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Encryption through Labeled Graphs Using Strong Face Bimagic Labeling

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Abstract

Modern cryptography is heavily based on mathematical theory and secure communication. It has been recognized that encryption and decryption mostly emerges from mathematical disciplines. In this paper we present a new combinatorial technique to encrypt and decrypt twin numbers through labeled graphs using strong face bimagic labeling.

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Keywords: graph labeling, bimagic, strong face graph, encryption and decryption

1 Introduction

The concept of graph labeling was introduced by Rosa 1967 in [6]. A labeling of a graph G is any mapping that sends a certain set of graph elements to a certain set of positive integers. If the domain is the vertex set, or the edge set respectively, the labeling is called a vertex labeling, or an edge labeling, respectively. If the domain is $V(G) \cup E(G)$ then the labeling is called total labeling.

In 2004 Babujee [2] introduced the notion of edge bimagic total labeling and also studied in [3] that a generalization of super edge magic total labeling in which there exists two distinct constants k_1 and k_2 such that the edge weights involved in this labeling are either equal to k_1 or k_2. An edge bimagic labeling is of interest for graphs that do not have any super edge magic total labeling. More precisely, a graph G with p vertices and q edges is said to be edge bimagic total if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, \ldots, p + q\}$ such that $f(u) + f(v) + f(uv) = k_1$ or k_2.

In 2015 Mohammed Ali and Babujee [1] introduced the concept of strong face graph and studied bimagic on strong face plane graphs.

In [1], it is proved that the strong face wheel graph W_n^* admits $(1, 1, 0)$ super bimagic labeling for every $n \geq 4$, with two magic constants $K_1 = 15n + 9$ and $K_2 = 15n + 8$.

Encryption by using labeling was first introduced by J. Baskar Babujee and S. Babitha by doing pair labeling for vertices and edges in [4].

In this paper we will use the technique of strong face graph to encrypt two numbers using the idea of digits number.

2 Main Results

Definition 2.1. *[1] Let G be a simple, connected, plane graph. A strong face graph G^* is obtained from G by adding a new vertex to every face of G except the external face and joining this vertex with all vertices surrounding that face, so that all faces of a new graph G^* are isomorphic to the cycle C_3.*

Moreover, if the faces of original graph G itself are C_3, then the number of faces increases twice.

Encryption and Decryption twin numbers using strong face wheel graph W_n^*.

Algorithm 2.2. (Encryption)

Input: *The two positive secret numbers S_1 and S_2, where S_1 have n digits and S_2 have m digits, $n \geq m$.*
Output: *Encrypted labeled strong face wheel graph W_n^*.*
begin
Step 1:

$V_1(W_n^*) = \{u, v_i : i = 1, 2, \ldots, n\}$, $V_2(W_n^*) = \{u_i : i = 1, 2, \ldots, n\}$,
$V(W_n^*) = V_1(W_n^*) \cup V_2(W_n^*)$,

$E_1(W_n^*) = \{uv_i : i = 1, 2, \ldots, n\} \cup \{v_1v_n, v_iv_{i+1} : i = 1, 2, \ldots, n-1\}$,
$E_2(W_n^*) = \{uu_i, u_iv_i : i = 1, 2, \ldots, n\}$
$\cup\{u_nv_1, u_iv_{i+1} : i = 1, 2, \ldots, n-1\}$,
$E(W_n^*) = E_1(W_n^*) \cup E_2(W_n^*)$,

Step 2: *Define a bijection* $f : V(W_n^*) \cup E(W_n^*) \to \{1, 2, \ldots, 7n+1\}$ *such that*
$f(u) = 1$,
$f(u_nv_1) = 4n + 1$,
$f(v_nv_1) = 6n + 2$,
for $i = 1, 2, \ldots, n$,
$f(v_i) = i + 1$,
$f(u_i) = n + 1 + i$,
$f(u_iv_i) = 4n + 2 - 2i$,
$f(uu_i) = 5n + 2 - i$,
$f(uv_i) = 5n + 1 + i$,
for $i = 1, 2, \ldots, n-1$,
$f(u_iv_{i+1}) = 4n + 1 - 2i$,
$f(v_iv_{i+1}) = 6n + 2 + i$,

Step 3: *Split the first secret number* S_1 *into* n *digits,* $S_1 = d_1d_2\ldots d_n$, *where* $d_1, d_2, \ldots, d_n$, *are the first, second,* $\ldots$, *last digits of* S_1 *respectively,*

Step 4: *Split the second secret number* S_2 *into* n *digits* $(n \geq m)$, *such that if* $m = n$, *then all the digits of* S_2, $e_i \geq 0$, *for* $1 \leq i \leq n$, *and if* $m < n$, *then the digits* $e_{m+1}, e_{m+2}, \ldots, e_n$, *are blank spaces,*

Step 5: *Define a function* $g : V(W_n^*) \to N$ *such that*
$g(v_i) = f(v_i) + (n+1) + d_i$ *for* $i = 1, 2, \ldots, n$,

$$g(u_i) = \begin{cases} f(u_i) + 2n + e_i & \text{for } i = 1, 2, \ldots, n \text{ if the digit } e_i \geq 0, \\ f(u_i) & \text{for } i = 1, 2, \ldots, n \text{ if the digits are} \\ & \text{blank spaces.} \end{cases}$$

end.

Algorithm 2.3. (Decryption)

Input: *Total labeled strong face wheel graph* W_n^* *with twin secret numbers as a vertices labeling.*

Output: *The two secret numbers* S_1 *and* S_2.

begin

Step 1: *Create vertex labeled matrix* $A_{n\times 2}$, *where*

$$a_{ij} = \begin{cases} g(v_i) & \text{for } j = 1, i = 1, 2, \ldots, n, \\ g(u_i) & \text{for } j = 2, i = 1, 2, \ldots, n \end{cases}$$

Step 2: *Construct a matrix* $B_{n\times 2}$, *where*

$$b_{ij} = \begin{cases} n + 1 & \text{for } j = 1, i = 1, 2, \ldots, n, \\ 3n - 1 & \text{for } j = 2, i = 1, 2, \ldots, n \end{cases}$$

Step 3: *Construct a matrix* $C_{n\times 2}$*, where* $c_{ij} = i + j$ *for* $j = 1, 2,$ $i = 1, 2, \dots, n,$

Step 4: *Calculate the matrix* $H_{n\times 2} = A_{n\times 2} - M_{n\times 2}$*, where* $m_{ij} = b_{ij} + c_{ij}$ *for* $j = 1, 2,\ i = 1, 2, \dots, n$

Step 5: *Calculate the two secret numbers,* $S_1 = d_1 d_2 d_n = h_{11} h_{21} \dots h_{n1}$ *and* $S_2 = e_1 e_2 \dots e_n = h_{12} h_{22} \dots h_{n2}$*, respectively and ignore all negative values.*

end.

Illustration for Encryption and Decryption Algorithm

Let $S_1 = 274011$ and $S_2 = 3050$, be two secret numbers.
Since the digits of $S_1 = n = 6$, digits of $S_2 = m = 4$, take a strong face wheel graph W_6^*.

- As per step 1 of algorithm 2.2, the vertex set and edge set of W_6^* is defined as

  $V(W_6^*) = V_1(W_6^*) \cup V_2(W_6^*)$, where $V_1(W_6^*) = \{u, v_i : i = 1, 2, \dots, 6\}$,

  $V_2(W_6^*) = \{u_i : i = 1, 2, \dots, 6\}$ and $E(W_6^*) = E_1(W_6^*) \cup E_2(W_6^*)$, where

  $E_1(W_6^*) = \{uv_i : i = 1, 2, \dots, 6\} \cup \{v_1 v_6, v_i v_{i+1} : i = 1, 2, \dots, 5\}$,

  $E_2(W_6^*) = \{uu_i, u_i v_i : i = 1, 2, \dots, 6\} \cup \{u_6 v_1, u_i v_{i+1} : i = 1, 2, \dots, 5\}$.

- As per step 2 of algorithm 2.2, define a bijection $f : V(W_6^*) \cup E(W_6^*) \rightarrow \{1, 2, \dots, 42 + 1\}$ such that

  $f(u) = 1$, where u is a center vertex of W_6^*,

  $f(u_n v_1) = 4n + 1 \Rightarrow f(u_6 v_1) = 25$,

  $f(v_n v_1) = 6n + 2 \Rightarrow f(v_6 v_1) = 38$,

  for $i = 1, 2, \dots, 6$,

  $f(v_i) = i + 1$,

  $f(u_i) = (n + 1) + i = 7 + i$,

  $f(u_i v_i) = 4n + 2 - 2i = 26 - 2i$,

  $f(uu_i) = 5n + 2 - i = 32 - i$,

  $f(uv_i) = 5n + 1 + i = 31 + i$,

  for $i = 1, 2, \dots, 5$,

  $f(u_i v_{i+1}) = 4n + 1 - 2i = 25 - 2i$,

  $f(v_i v_{i+1}) = 6n + 2 + i = 38 + i$.

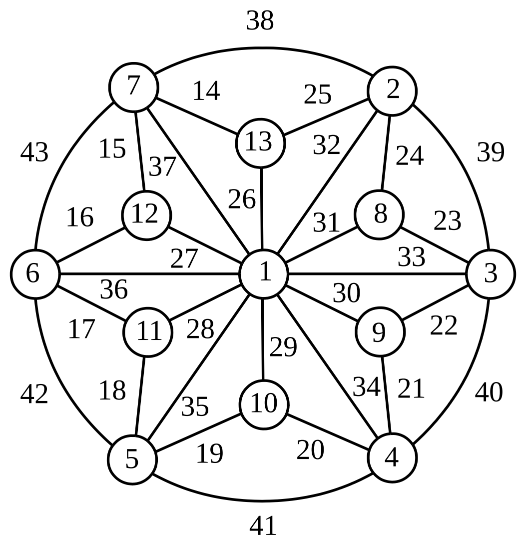

Figure 1:

It is easy to observe that the strong face wheel graph W_6^* given in figure 1 admits face bimagic labeling with two different magic constants $K_1 = 15n+9 = 99$, $K_2 = 15n + 8 = 98$.

Encryption: Now to encrypt twin secret numbers S_1, S_2, as per step 3 and 4 of algorithm 2.2, split these numbers into n digits for each one, S_1 have six digits, so that let d_i for $i = 1, 2, \dots, 6$, is the digits of S_1, and S_2 have four digits, so that let e_i for $i = 1, 2, \dots, 6$, are the digits of S_2, where the last two digits of S_2 are blank spaces. Hence

$d_1 = 2$, $d_2 = 7$, $d_3 = 4$, $d_4 = 0$, $d_5 = 1$ and $d_6 = 1$, while $e_1 = 3$, $e_2 = 0$, $e_3 = 5$, $e_4 = 0$, e_5 and e_6 are blank spaces.

- As per step 5 of algorithm 2.2, define a function $g : V(W_6^*) \to N$ such that

  $$g(v_i) = f(v_i) + (n + 1) + d_i \text{ for } i = 1, 2, \dots, 6,$$

  $$g(u_i) = \begin{cases} f(u_i) + 2n + e_i & \text{for } i = 1, 2, \dots, 4, \\ f(u_i) & \text{for } i = 5, 6, \end{cases}$$

Hence the new graph given in figure 2 is encrypted labeled strong face wheel graph W_6^*, with twin secret numbers $S_1 = 274011$ and $S_2 = 3050$,

Decryption: Consider the encrypted labeled strong face wheel graph W_6^*, given in figure 2.

- As per step 1 of algorithm 2.3, Find the matrix $A_{n\times 2}$, where

  $$a_{ij} = \begin{cases} g(v_i) & \text{for } j = 1, i = 1, 2, \dots, n, \\ g(u_i) & \text{for } j = 2, i = 1, 2, \dots, n, \end{cases}$$

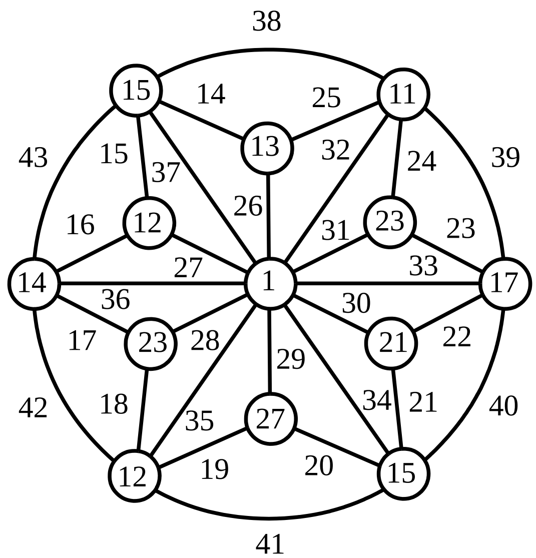

Figure 2:

$$A_{n\times 2} = \begin{bmatrix} g(v_1) & g(u_1) \\ g(v_2) & g(u_2) \\ \vdots & \vdots \\ g(v_n) & g(u_n) \end{bmatrix} \Rightarrow A_{6\times 2} = \begin{bmatrix} 11 & 23 \\ 17 & 21 \\ 15 & 27 \\ 12 & 23 \\ 14 & 12 \\ 15 & 13 \end{bmatrix}$$

- As per step 2 and 3 of algorithm 2.3, construct a matrix $B_{n\times 2}$ and $C_{n\times 2}$, where

  $$b_{ij} = \begin{cases} n+1 & \text{for } j=1, i=1,2,\ldots,n, \\ 3n-1 & \text{for } j=2, i=1,2,\ldots,n, \end{cases}$$

  And $c_{ij} = i+j$ for $j=1,2,\ i=1,2,\ldots,n$, respectively, thus

$$B_{n\times 2} = \begin{bmatrix} n+1 & 3n-1 \\ n+1 & 3n-1 \\ \vdots & \vdots \\ n+1 & 3n-1 \end{bmatrix} \Rightarrow B_{6\times 2} = \begin{bmatrix} 7 & 17 \\ 7 & 17 \\ 7 & 17 \\ 7 & 17 \\ 7 & 17 \\ 7 & 17 \end{bmatrix}$$

$$C_{n\times 2} = \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 2+2 \\ \vdots & \vdots \\ n+1 & n+2 \end{bmatrix} \Rightarrow C_{6\times 2} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 6 \\ 6 & 7 \\ 7 & 8 \end{bmatrix}$$

- As per step 4 of algorithm 2.3, calculate $H_{n\times 2} = A_{n\times 2} - M_{n\times 2}$, where $m_{ij} = b_{ij} + c_{ij}$

  for $j = 1, 2$, $i = 1, 2, \ldots, n$. Thus

$$H_{n\times 2} = \begin{bmatrix} g(v_1) & g(u_1) \\ g(v_2) & g(u_2) \\ \vdots & \vdots \\ g(v_n) & g(u_n) \end{bmatrix} - \begin{bmatrix} (n+1)+(2) & (3n-1)+(3) \\ (n+1)+(3) & (3n-1)+(4) \\ \vdots & \vdots \\ (n+1)+(n+1) & (3n-1)+(n+2) \end{bmatrix} \Rightarrow$$

$$H_{6\times 2} = \begin{bmatrix} 11 & 23 \\ 17 & 21 \\ 15 & 27 \\ 12 & 23 \\ 14 & 12 \\ 15 & 13 \end{bmatrix} - \begin{bmatrix} 9 & 20 \\ 10 & 21 \\ 11 & 22 \\ 12 & 23 \\ 13 & 24 \\ 14 & 25 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 7 & 0 \\ 4 & 5 \\ 0 & 0 \\ 1 & -12 \\ 1 & -13 \end{bmatrix}$$

- As per step 5 of algorithm 2.3, $S_1 = d_1 d_2 \ldots d_6 = h_{11} h_{21} \ldots h_{61} = 274011$ and $S_2 = e_1 e_2 \ldots e_4 = h_{12} h_{22} h_{32} h_{42} = 3050$.

3 Conclusion

We have exhibited encryption of twin numbers using strong face wheel graphs. This will be useful to communicate secretly the twin passwords or pin numbers to a single graph. Note that by changing the definition of the function in step 5 of algorithm 2.2, and step 2, 3 and 4 in algorithm 2.3, one can easily create different ways of encryption.

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