \phi_A$.

(6) Since $(F, A)\tilde{\cap}(G, A)\tilde{\subseteq}(F, A)$ and $(F, A)\tilde{\cap}(G, A)\tilde{\subseteq}(G, A)$, by part 4 $Int_{\beta c}((F, A)$ $\tilde{\cap}(G, A)\tilde{\subseteq}Int_{\beta c}(F, A)$ and $Int_{\beta c}((F, A)\tilde{\cap}(G, A)\tilde{\subseteq}Int_{\beta c}(G, A)$. Hence $Int_{\beta c}((F, A)\tilde{\cap}$ $(G, A))\tilde{\subseteq}Int_{\beta c}(F, A)\tilde{\cap}Int_{\beta c}(G, A)$.

(7) Since $(F,A)\tilde{\subseteq}(F,A)\tilde{\cup}(G,A)$ and $(G,A)\tilde{\subseteq}(F,A)\tilde{\cup}(G,A)$, by part 4 $Int_{\beta c}(F,A)\tilde{\subseteq} Int_{\beta c}((F,A)\tilde{\cup}(G,A))$ and $Int_{\beta c}(G,A)\tilde{\subseteq}Int_{\beta c}((F,A)\tilde{\cup}(G,A))$. Hence, $Int_{\beta c}(F,A)\tilde{\cup} Int_{\beta c}(G,A)\tilde{\subseteq}Int_{\beta c}((F,A)\tilde{\cup}(G,A))$. □

Remark 4.3. *The converse of parts* (4) *and* (5) *and the reverse inclusions of parts* (6) *and* (7) *need not be true in general as illustrated in the following examples:*

Example 4.4. $[Int_{\beta c}(F,A)\tilde{\subseteq}Int_{\beta c}(G,A) \nRightarrow (F,A)\tilde{\subseteq}(G,A)]$
Let $X = \{a,b,c\}$ and $A = \{e\}$ with a soft topology $\tau = \{\phi_A, X_A, (e,\{a\})\}$, then if $(F,A) = (e,\{a\})$ and $(G,A) = (e,\{b\})$, we have $Int_{\beta c}(F,A) = Int_{\beta c}(F,A) = \phi_A$. But $(F,A)\tilde{\nsubseteq}(G,A)$.

Example 4.5. $[Int_{\beta c}(F,A)\tilde{\cap}Int_{\beta c}(G,A) = \phi_A \nRightarrow (F,A)\tilde{\cap}(G,A) = \phi_A]$
In Example 4.4, if $(F,A) = (e,\{a\})$ and $(G,A) = (e,\{a,b\})$, then $Int_{\beta c}(F,A) = Int_{\beta c}(F,A) = \phi_A$ which implies that, $Int_{\beta c}(F,A)\tilde{\cap}Int_{\beta c}(G,A) = \phi_A$. But $(F,A)\tilde{\cap}(G,A) = (e,\{a\}) \neq \phi_A$.

Example 4.6. $[Int_{\beta c}((F,A)\tilde{\cup}(G,A))\tilde{\nsubseteq}Int_{\beta c}(F,A)\tilde{\cup}Int_{\beta c}(G,A)]$
In Example 4.4, if $(F,A) = (e,\{a\})$ and $(G,A) = (e,\{b,c\})$, then $Int_{\beta c}((F,A) = Int_{\beta c}(G,A) = \phi_A$. But $(F,A)\tilde{\cup}(G,A) = X_A$ and $Int_{\beta c}((F,A)\tilde{\cup}(G,A)) = X_A$.

Example 4.7. $[Int_{\beta c}(F,A)\tilde{\cap}Int_{\beta c}(G,A)\tilde{\nsubseteq}Int_{\beta c}((F,A)\tilde{\cap}(G,A))]$
Consider a soft topological space $\widetilde{X}$ in Example 3.5. If $(F,A) = (e,\{a,c\})$ and $(G,A) = (e,\{b,c\})$, then $Int_{\beta c}((F,A) = (e,\{a,c\})$ and $Int_{\beta c}((G,A) = (e,\{b,c\})$ which implies that, $Int_{\beta c}(F,A)\tilde{\cap}Int_{\beta c}(G,A) = (e,\{c\})$. But $(F,A)\tilde{\cap} (G,A) = (e,\{c\})$ and $Int_{\beta c}((F,A)\tilde{\cap}(G,A)) = \phi_A$.

Definition 4.8. Let $\widetilde{X}$ be a soft topological space over X and (B,A) a soft set over X. The *soft βc-closure* of (B,A), denoted by $Cl_{\beta c}(B,A)$, is the intersection of all soft βc-closed supersets of (B,A). Clearly $Cl_{\beta c}(B,A)$ is the smallest soft βc-closed set in $\widetilde{X}$ which contains (B,A).

Theorem 4.9. *Let $\widetilde{X}$ be a soft topological space, (F,A) a soft set over X and $x_e\tilde{\in}X_A$. Then, the following two statement are equivalent:*

(1) $x_e\tilde{\in}Cl_{\beta c}(F,A)$.

(2) For any soft βc-open set (G,A) over X containing x_e we have, $(F,A)\tilde{\cap}(G,A) \neq \phi_A$.

Proof. $(1 \Rightarrow 2)$ Suppose there exists a soft βc-open set (G,A) containing x_e such that $(F,A)\tilde{\cap}(G,A) = \phi_A$. Then $(F,A)\tilde{\subseteq}(G,A)^c$. Since $(G,A)^c$ is a soft βc-closed set, $Cl_{\beta c}(F,A)\tilde{\subseteq}(G,A)^c$. Hence $x_e\tilde{\notin}Cl_{\beta c}(F,A)$.

$(2 \Rightarrow 1)$ If $x_e\tilde{\notin}Cl_{\beta c}(F,A)$, then there exists a soft βc-closed set (C,A) such that $(F,A)\tilde{\subseteq}(C,A)$ and $x_e\tilde{\notin}(C,A)$. But $(C,A)^c$ is a soft βc-open set containing x_e, and $(F,A)\tilde{\cap}(C,A)^c\tilde{\subseteq}(F,A)\tilde{\cap}(F,A)^c = \phi_A$. □

The relations between the soft βc-closure and soft βc-interior can be considered in the following theorem:

Theorem 4.10. *Let $\widetilde{X}$ be a soft topological space over X and (M, A) be a soft set over X. Then*

(1) $(Cl_{\beta c}(M, A))^{c} = Int_{\beta c}((M, A)^c)$.

(2) $(Int_{\beta c}(M, A))^{c} = Cl_{\beta c}((M, A)^c)$.

(3) $Cl_{\beta c}(F, A) = (Int_{\beta c}(F, A)^c)^{c}$.

(4) $Int_{\beta c}(M, A) = (Cl_{\beta c}((M, A)^c))^{c}$.

Proof. (1) $(Cl_{\beta c}(M, A))^{c} = \left(\tilde{\bigcap}\{(F, A) : (F, A) \text{ is soft } \beta c\text{−closed and} (M, A)\tilde{\subseteq}(F, A)\}\right)^{c}$
$= \tilde{\bigcup}\{(F, A)^c : (F, A) \text{ is soft } \beta c\text{−closed and} (M, A)\tilde{\subseteq}(F, A)\} = \tilde{\bigcup}\{(F, A)^c :$
$(F, A)^c \text{ is soft } \beta c\text{−open and} (F, A)^c\tilde{\subseteq}(M, A)^c\} = Int_{\beta c}((M, A)^c)$.
The other parts can be proved in similar way. □

Theorem 4.11. *Let $\widetilde{X}$ be a soft topological space and let (F, A) and (G, A) be soft sets over X. Then*

(1) *(F, A) is a soft βc-closed set if and only if* $(F, A) = Cl_{\beta c}(F, A)$.

(2) $Cl_{\beta c}(\phi_A) = \phi_A$ *and* $Cl_{\beta c}(X_A) = X_A$.

(3) $Cl_{\beta c}(Cl_{\beta c}(F, A)) = Cl_{\beta c}(F, A)$.

(4) $(F, A)\tilde{\subseteq}(G, A)$ *implies* $Cl_{\beta c}(F, A)\tilde{\subseteq}Cl_{\beta c}(G, A)$.

(5) *If* $Cl_{\beta c}(F, A)\tilde{\cap}Cl_{\beta c}(G, A) = \phi_A$, *then* $(F, A)\tilde{\cap}(G, A) = \phi_A$

(6) $Cl_{\beta c}(F, A)\tilde{\cup}Cl_{\beta c}(G, A)\tilde{\subseteq}Cl_{\beta c}((F, A)\tilde{\cup}(G, A))$.

(7) $Cl_{\beta c}((F, A)\tilde{\cap}(G, A))\tilde{\subseteq}Cl_{\beta c}(F, A)\tilde{\cap}Cl_{\beta c}(G, A)$.

Proof. Using Theorem 4.10 and Theorem 4.2 □

Remark 4.12. *The converse of parts* (4) *and* (5) *and the reverse inclusions of* (6) *and* (7) *need not be true in general.*

Theorem 4.13. *Let $\widetilde{X}$ be a soft topological space, then for any soft set over X, we have the following:*

(1) *If (G, A) is a soft clopen set, then* $Cl_{\beta c}(F, A)\tilde{\cap}(G, A)\tilde{\subseteq}Cl_{\beta c}((F, A)\tilde{\cap}(G, A))$.

(2) *If (M, A) is a soft clopen set, then* $Int_{\beta c}((F, A)\tilde{\cup}(M, A))\tilde{\subseteq}Int_{\beta c}(F, A)\tilde{\cup}(M, A)$.

Proof. (1) Let $x_e\tilde{\in}Cl_{\beta c}(F,A)\tilde{\cap}(G,A)$. Then $x_e\tilde{\in}Cl_{\beta c}(F,A)$ and $x_e\tilde{\in}(G,A)$. Let (O,A) be any soft βc-open set containing x_e, then by Corollary 3.19, $(O,A)\tilde{\cap}(G,A)$ is a soft βc-open set containing x_e. Since $x_e\tilde{\in}Cl_{\beta c}(F,A)$, by Theorem 4.16, $((O,A)\tilde{\cap}(G,A))\tilde{\cap}(F,A)\neq\phi_A$ which implies that $(O,A)\tilde{\cap}((G,A)\tilde{\cap}(F,A))$ $\neq\phi_A$. Hence, by Theorem 4.16, $x_e\tilde{\in}Cl_{\beta c}((F,A)\tilde{\cap}(G,A))$.

(2) Direct using part (1) and Theorem 4.10. □

Remark 4.14. *The condition of being (G,A) and (M,A) are soft clopen sets is necessary in Theorem 4.13 as illustrated in the following example:*

Example 4.15. *Consider a soft topological space in Example 4.4. Let $(F,A)=(e,\{a\})$ and $(G,A)=(e,\{b\})$, then (G,A) is not a soft clopen set. Now $Cl_{\beta c}(F,A)\tilde{\cap}(G,A)=X_A\tilde{\cap}(G,A)=(G,A)\tilde{\nsubseteq}Cl_{\beta c}((F,A)\tilde{\cap}(G,A))=Cl_{\beta c}(\phi_A)=\phi_A$. Also, if $(M,A)=(e,\{b,c\})$, then (M,A) is not a soft clopen set. Now $Int_{\beta c}((F,A)\tilde{\cup}(M,A))=Int_{\beta c}(X_A)=X_A\tilde{\nsubseteq}Int_{\beta c}(F,A)\tilde{\cup}(M,A)=\phi_A\tilde{\cup}(M,A)=(M,A)$.*

5 Soft βc-continuous functions

Definition 5.1. Let (X,τ_1,A) and (Y,τ_2,B) be two soft topological spaces. A soft function $f_{pu}:SS(X)_A\to SS(Y)_B$ is called a *soft βc-continuous function* at a soft point $x_e\tilde{\in}X_A$ if for each soft open set (F,B) over Y containing $f_{pu}(x_e)$, there exists a soft βc-open set (G,A) over X containing x_e such that $f_{pu}(G,A)\tilde{\subseteq}(F,B)$. If f_{pu} is a soft βc-continuous at every soft point x_e over X, then it is called *soft βc-continuous* on X.

Remark 5.2. *Every soft βc-continuous function is a soft β-continuous function.*

Theorem 5.3. *A soft function $f_{pu}:SS(X)_A\to SS(Y)_B$ is soft βc-continuous if and only if the inverse image of every soft open set over Y is a soft βc-open set over X.*

Proof. Let f_{pu} be a soft βc-continuous function and (F,B) be a soft open set over Y. If $f_{pu}^{-1}(F,B)=\phi_A$, then $f_{pu}^{-1}(F,B)$ is a soft βc-open set over X. Else, let $x_e\tilde{\in}f_{pu}^{-1}(F,B)$. So $f_{pu}(x_e)\tilde{\in}(F,B)$. By soft βc-continuity of f_{pu}, there exists a soft βc-open set (G,A) over X containing x_e such that $f_{pu}(G,A)\tilde{\subseteq}(F,B)$. Hence, $x_e\tilde{\in}(G,A)\tilde{\subseteq}f_{pu}^{-1}(F,B)$ and therefore, by Theorem 3.13, $f_{pu}^{-1}(F,B)$ is a soft βc-open set over X. Conversely, let $x_e\tilde{\in}X_A$ and (F,B) be a soft open set over Y containing $f_{pu}(x_e)$. Then, $x_e\tilde{\in}f_{pu}^{-1}(F,B)$ which is a soft βc-open set. So $f_{pu}(f_{pu}^{-1}(F,B))\tilde{\subseteq}(F,B)$. Therefore f_{pu} is a soft βc-continuous function. □

Theorem 5.4. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function such that $\widetilde{X}$ is soft T_1-space. Then f_{pu} is a soft βc-continuous function if and only if f_{pu} is a soft β-continuous function.*

Proof. For any soft open set (F, A) over Y we have, $f_{pu}^{-1}(F, A)$ is a soft β-open set over X, and by Theorem 2.4, $f_{pu}^{-1}(F, A)$ is a soft βc-open set. Therefore, f_{pu} is a soft βc-continuous function. The converse is direct. □

Theorem 5.5. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is a soft βc-continuous if and only if f_{pu} is a soft β-continuous function and for each $x_e \tilde{\in} X_A$ and each soft open set (F, B) over Y such that $f_{pu}(x_e) \tilde{\in} (F, B)$, there exists a soft closed set (C, A) over X containing x_e such that $f_{pu}(C, A) \tilde{\subseteq} (F, B)$.*

Proof. Let $x_e \tilde{\in} X_A$ such that $f_{pu}(x_e) \tilde{\subseteq} (F, B)$. Then, there exists a soft βc-open set (G, A) over X such that $x_e \tilde{\in} (G, A)$ and $f_{pu}(G, A) \tilde{\subseteq} (F, B)$. But (G, A) is a soft βc-open set which implies that there exists a soft closed set (C, A) over X such that $x_e \tilde{\in} (C, A) \tilde{\subseteq} (G, A)$ and hence, $f_{pu}(C, A) \tilde{\subseteq} (F, A)$. Conversely, if (F, A) be a soft open set over Y, then by assumption, $f_{pu}^{-1}(F, A)$ is soft β-open set and for any $x_e \tilde{\in} f_{pu}^{-1}(F, A)$ we have, $f_{pu}(x_e) \tilde{\in} (F, A)$ and so, there exists a soft closed set (C, A) over X containing x_e such that $f(C, A) \tilde{\subseteq} (F, A)$. So, $x_e \tilde{\in} (C, A) \tilde{\subseteq} f_{pu}^{-1}(F, B)$. Therefore, $f_{pu}^{-1}(F, B)$ is a soft βc-open set over X. □

Theorem 5.6. *Let (X, τ_1, A) and (Y, τ_2, A) be two soft topological spaces and consider $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function. Then, the following are equivalent:*

(1) f_{pu} is a soft βc-continuous function.

(2) $f_{pu}^{-1}(G, B)$ is a soft βc-open set over X for any soft open set (G, B) over Y.

(3) $f_{pu}^{-1}(C, B)$ is a soft βc-closed set over X for any soft closed set (C, B) over Y.

(4) $f_{pu}\left(Cl_{\beta c}(M, A)\right) \tilde{\subseteq} \overline{f_{pu}(M, A)}$ for any soft set (M, A) over X.

(5) $Cl_{\beta c}\left(f_{pu}^{-1}(K, B)\right) \tilde{\subseteq} f_{pu}^{-1}\left(\overline{(K, B)}\right)$ for any soft set (K, B) over Y.

(6) $f_{pu}^{-1}\left((M, B)^{\circ}\right) \tilde{\subseteq} Int_{\beta c}\left(f_{pu}^{-1}(M, B)\right)$ for any soft set (M, B) over Y.

Proof. $(1 \Rightarrow 2)$ Follows from Theorem 5.3.

$(2 \Rightarrow 3)$ Let (C, B) be any soft closed set over Y then $(C, B)^c$ is a soft open set. By part (2), we have, $f_{pu}^{-1}((C, A)^c) = (f_{pu}^{-1}(C, A))^c$ is a soft βc-open set over X. Hence, $f_{pu}^{-1}(C, A)$ is a soft βc-closed set over X.

$(3 \Rightarrow 4)$ Let (M, A) be a soft set over X. Then, $f_{pu}(M, A)\tilde{\subseteq}\overline{f_{pu}(M, A)}$. Hence, by part (3) we have, $f_{pu}^{-1}\left(\overline{f_{pu}(M, A)}\right)$ is a soft βc-closed set over X and $(M, A)\tilde{\subseteq}f_{pu}^{-1}\left(\overline{f_{pu}(M, A)}\right)$. Therefore, $Cl_{\beta c}(M, A)\tilde{\subseteq}f_{pu}^{-1}\left(\overline{f_{pu}(M, A)}\right)$ and hence, $f_{pu}(Cl_{\beta c}(M, A))\tilde{\subseteq}\overline{f_{pu}(M, A)}$.

$(4 \Rightarrow 5)$ Let (K, B) be any soft set over Y. Then, $f_{pu}^{-1}(K, B)$ is a soft set over X and by using part (4) we have, $f_{pu}(Cl_{\beta c}(f_{pu}^{-1}(K, B)))\tilde{\subseteq}\overline{f_{pu}(f_{pu}^{-1}(K, B))}\tilde{\subseteq}\overline{(K, B)}$.

$(5 \Rightarrow 6)$ Let (K, B) be a soft set over Y. By part (5) to $(K, A)^c$, we have $Cl_{\beta c}\left(f_{pu}^{-1}((K, A)^c)\right)\tilde{\subseteq}f_{pu}^{-1}\left(\overline{(K, A)^c}\right)$ which implies that, $Cl_{\beta c}\left(f_{pu}^{-1}((K, B)^c)\right)\tilde{\subseteq}$ $f_{pu}^{-1}\left(((K, B)^\circ)^c\right)$ and so, $\left(Int_{\beta c}(f_{pu}^{-1}(K, B))\right)^c\tilde{\subseteq}\left(f_{pu}^{-1}((K, B)^\circ)\right)^c$. Hence $f_{pu}^{-1}((K, B)^\circ)$ $\tilde{\subseteq}Int_{\beta c}(f_{pu}^{-1}(K, B))$.

$(6 \Rightarrow 1)$ Let $x_e\tilde{\in}X_A$ and let (F, B) be any soft open set over Y containing $f_{pu}(x_e)$, then $x_e\tilde{\in}f_{pu}^{-1}(F, B)$ and $f_{pu}^{-1}(F, B)$ is a soft set over X. By part (6), $f_{pu}^{-1}(F, B) = f_{pu}^{-1}((F, B)^\circ)\tilde{\subseteq}Int_{\beta c}(f_{pu}^{-1}(F, B))$. Therefore, $f_{pu}^{-1}(F, B)$ is a soft βc-open set over X which contains x_e and $f_{pu}(f_{pu}^{-1}(F, B))\tilde{\subseteq}(F, B)$. Hence, f_{pu} is a soft βc-continuous function. □

Definition 5.7. A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is called:

(1) *soft βc-irresolute* if for every soft βc-open set (G, B) over Y, $f_{pu}^{-1}(G, A)$ is a soft βc-open set over X.

(2) *soft βc-open* if for every soft βc-open set (H, A) over X, $f_{pu}(H, A)$ is a soft βc-open set over Y.

(3) *soft βc-closed* if for every soft βc-closed set (H, A) over X, $f_{pu}(H, A)$ is a soft βc-closed set over Y.

Theorem 5.8. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ and $g_{qv} : SS(Y)_B \to SS(Z)_C$ be two soft functions, then the following properties hold:*

(1) If f_{pu} is a soft βc-continuous and g_{qv} is a soft continuous, then $g_{qv} \circ f_{pu}$ is a soft βc-continuous.

(2) If f_{pu} is a soft βc-irresolute and g_{qv} is a soft βc-continuous, then $g_{qv} \circ f_{pu}$ is a soft βc-continuous.

Proof. (1) Let (F, C) be a soft open set over Z. Then $g_{qv}^{-1}(F, C)$ is a soft open set over Y. Since f_{pu} is a soft βc-continuous, $f_{pu}^{-1}(g_{qv}^{-1}(F, C))$ is a soft βc-open set over X. Hence, $(g_{qv} \circ f_{pu})^{-1}(F, C) = f_{pu}^{-1}(g_{qv}^{-1}(F, C))$ is a soft βc-open set over X. Therefore, $g_{qv} \circ f_{pu}$ is a soft βc-continuous.

We can prove part (2) in similar way. □

Theorem 5.9. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ and $g_{qv} : SS(Y)_B \to SS(Z)_C$ be two soft functions. If f_{pu} is a soft βc-open and surjective and $g_{qv} \circ f_{pu}$ is a soft βc-continuous, then g_{qv} is a soft βc-continuous.*

Proof. Let (F, C) be a soft open set over Z. Since $g_{qv} \circ f_{pu}$ is a soft βc-continuous, $(g_{qv} \circ f_{pu})^{-1}(F, C) = f_{pu}^{-1}(g_{qv}^{-1}(F, C))$ is a soft βc-open set over X. Since f_{pu} is a soft βc-open and surjective, then $f_{pu}(f_{pu}^{-1}(g_{qv}^{-1}(F, C))) = g_{qv}^{-1}(F, C)$ is a soft βc-open set over Y. Hence, g_{qv} is a soft βc-continuous. □

Corollary 5.10. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft βc-open, a soft βc-irresolute and surjective function and let $g_{qv} : SS(Y)_B \to SS(Z)_C$ a soft functions. Then, $g_{qv} \circ f_{pu} : SS(X)_A \to SS(Z)_C$ is a soft βc-continuous if and only if g_{qv} is a soft βc-continuous.*

Theorem 5.11. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function such that $\widetilde{X}$ is a soft locally indiscrete space. Then, f_{pu} is a soft semi-continuous function if and only if f_{pu} is a soft βc-continuous function.*

Proof. For any soft open set (V, A) over Y, $f_{pu}^{-1}(V, A)$ is a soft semi-open set over X. Since $\widetilde{X}$ is a soft locally indiscrete space, by Theorem 3.18, $f_{pu}^{-1}(V, A)$ is a soft βc-open set over X. Therefore, f_{pu} is a soft βc-continuous function. Conversely, If f_{pu} is a soft βc-continuous function, then for any soft open set (V, A) over Y, $f_{pu}^{-1}(V, A)$ is a soft βc-open set over X which implies that, $f_{pu}^{-1}(V, A) = \tilde{\bigcup}_{x_e \tilde{\in} f_{pu}^{-1}(V,A)}(F_{x_e}, A)$ where (F_{x_e}, A) is a soft closed set over X for each $x_e \tilde{\in} f_{pu}^{-1}(V, A)$. Since $\widetilde{X}$ is a soft locally indiscrete space, $f_{pu}^{-1}(V, A)$ is a soft open set over X because it is a union of soft open sets over X. Hence, $f_{pu}^{-1}(V, A)$ is a soft semi-open set over X. Therefore f_{pu} is a soft semi-continuous function. □

Corollary 5.12. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function such that $\widetilde{X}$ is a soft locally indiscrete space. Then*

(1) f_{pu} is a soft continuous function if and only if f_{pu} is a soft βc-continuous function.

(2) f_{pu} is a soft α-continuous function if and only if f_{pu} is a soft βc-continuous function

References

[1] A. Kharal and B. Ahmad, Mappings on Fuzzy Soft Classes, *Advances in Fuzzy Systems*, **2009** (2009), 1-6. https://doi.org/10.1155/2009/407890

[2] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, *Annal of Fuzzy Mathematics and Informatics*, **7** (2014), no. 2, 181-196.

[3] B. P. Varol, A. Shostak and H. Aygun, A New approach to soft topology, *Hacettepe Journal of Mathematics and Statistics*, **41** (2012), no. 5, 731-741.

[4] C. G. Aras, A. Sonmez, H. Çakalli, On soft mappings, (2013), arXiv.org/abs/1305.4545.

[5] D. Molodtsov, Soft Set Theory-First Results, *Computer and Mathematics with Applications,* **37** (1999), 19-31.
https://doi.org/10.1016/s0898-1221(99)00056-5

[6] D. N. Georgiou, A. C. Megaritis and V. I. Petropoulos, On Soft Topological Spaces, *Applied Mathematics and Information Sciences*, **7** (2013), no. 5, 1889-1901. https://doi.org/10.12785/amis/070527

[7] I. Arockiarani and A. Selvi, On Soft slightly πg-continuous functions, *Journal of Progressive Research in Mathematics*, **3** (2015), no. 2, 168-174.

[8] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, **3** (2012), no. 2, 171-185.

[9] J. Mahanta, P. K. Das, On soft topological space via semiopen and semiclosed soft sets, *Kyungpook Mathematical Journal*, **54** (2014), no. 2, 221-236. https://doi.org/10.5666/kmj.2014.54.2.221

[10] M. Akdag and A. Ozkan, Soft α-open sets and Soft α-Continuous Functions, *Abstract and Applied Analysis*, (2014), 1-7.
https://doi.org/10.1155/2014/891341

[11] M. Akdag and A. Ozkan, Soft b-open sets and Soft b-continuous functions, *Mathematical Sciences*, **8** (2014), 124.
https://doi.org/10.1007/s40096-014-0124-7

[12] M. Irfan Ali, F. Feng, L. Xiaoyan and W. Min, On some new operations in soft set theory, *Computer and Mathematics with Applications*, **57** (2009), no. 9, 1547-1553. https://doi.org/10.1016/j.camwa.2008.11.009

[13] M. Shabir and M. Nas, On soft topological spaces, *Computer and Mathematics with Applications*, **61** (2011), no. 7, 1786-1799.
https://doi.org/10.1016/j.camwa.2011.02.006

[14] N. Çağman, S. Karataş and S. Enginoglu, Soft topology, *Computer and Mathematics with Applications*, **62** (2011), no. 1, 351-358. https://doi.org/10.1016/j.camwa.2011.05.016

[15] P. K. Maji, R. Biswas and A. R. Roy, Soft Set Theory, *Computer and Mathematics with Applications*, **45** (2003), no. 4-5, 555-562. https://doi.org/10.1016/s0898-1221(03)00016-6

[16] S. Das and S. K. Samanta, Soft metric, *Annals of Fuzzy Mathematics and Informatics*, **6** (2013), no. 1, 77-94.

[17] S. Hussain and B. Ahmad, Soft separation axioms in soft topological spaces, *Hacettepe Journal of Mathematics and Statistics*, **3** (2015), 559-568.

[18] S. Yuksel, N. Tozlu and Z. G. Ergul, Soft Regular Generalized Closed Sets in Soft Topological Spaces, *International Journal of Mathematical Analysis*, **8** (2014), no. 8, 355-367. https://doi.org/10.12988/ijma.2014.4125

[19] Y. Yumak, A. K. Kaymakci, Soft β-open sets and the applications, (2013), arXiv.org/abs/1312.6964.

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